Gravitational collapse and the vacuum energy

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Abstract. To explain the accelerated expansion of the universe, models with interacting dark components (dark energy and dark matter) have been considered recently in the literature. Generally, the dark energy component is physically interpreted as the vacuum energy of the all fields that fill the universe. As the other side of the same coin, the influence of the vacuum energy on the gravitational collapse is of great interest. We study such collapse adopting different parameterizations for the evolution of the vacuum energy. We discuss the homogeneous collapsing star fluid, that interacts with a vacuum energy component, using the stiff matter case as example. We conclude this work with a discussion of the Cahill-McVittie mass for the collapsed object.

1. Introduction
In recent years there have been a number of important discoveries relevant to the fields of cosmology and relativistic astrophysics. A couple months ago the Nobel prize for 2013 was awarded for the prediction and discovery of the Higgs boson. The Higgs boson has a special place in the inflationary model of the early universe. The research of Vera Rubin, which predicted the presence of dark matter in the galaxies through the study of the orbital velocities of the interstellar matter in the galaxies, has been substantiated through the discovery of the accelerated expansion of the universe. Indeed, the data from distant supernovae that took place at two different times provide the evidence for the accelerated expansion of the universe.

Within the framework of general relativity, we must provide, quite generally, a negative pressure to the cosmic fluid in order to explain the increase of the expansion velocity of the universe. Among the several models that can explain the accelerated expansion of the universe the most popular one is the Λ-CDM (cosmological constant plus cold dark matter) model. Alternative models consider a time-dependent cosmological term, in order to avoid the discrepancy between the values of acceleration measured by the cosmological observations and those estimated from the quantum field theory. Independent of the details of these models, the cosmological term in the framework of the general relativity theory is generally interpreted as the vacuum energy of the all fields that fill the universe.

We view the cosmic expansion and the gravitational collapse as different sides of the same coin. From this standpoint, a study of the influence of the vacuum energy on the gravitational collapse of a fluid star emerges naturally. Thus, in this contribution, we study the final fate of a collapsing star in a model which contains vacuum energy and a matter fluid. We will not specify the matter components individually. Apart from this, we will find the collapsed mass using the Cahill-McVittie definition.

2. Basic equations of the model

In this study we assume that the space-time is homogeneous and isotropic and is governed by the metric

$$ds^2 = dt^2 - R^2 \left(dr^2 + r^2 d\Omega^2 \right), \tag{1}$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$, and the function R depends only of the time. In this aspect we follow the pioneering collapse model described by Oppenheimer [1].

The exterior star space-time does not have a direct influence on the collapse process. Hence, the interior space-time of the star is the important geometric object of our study, which is governed by the metric given by Eq. (1).

The appearance of the apparent horizon is the main characteristics that decides on the final outcome of the collapse process which could be either a black hole or a naked singularity. If the apparent horizon is formed before the singularity, the final fate of the collapse process is a black hole. In the opposite case we have the formation of a naked singularity.

Let us begin this work by considering the Einstein's field equations. For a fluid star with pressure and density and for the space-time described by the metric above, they can be written as:

$$G^{\mu\nu} = \frac{8\pi G}{c^4} \left[T^{\mu\nu} + \frac{\Lambda}{8\pi G} g^{\mu\nu} \right], \tag{2}$$

where $G^{\mu\nu}$ is the Einstein tensor, $T^{\mu\nu}$ is the energy-momentum tensor, G is the newtonian gravitational constant, and c is the light velocity. On the other hand, according to the Bianchi identities the covariant derivative of the Einstein tensor is identically null ($G^{\alpha\beta}_{;\beta} = 0$). Consequently, the Einstein field equations can be written in the form of a conservation equation

$$u_\mu T^{\mu\nu}_{;\nu} = -u_\mu \left(\frac{\Lambda g^{\mu\nu}}{8\pi G} \right)_{;\nu}, \tag{3}$$

where the coupling of the stellar fluid to the vacuum component assumes the form

$$\dot{\rho}_f + 3\frac{\dot{R}}{R} \left(\rho_f + P_f \right) = -\dot{\rho}_v, \tag{4}$$

where u_μ, ρ_f, ρ_v and P_f are the four-velocity of an element of the star fluid, the energy density of the material component, the vacuum energy density (ρ_Λ), and the fluid's material pressure, respectively. For the energy momentum tensor of the fluid we adopt the popular perfect fluid from

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu - Pg^{\mu\nu}, \tag{5}$$

and we assume the usual form of the equation of state for the vacuum component $P_\Lambda = -\rho_\Lambda$. This form was introduced by Y. Zeldovich [2] in the discussion of the properties of a Higgs field. An interesting discussion of the inclusion of the Higgs field into the theory of elementary particles and the vacuum energy can be found in Ref. [3].

At this point we can write Einstein's field equations in the following form

$$\frac{8\pi G}{c^4} \rho_f + \Lambda(t) = 3H^2, \tag{6}$$

$$\frac{8\pi G}{c^4} p_f - \Lambda(t) = -2\dot{H} - 3H^2, \tag{7}$$

where H is the Hubble function ($H = \dot{R}/R$).

To integrate Einstein's field equations we need to establish an expression for the vacuum component. This is in full analogy to the $\Lambda(t)$-CDM cosmological models, which require a law for the decay of the vacuum.

## 3. Gravitational collapse and the vacuum energy

There are a large number of models in the literature, which propose different $\Lambda(t)$ decay laws, see for example Refs. [4], [5], and citations therein. In this work, we begin our study by using the law proposed by Wang and Meng [6]. The decay law for the vacuum energy that interacts with the matter fluid, given by Eq. (4), is model independent. The authors assume that the energy density of the matter component will dilute at a rate that is different from the usual one $\rho_f = \rho_{f0}R^{-3}$, namely $\rho_f = \rho_{f0}R^{-3+\epsilon}$. The parameter ϵ represents the deviation from the standard case, where the interaction between the matter fluid and the vacuum energy does not exist. In some sense, this assumption is independent of the adopted model, but in several cases it can mimic a specific model. For example, substituting for the parameter $\epsilon = 3\beta$, we recover the homogeneous and isotropic collapse of dust, where the decaying vacuum energy is proportional to the square of Hubble function. This model was studied in Ref. [7]. The relation between the approach adopted by Wang and Meng and the model with Λ-term which depends on the square of the Hubble function can be see by integrating the energy conservation equation, given by Eq. (4), assuming that the matter fluid and the vacuum energy components have identical functional dependences.

In this brief study, we will exemplify the collapse process, using the equation of state $P_f = \rho_f$, which corresponds to the "stiff" matter. Naturally, this equation of state is more realistic than that of the dust fluid form the point of view of the collapse process. Hence, redefining the parameter $\epsilon = 6\beta - 3$, we obtain the collapse process for the stiff matter case, with the decay of the vacuum energy proportional to the square of the Hubble function, namely $\Lambda = 3\beta H^2$.

With this dependence for the vacuum energy density, and using the Einstein field equations, a direct integration gives

$$R(t) = R_i[3(1-\beta)H_i(t_c - t)]^{\frac{1}{3(1-\beta)}} \, , \tag{8}$$

where t_c is the collapse time and we used the initial conditions $R(t=0) = R_i$, $H(t=0) = -H_i$. In the Fig. 1 we display the influence of the vacuum component on the scale factor. We can
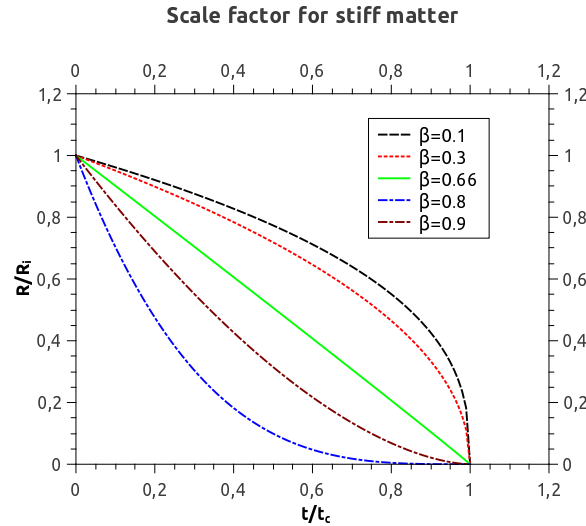
Figure 1. The evolution of the scale factor for stiff matter. We consider different values for the β-parameter. Note the interesting change of concavity that occurs at $\beta = 2/3$. The concavity is linked to the second derivative of the scale factor and it correspond to the acceleration of the fall of comoving fluid element.

write an expression for the collapse time, using the scale factor above and recalling that we are

dealing with stiff matter case, in the following form

$$t_c = \frac{H_i^{-1}}{3(1-\beta)}\,.$$

(9)

Next we ask what is in the final stage of the collapse process - a black hole or a naked singularity? To answer this question we look for the apparent horizon formation moment, that are space-like surfaces with future pointing towards the converging null geodesics on both sides of the surface [8]. If the apparent horizon dresses the singularity we have the formation of a black hole. If the apparent horizon is formed after the singularity is reached, we have the appearance of a naked singularity (see Fig. 2).

The condition for the appearance of the apparent horizon is given by [8]

$$K_{,\alpha}K_{,\beta}g^{\alpha\beta} = \left(r\dot{R}\right)^2 - 1 = 0\,,$$

(10)

where $()_{,x} = \frac{\partial}{\partial x}$ and $K(t,r) = rR(t)$.

Since initially the fluid star is not trapped the condition

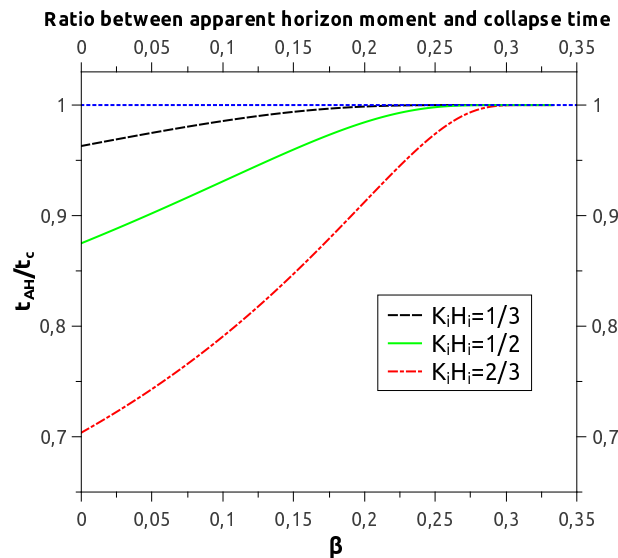$$K_{,\alpha}K_{,\beta}g^{\alpha\beta} = \left[r\dot{R}\left(t_i\right)\right]^2 - 1 < 0\,,$$

(11)

must be obeyed, which implies that $0 < K_i H_i < 1$.

Finally, to calculate the collapsed mass we use the Cahill-McVittie definition [9], which is given by

$$m(t,r) = \frac{1}{2}K\left(1 + K_{,\alpha}K_{,\beta}g^{\alpha\beta}\right) = \frac{1}{2}K\dot{K}^2\,.$$

(12)

Summarizing, the apparent horizon condition, which is given by Eq. (10), and the mass function

Figure 2. Ratio between the apparent horizon appearance moment and the collapse time for stiff matter. In the considered interval of the β-parameter the cosmic censorship conjecture is valid and the singularity is dressed. Hence, the final fate of the collapse process is a black hole.

for the stiff matter, for a homogeneous and isotropic collapse, are given by:

$$\dot{K} = K_i H_i \left[1 - 3(1-\beta)H_i t_{AH}\right]^{\frac{3\beta-2}{3(1-\beta)}} = 1, \tag{13}$$

$$m(r,t) = \frac{1}{2}K_i^3 H_i^2 \left\{1 - 3(1-\beta)H_i t\right\}^{\frac{2\beta-1}{1-\beta}}, \tag{14}$$

where t_{AH} is the apparent horizon formation moment:

$$t_{AH} = \frac{H_i^{-1}}{3(1-\beta)}\left[1 - (K_i H_i)^{\frac{3(1-\beta)}{2-3\beta}}\right], \tag{15}$$

or equivalently [see Eq. (9)]

$$\frac{t_{AH}}{t_c} = \left[1 - (K_i H_i)^{\frac{3(1-\beta)}{2-3\beta}}\right]. \tag{16}$$

Now we turn to the determination of the collapsed mass. The total collapsed mass can be defined through the value of the mass at the moment of formation of the apparent horizon (t_{AH}). In other words, the collapsed mass is truly the upper limit of the matter that can be accumulated inside the surface determined by the apparent horizon. Therefore, we write the expression for the collapsed mass as

$$M(\tau_{AH}) = \frac{1}{2}K_i^{3/2} H_i^{1/2}(K_i H_i)^{\frac{3\beta/2}{2-3\beta}}, \tag{17}$$

where $M_s(\tau_{AH}) = \frac{1}{2}K_i^{3/2} H_i^{1/2}$ is the mass of a pure stiff fluid (fluid without interaction with the vacuum energy component, that is $\beta = 0$). The expression above can be obtained by substituting Eq. (15) into Eq. (14).
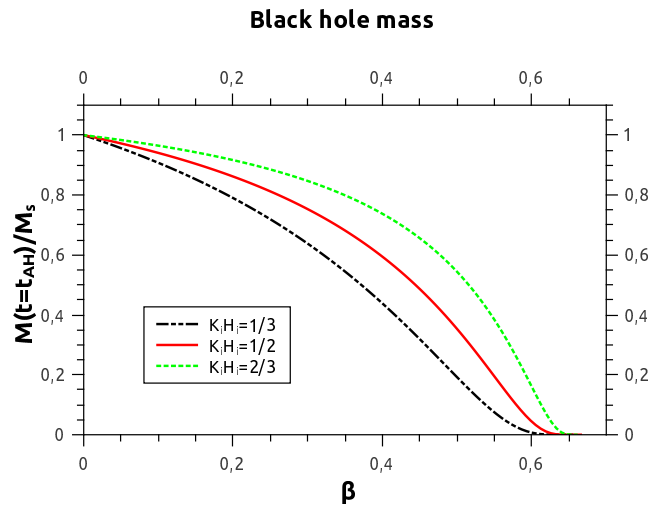
Figure 3. Collapsed mass of the black hole formed. We display the ratio between the mass of the black hole formed and the mass of the pure stiff fluid (without influence of the vacuum component). In the interval considered for the β-parameter the cosmic censorship conjecture remains valid. This is also seen in the graph for the apparent horizon moment (Fig. 2). We adopt the mass definition by Cahill-McVittie. The mass of the black hole formed diminishes with increase of the vacuum energy.

4. Final remarks

We discussed the influence of the vacuum energy on the gravitational collapse of a stiff fluid. We begun the discussion by adopting an approach which is analogous to the one adopted in the work of Wang and Meng [6]. However, because the parameter that represents the deviation from the standard case (which is the case without the influence of the vacuum energy and with null pressure) can mimic several laws for the cosmological term, we were able to find the relation between the Wang-Meng model and the model which has the Λ-term proportional to the square of the Hubble function.

For the stiff matter case, we establish the value for the β-parameter for which the cosmic censorship conjecture is valid, namely $\beta = 2/3$. This value appears in the graph for the scale factor (Fig. 1), where we observed that the change of the concavity of the graph is related to the acceleration of the fall of the comoving fluid element. In the graph for the apparent horizon formation condition (Fig. 2) we see that for $\beta > 2/3$ the singularity is not dressed.

Finally, in the graph for the black hole mass (Fig. 3), we note that for $\beta = 2/3$ we can obtain a black hole with null mass, that is a consequence of the inclusion in the energy momentum tensor of the vacuum component, and more specifically, of the vacuum pressure. Note that, null concavity is attained at the same value of the β-parameter as in the graph for the scale factor. Naturally, the relation between both is an old acquaintance of ours: the Newton's second law that relates the mass and acceleration. The statements above can be made more clear by rewriting the Einstein field equations as

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{c^4}\left(\rho + 3P\right) .$$

This expression is null for stiff matter and $\beta = \frac{2}{3}$.

On the another hand, it is also interesting to compare the collapse time for a dust fluid and the stiff matter, which are given by [7] [see Eq. (9)]

$$t_c(\text{dust}) = \frac{2}{3H_1(1-\beta)}, \qquad t_c(\text{stiff}) = \frac{1}{3H_1(1-\beta)}. \tag{18}$$

The difference between the both collapse times, furnish, theoretically, the necessary time for a dust fluid to reach the stiff matter state. Note that the time for dust cloud to become a ball of stiff matter is equal to the time needed for the stiff ball collapse.

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