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# An almost Einsteinian theory of gravitation

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**Abstract.** A new version of relativistic theory of gravitation is proposed, in which the GR is supplemented with two postulates. Owing to these postulates it turns possible to introduce into consideration a covariant energy-momentum tensor of gravitational field. Additionally the influence of Universe on the gravitating system under consideration is taken into account. The equations describing the gravitation field in the framework of proposed tensor mono-metric theory of gravitation are derived. In its simplest version the proposed theory contains nine free parameters. At a particular choice of these parameters the theory is reduced to GR with a cosmological term.

## 1. Introduction

Yet after almost 100 years since GR creation: (a) some important conclusions of GR (e.g., the black holes, gravitational waves) have not been directly confirmed by observational data, (b) there still exist some unresolved problems in GR (e.g., the lack in the theory of the covariant energy-momentum tensor of the gravitational field), and some important phenomena have been discovered (e.g., accelerated expansion of Universe and the associated problem of "dark energy"), that had not found any consistent and non-contradictory explanation in the frameworks of GR. It is the desire to bypass the above difficulties that justifies the efforts for development of more accurate, improved or, possibly, the modified GR.

There is always several directions of search. We indicate only a few variants of relativistic theories of gravitation discussed in the science literature:

(a) Scalar-tensor theories of gravitation, in which the gravitation «constant» $G(x^{\mu})$ is a function of space-time coordinates $x^{\mu}$ (see, e.g., [1,2]),

(b) Bimetric theories of gravitation, in which along with the principal metric $g_{ik}$ there is another "background" metric $\gamma_{ik}$. In these theories the operation of covariant differentiation with respect to the background metric $\gamma_{ik}$ plays a crucial role (see, e.g., [3-9]).

(c) The theories in which: (i) the contributions of the summands that are nonlinear in Riemann tensor $R_{iklm}$, its convolutions and covariant derivatives are taken into account, (ii) the space-time is multi-dimensional and other theories of gravitation [10].

In this paper a modified version of GR is proposed, in which *the GR is supplemented with two postulates*.

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## 2. The Tensor Mono-metric Theory of Gravitation (TMTG)

*2.1. The principal provisions of the proposed theory*

As in GR, in the proposed Tensor Mono-metric Theory of Gravitation (TMTG) (a) *the space-time is described by one metric tensor* $g^{ik}$. Then as in GR, $g^{ik}$ describes the gravitational field as well as also the effects connected with the choice of non-inertial reference system. Thus, as in GR (b) only a part of $g^{ik}$ describes the gravitational field. If *one denotes this part of metric tensor* as $\varphi^{ik}$, then

$$g^{ik} = g_*^{ik} + \varphi^{ik} . \tag{1}$$

The first summand in (1) *is not related to the gravitational field* and is obtained from $g^{ik}$, when in the space-time there is no gravitating system in question: $\varphi^{ik} = 0$ ($g_*^{ik}$ is defined, e.g., by initial conditions of the problem under consideration and the choice of the reference frame). And, at last, as in GR, (c) the influence of gravitation on the matter and on non-gravitational fields is described by metrical tensor $g^{ik}$.

In TMTG it is additionally assumed in comparison with GR that (A) $\varphi^{ik}$ is a covariant symmetrical second rank tensor. So, $g_*^{ik}$ is also a covariant tensor that does not change at variation of gravitational field action with respect to $\varphi^{ik}$. In TMTG it is also assumed that (B) *the effect of gravitational field* on the matter, non-gravitational fields and *on itself is produced in the same way* (via metrical tensor $g^{ik}$). For this reason the covariant density of gravitational field Lagrangian $L_g$ must depend not only on $\varphi^{ik}$ and covariant derivatives of this tensor, but also on $g^{ik}$:

$$L_g = L_g(g^{\alpha\beta}, \varphi^{\mu\nu}, \varphi^{\mu\nu}{}_{;\lambda}) . \tag{2}$$

It follows from the aforesaid that the total action of gravitating system is

$$S = S_g + S_M^* = c^{-1} \int \left( L_g + L_M^* \right) \sqrt{-g} d^4 x , \tag{3}$$

where $L_M^*$ is the covariant density of the Lagrangian of the matter and non-gravitation fields, $c$ is the velocity of light.

*2.2. Equations of the gravitational field*

First let us find the variation of the first term in (3):

$$c\delta S_g = \int \delta(L_g \sqrt{-g}) d^4 x = 0.5 \int (-G_{(ik)} \delta\varphi^{ik} + T_{(ik)}^{g_*} \delta g^{ik}) \sqrt{-g} d^4 x , \tag{4}$$

where $G_{(ik)}$ tensor is obtained by means of variation of $S_g$ only with respect to $\varphi^{ik}$, and $T_{(ik)}^{g_*}$ tensor – by variation of $S_g$ only with respect to $g^{ik}$, the brackets in $(ik)$ implying the operation of symmetrization in *i* and *k* indices. In particular,

$$c\delta S_g = 0.5 \int T_{(ik)}^{g_*} \delta g^{ik} \sqrt{-g} d^4 x \tag{5}$$

if $\varphi^{ik}$ is not varied, and the metrical tensor $g^{ik}$ is varied independently of $\varphi^{ik}$.

The equations of gravitational field are obtained based on the principle of least action:

$$\delta S = \delta S_g + \delta S_M^* = 0 . \tag{6}$$

Subjected to variation is the gravitational field, i.e., the quantities $\varphi^{ik}$. At the substitution of (4) to (6) it follows in view of the equality $\delta g^{ik} = \delta\varphi^{ik}$ (see (1)) that

$$G_{(ik)} = T_{(ik)}^{M_*} + T_{(ik)}^{g_*} . \tag{7}$$

These are the equations of gravitational field, - the main equations of TMTG. Here $T^{M*}_{ik}$ is the covariant energy-momentum tensor of the matter and of non-gravitational fields determined by the equality

$$c\delta S^{*}_{M} = 0.5\int T^{M*}_{(ik)}\delta g^{ik}\sqrt{-g}d^4x \tag{8}$$

(see [11]).

Comparing (5) and (8) one ascertains that the *covariant symmetrical tensor* $T^{g*}_{(ik)}$ is determined just as the energy-momentum tensor of the matter and of non-gravitating fields is determined in GR. In this sense $T^{g*}_{(ik)}$ is a *covariant tensor of the energy-momentum of the gravitational field.*

*2.3 Covariant conservation laws*

The action $S_g$ is a scalar and hence its variation (4) must be zero:

$$\delta S_g = 0, \tag{9}$$

in case of transformation of coordinates

$$x'^i = x^i + \zeta^i(x^\mu), \tag{10}$$

where $\zeta^i$ are small values. Here

$$\delta g^{ik}(x^\mu) = \zeta^{i;k}(x^\mu) + \zeta^{k;i}(x^\mu)$$
$$\delta\varphi^{ik}(x^\mu) = \varphi^{in}\partial\zeta^k/\partial x^n + \varphi^{nk}\partial\zeta^i/\partial x^n - \zeta^n\partial\varphi^{ik}/\partial x^n \tag{11}$$

in the linear approximation in $\zeta^i$. At the substitution of these expressions to (4) it follows from (9) in view of the arbitrariness of $\zeta^i$ that

$$T^{g*}_{(im);n}g^{mn}\sqrt{-g} = \partial G_{(im)}\phi^{mn}\sqrt{-g}/\partial x^n + 0.5\sqrt{-g}G_{(mn)}\partial\phi^{mn}/\partial x^i. \tag{12}$$

In GR based on condition

$$\delta S^{*}_{M} = 0 \tag{13}$$

and transformation of coordinates (10) the covariant conservation law

$$T^{M*}_{(im);n}g^{mn} = 0 \tag{14}$$

is analogously deduced for matter and non-gravitating fields [11].

The conservation law (12) is not reduced to equality $T^{g*}_{(im);n}g^{mn} = 0$ and in this sense it differs from an analogous law (14). The difference is due to the fact that the gravitational field is the *only field that interacts with matter and other* (*non-gravitational*) *fields by means of metrical tensor* $g^{ik}$. It should be added here that an equation

$$(T^{g*}_{(im)} - G_{(im)})_{;n}g^{mn} = 0 \tag{15}$$

follows from (7) and (14). Using this equation one can rewrite the differential conservation law (12) in an explicitly covariant form.

Really, let us consider the space-time when there is no gravitating system under consideration. In this space-time (the "initial" space-time) the square of linear element

$$ds_0^2 = g_{(0)ik}dx^i dx^k. \tag{16}$$

Here we shall identify the contravariant components $g^{ik}_{(0)}$ of the metrical tensor $g_{(0)ik}$ with $g^{ik}_{*}$:

$$g^{ik}_{(0)} = g^{ik}_{*}, \tag{17}$$

and denote the operation of covariant differentiation in such a space-time by the vertical bar (as opposed to the symbol ";", designating the operation of covariant differentiation in the space-time

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with metrical tensor $g_{ik}$). Using the introduced operation of covariant differentiation in the "initial" space-time, as well as (15), we can write the differential conservation law (12) in the following clearly covariant form:

$$[\eta(T^{M*}_{(im)} + T^{g*}_{(im)})]_{|n} g^{mn}_{*} = 0\,, \tag{18}$$

where $\eta = \sqrt{g / g_{(0)}}$ is a scalar quantity, and $g_{(0)}$ is the determinant of metrical tensor $g_{(0)ik}$. Thus, the differential conservation law that follows from the invariance of action $S_g$ with respect to the transformation of coordinates $x^{\mu}$, may be represented in the form of covariant conservation law for the total energy-momentum tensor of the matter and all fields (including the gravitational field).

## 3. Influence of the Universe

The total density of the Lagrangian of matter and non-gravitational fields is

$$L^{*}_{M} = L_M(g^{\alpha\beta}, q, \partial q / x^{\lambda}) + L_{UN}(g^{\alpha\beta}, p, \partial p / \partial x^{\lambda})\,, \tag{19}$$

where the density of Lagrangian $L_M$ describes the contribution of gravitating system under consideration, and the density of Lagrangian $L_{UN}$ describes the contribution of the "remaining part of Universe". $L_{UN}$ is a certain function of quantities $p$ describing the state of this "remaining part of Universe", and of time and space derivatives of $p$. As the metrical tensor $g^{\alpha\beta}$ enters into the equations of motion

$$\left(\frac{\partial}{\partial x^{\lambda}} \frac{\partial}{\partial p_{,\lambda}} - \frac{\partial}{\partial p}\right) L_{UN} \sqrt{-g} = 0 \tag{20}$$

the solutions of these equations must depend not only on space-time coordinates of $x^{\lambda}$, but also on $g^{\alpha\beta}$:

$$p = p(g^{\alpha\beta}, x^{\lambda})\,. \tag{21}$$

After substitution of these expressions to $L_{UN}$ one finds

$$L^{*}_{M} = L_M(g^{\alpha\beta}, q, \partial q / x^{\lambda}) + L_{UN}[g^{\alpha\beta}, x^{\lambda}]\,. \tag{22}$$

According to this equality the covariant energy-momentum tensor of the matter and of non-gravitational fields splits into two summands:

$$T^{M*}_{(ik)} = T^{M}_{(ik)} + W_{(ik)}\,, \tag{23}$$

where

$$\delta \int L_{UN}[g^{\alpha\beta}, x^{\lambda}] \sqrt{-g}\, d\Omega = 0.5 \int W_{(ik)} \delta g^{ik} \sqrt{-g}\, d\Omega\,. \tag{24}$$

Here $T^{M}_{(ik)}$ is the energy-momentum tensor of matter and non-gravitational fields for the gravitating system under consideration, and $W_{(ik)}$ is the tensor describing the contribution of the "remaining part of Universe". As a result one may rewrite the field equations (7) of TMTG in the following form:

$$G_{(ik)} = T^{M}_{(ik)} + T^{g*}_{(ik)} + W_{(ik)}\,. \tag{25}$$

## 4. The simplest version of the theory

Now consider the "simplest" version of TMTG, in which the density of Lagrangian of the gravitational field (2) is square-law dependent on $\varphi^{\mu\nu}$ and $\varphi^{\mu\nu}{}_{;\lambda}$:

$$L_g = (a_1 \varphi^{im;r} \varphi^{kn;s} + a_2 \varphi^{im;r} \varphi^{ks;n}) g_{ik} g_{mn} g_{rs} + a_3 \varphi_{,n} \varphi^{,n} + a_4 \varphi_{,n} \varphi^{nr}{}_{;r} +$$

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$$+a_5\varphi^{mr}{}_{;r}\varphi^{ns}{}_{;s}g_{mn}+a_6\varphi^2+a_7\varphi^{im}\varphi^{kn}g_{ik}g_{mn}\,, \qquad (26)$$

where $\varphi = g_{\alpha\beta}\varphi^{\alpha\beta}$, and the indices are raised and descended with the help of metrical tensor $g^{ik}$ and of its covariant components $g_{ik}$. In (26) there are seven free parameters $a_i$.

Generally speaking, the density of Lagrangian $L_{UN}[g^{\alpha\beta}, x^{\lambda}]$ is determined by the tensor $R_{tklm}$ of space-time curvature, by its convolutions and covariant derivatives. In the simplest case

$$L_{UN} = b_1 + b_2 R\,, \qquad (27)$$

where $b_1$ and $b_2$ are the free parameters of the theory. Here it follows from (24) that

$$W_{(ik)} = -b_1 g_{ik} + 2b_2(R_{ik} - 0.5 g_{ik} R)\,. \qquad (28)$$

Consequently the equations of gravitational field (25) assume the form

$$G_{(ik)} = T^M_{(ik)} + T^{g_*}_{(ik)} - b_1 g_{ik} + 2b_2(R_{ik} - 0.5 g_{ik} R)\,. \qquad (29)$$

The final explicit expressions for $G_{(ik)}$ and $T^{g_*}_{(ik)}$ are not given here for brevity.

*4.1. Partial choice of the values of free parameters of TMTG*

In case of a partial choice of the values of free parameters of theory:

$$a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0\,, \qquad b_1 = \frac{c^4}{8\pi G}\Lambda\,, \qquad b_2 = -\frac{c^4}{16\pi G} \qquad (30)$$

(29) is reduced to the Einstein equations with the cosmological term:

$$R_{ik} - 0.5 g_{ik} R + g_{ik}\Lambda = 8\pi G T^M_{(ik)} / c^4 \qquad (31)$$

($G$ is the gravitation constant and $\Lambda$ is the cosmological constant).

*4.2. Comparison with observation data*

In case of (30) the problem of matching TMTG with observation data is reduced to the problem of matching GR with these data. When $a_i$, $b_1$ and $b_2$ insignificantly differ from (30) one can consider TMTG as an “almost Einsteinian theory of gravitation”. The problem consists only in the following: “To what extent these insignificant differences from (30) are essential for astrophysical applications of TMTG?” The question is not idle since in (26) $a_6$ and $a_7$ determine the energy of gravitational field that is constant in time and in space. The difference of $a_6$ and $a_7$ from zero may be crucial for models of Universe within the frameworks of TMTG.

In future it is supposed to determine the PPN-parameters of TMTG to afford the determination of the limits of permissible values of $a_i$, $b_1$ and $b_2$.

*4.3. Comparison with bimetric theories of gravitation*

The secondary (“background”) metric $\gamma_{ik}$ of the space-time mentioned in Introduction, plays an important part in bimetric theories of gravitation. For instance, the determination of the action of gravitational field is based in these theories on the operation of covariant differentiation with respect to the background metric $\gamma_{ik}$. In (18) we had also introduced the second (“initial”) metrical tensor $g^{ik}_{(0)}$. However, in contrast with the bimetric theories, the operation of covariant differentiation with respect to an “initial” metrical tensor $g^{ik}_{(0)}$ in the action of gravitational field is not generally employed (see (2),(4)).

## 5. Conclusions

In the present work a modified version of GR, named the Tensor Mono-metric Theory of Gravitation (TMTG), is proposed, in which the GR is supplemented with two postulates. It is first assumed that the part of metrical tensor describing the gravitational field, is a *second rank covariant tensor.* Second, it is assumed that the gravitational field acts on the matter, on non-gravitational fields and on itself *in a similar way* (through the metrical tensor). This permitted an introduction of a second rank *covariant* symmetrical *energy-momentum tensor for the gravitational field* that is defined in exactly the same way as the energy-momentum tensor of the matter and non-gravitational fields is defined in GR. And, at last, in TMTG an allowance is made for the influence of the "remaining parts of Universe" on the gravitating system under consideration.

Further, the equations describing the gravitational field in the framework of TMTG were derived and then the differential law of conservation, stemming from an invariance of gravitational field action with respect to transformation of four-dimensional coordinates of space-time, have been obtained. This conservation law is represented in the covariant form as the conservation law for the *total energy-momentum tensor of the matter and of all fields including the gravitational field.*

In the "simplest" version of TMTG it contains nine free parameters. At a particular choice of these parameters the theory is reduced to GR with a cosmological term. Two of nine parameters specify the energy of constant gravitational field (constant in time and space), and, hence, may be crucial for models of Universe in TMTG.

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