

Equation of state of hypernuclear matter: tuning hyperon–scalar-meson couplings

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Abstract. We discuss to which extent the modifications of the hyperon–scalar-meson coupling constants affect the equation of state (EoS) hypernuclear matter. The study is carried out within a relativistic density functional theory. The nucleonic matter is described in terms of a density-dependent parametrization of nucleon-meson couplings, whereas the hyperon–meson couplings are deduced from the octet model. We identify the parameter space of hyperon-meson couplings for which massive stellar configurations with $M \leq 2.25M_{\odot}$ exist. We also discuss the EoS at finite temperatures with and without of a trapped neutrino component and show that neutrinos stiffen the EoS and change qualitatively the composition of stellar matter.

1. Introduction

The recent observations of two-solar-mass pulsars in binary orbits with white dwarfs [1, 2] spurred an intensive discussion of the phase structure of dense matter, which is consistent with the implied observational lower bound on the maximum mass of any sequence of compact stars based on the unique equation of state (hereafter EoS) of dense matter. In this article we review and summarize the key result of our study of hypernuclear matter in the context of these observations of massive compact stars [3].

Large central densities achieved in massive compact stars may require substantial population of heavy baryons (hyperons), because these become energetically favorable once the Fermi energy of neutrons becomes of the order of their rest mass. The onset of hyperons (and more generally any new constituent) reduces the degeneracy pressure of a cold thermodynamic ensemble. Therefore the EoS becomes softer than in the absence of the hyperons (or any other constituent). This decreases the maximum mass of a compact stars to values which contradict the observation of massive compact stars in nature. The controversy between the theory and observations is the “hyperonization puzzle” in compact stars.

Hyperons in dense nuclear matter have been studied using a number of methods, including Lagrangian based relativistic density functional methods [4, 5, 6, 7, 8, 9, 10, 11, 12] with coupling parameters fixed by the nuclear phenomenology, as well as models based on hyperon-nucleon potentials (for recent work see [13, 14]). While the potential models fail to produce heavy enough stars and are most likely accurate at densities not much larger than the nuclear saturation density, the relativistic Lagrangian based models are suitable candidates for extrapolation to high density regime.

The hypernuclear EoS was investigated [3] by us recently in the framework of the density-dependent relativistic density functional method. In particular, we focus on the sensitivity of



the EoS of hypernuclear matter to the unknown hyperon–scalar-meson couplings. These are constrained only by imposing SU(6) symmetry breaking and the nonet mixing. Within this framework, we argue that the parameters can be tuned such that two-solar massive hyperonic compact stars can exist. The EoS and composition of matter were also studied at finite temperatures relevant for the hot proto-neutron star stage of evolution and it was shown that the neutrino component stiffens the EoS of hypernuclear matter and substantially changes the composition of matter [3].

2. Theoretical model and choice of couplings

The relativistic Lagrangian density of our model reads

$$\begin{aligned}\mathcal{L} = & \sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega B} \omega_\mu - \frac{1}{2} g_{\rho B} \boldsymbol{\tau} \cdot \boldsymbol{\rho}_\mu \right) - (m_B - g_{\sigma B} \sigma) \right] \psi_B \\ & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \boldsymbol{\rho}^{\mu\nu} \boldsymbol{\rho}_{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}^\mu \cdot \boldsymbol{\rho}_\mu \\ & + \sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu \partial_\mu - m_\lambda) \psi_\lambda - \frac{1}{4} F^{\mu\nu} F_{\mu\nu},\end{aligned}\quad (1)$$

where the B -sum is over the $J^P = \frac{1}{2}^+$ baryon octet, ψ_B are the baryonic Dirac fields with masses m_B . The meson fields σ, ω_μ and $\boldsymbol{\rho}_\mu$ mediate the interaction among baryon fields, $\omega_{\mu\nu}$ and $\boldsymbol{\rho}_{\mu\nu}$ represent the field strength tensors of vector mesons and m_σ, m_ω , and m_ρ are their masses. The baryon-meson coupling constants are denoted by g_{mB} . The last line of Eq. (1) stands for the contribution of the free leptons, where the λ -sum runs over the leptons e^-, μ^-, ν_e and ν_μ with masses m_λ . The last term is the electromagnetic energy density. The contribution of neutrinos is included at temperatures above those at which the neutrinos decouple from matter, which is of order of several MeV.

The nucleon–meson coupling constants have been taken according to the DD-ME2 density-dependent parameterization [15]. The density dependence of the couplings implicitly takes into account many-body correlations among nucleons which are beyond the mean-field approximation. The nucleon-meson coupling constants are parametrized as $g_{iN}(\rho_B) = g_{iN}(\rho_0) h_i(x)$, for $i = \sigma, \omega$, and $g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0) \exp[-a_\rho(x - 1)]$ for the $\boldsymbol{\rho}_\mu$ -meson, where ρ_B is the baryon density, ρ_0 is the saturation density, $x = \rho_B/\rho_0$ and the explicit form of the functions $h_i(x)$ and the values of couplings can be found elsewhere [3, 15]. The pressure and energy density of the model is further supplemented from the contribution coming from the so-called rearrangement self-energy [16], which guarantees the thermodynamical consistency.

In order to fix the hyperon–meson couplings we consider the SU(3)-flavor symmetric octet model. Due to the universal coupling of the $\boldsymbol{\rho}_\mu$ meson to the isospin current [17] and the ideal mixing between the ω and ϕ mesons [18], the couplings between hyperons and vector mesons are as follows

$$\begin{aligned}g_{\Xi\rho} &= g_{N\rho}, & g_{\Sigma\rho} &= 2g_{N\rho}, & g_{\Lambda\rho} &= 0, \\ g_{\Xi\omega} &= \frac{1}{3}g_{N\omega}, & g_{\Sigma\omega} &= g_{\Lambda\omega} = \frac{2}{3}g_{N\omega}.\end{aligned}\quad (2)$$

Within the octet model the baryon-scalar mesons couplings of the scalar octet can be expressed in terms of only two parameters, the nucleon– a_0 meson coupling constant g_S and the $F/(F + D)$ ratio of the scalar octet [19]. By considering the mixing with the scalar singlet state, one is then left with the following relation between the coupling of the baryons with the σ -meson [3]:

$$2(g_{N\sigma} + g_{\Xi\sigma}) = 3g_{\Lambda\sigma} + g_{\Sigma\sigma}, \quad (3)$$

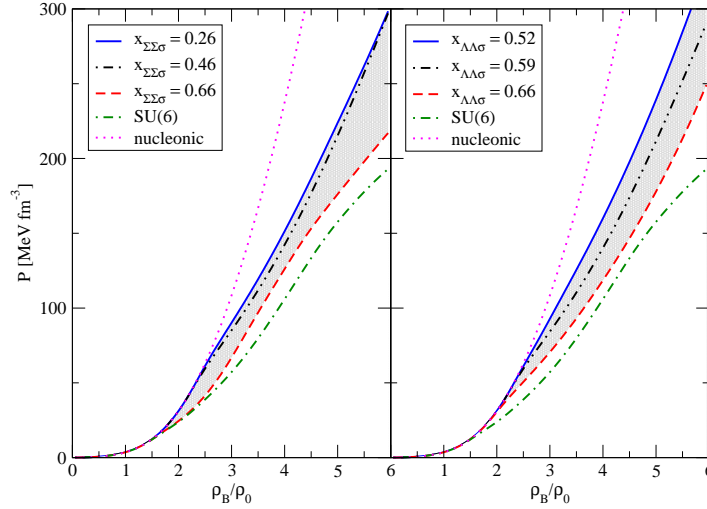


Figure 1. Equations of state of hypernuclear matter for a range of values of hyperon- σ meson couplings defined in terms of $x_{H\sigma} = g_{H\sigma}/g_{N\sigma}$, $H \in \Lambda, \Sigma$. The nucleonic EoS (dotted line, magenta online) and hyperonic EoS with SU(6) quark model couplings (dot-dashed line, green online) are shown as a reference. The nucleonic coupling constants correspond to the DD-ME2 parametrization [15]; the hyperon-vector meson couplings are fixed as explained in the text. Left panel: we assume $x_{\Lambda\sigma} = 0.58$, as in the NSC potential model, and a range $0.26 \leq x_{\Sigma\sigma} \leq 0.66$ which generates the shaded area. Right panel: we assume $x_{\Sigma\sigma} = 0.448$ and a range $0.26 \leq x_{\Lambda\sigma} \leq 0.66$. The cases $x_{\Sigma\sigma} = 0.46$ (left panel) and $x_{\Lambda\sigma} = 0.59$ (right panel), shown by dash-double-dotted (black online) lines, fit the depth of the potentials of the Σ^- and Λ hyperons in nuclear matter at saturation.

We assume that the hyperon coupling constants must be positive and less than the nucleon coupling constant. Then, by solving Eq. (3) for one of the dependent hyperon- σ meson coupling constant, say $g_{\Xi\sigma}$, we obtain

$$g_{N\sigma} \leq \frac{1}{2}(3g_{\Lambda\sigma} + g_{\Sigma\sigma}) \leq 2g_{N\sigma}. \quad (4)$$

We further proceed by first fixing the value of $g_{\Lambda\sigma}$ coupling constant at the value provided by the Nijmegen Soft Core (NSC) hypernuclear potential [20] and varying the range of couplings $g_{\Sigma\sigma}$ within the limits provided by Eq. (4) and then we repeat the calculations by interchanging the role of Σ and Λ hyperons. We also studied the case where one of couplings is fixed to the depth of the potential of the Σ^- and Λ hyperons in nuclear matter at the saturation density.

3. Results

The dependence of the EoS on the variation of the hyperon-scalar meson coupling at $T = 0$ is shown in Fig. 1 for a range of parameter space (for details see the figure caption). It is clearly seen that the hyperonization of matter softens the nucleonic EoS. The softening is smallest for the lowest possible values of the couplings of the hyperons to the scalar mesons.

Fig. 2 shows the particle fractions of fermions at zero temperature, for the limiting cases of the SU(6) quark model couplings (left panel) and the stiffest hypernuclear EoS (right panel).

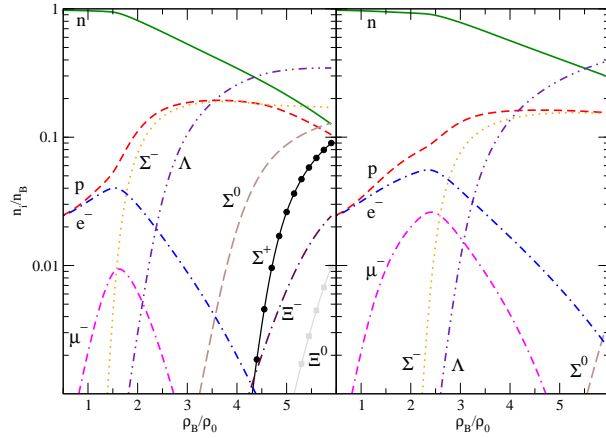


Figure 2. Particle fractions in hypernuclear matter at $T = 0$. Left panel: hyperon–scalar meson couplings are fixed as in the SU(6) symmetric quark model. Right panel: a stiff hypernuclear EoS from our parameter space with $x_{\Sigma\sigma} = 0.448$ and $x_{\Lambda\sigma} = 0.52$.

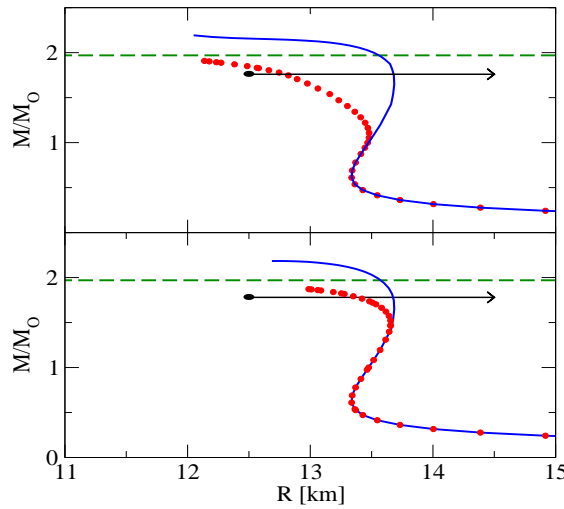


Figure 3. The mass–radius relations for compact hypernuclear stars at zero temperature. The solid (blue) lines show the cases $x_{\Sigma\sigma} = 0.26$ and $x_{\Lambda\sigma} = 0.58$ (upper panel) and $x_{\Sigma\sigma} = 0.52$ and $x_{\Lambda\sigma} = 0.448$ (lower panel). The (red) dots show the cases $x_{\Sigma\sigma} = 0.66$ and $x_{\Lambda\sigma} = 0.58$ (upper panel) and $x_{\Sigma\sigma} = 0.66$ and $x_{\Lambda\sigma} = 0.448$ (lower panel). The dash-dotted (green) line shows the observational lower limit on the maximum mass $1.97M_{\odot}$. The arrow shows the mass-radius constraint of Ref. [21] at 2σ level, which is $M = 1.76M_{\odot}$ and $R \geq 12.5$ km.

The first EoS is characterized by large hyperon–scalar-meson couplings, whereas the second by small ones. Thus, the larger are the hyperon–scalar-meson couplings the more favorable is the formation of hyperons and the softer is the EoS. The mass radius relation for these EoS is shown in Fig. 3, which demonstrates the large enough masses can be achieved within the

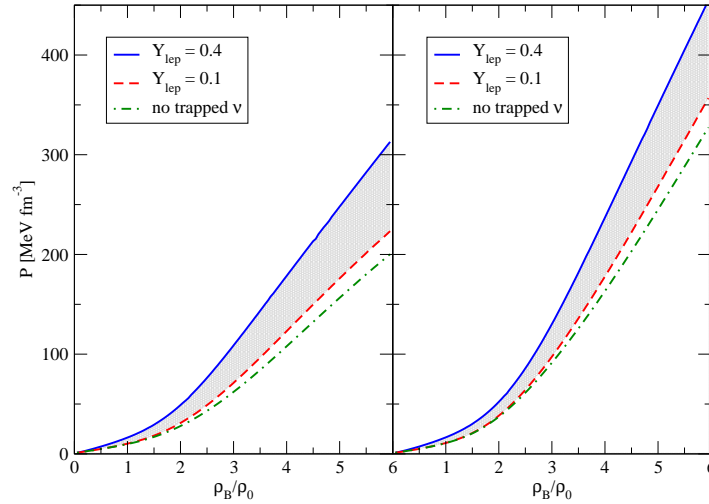


Figure 4. Equation of state in the DD-ME2 parametrization [15] at finite temperature, $T = 50$ MeV. Left panel: the hyperon–scalar meson coupling constants are fixed by the quark model. In this figure the dependence on the presence of trapped neutrinos is shown. The dot-dashed line (green online) corresponds to the case without neutrinos. The presence of neutrinos make the EoS stiffer. The dashed region represents the variation of the EoS with the lepton fraction. The dashed line (red online) corresponds to a lepton fraction $Y_L = 0.1$ and the full line (blue online) to $Y_L = 0.4$. Right panel: same as left, in which the coupling constants are the one of the stiffest case considered, corresponding to $g_{\Sigma\sigma} = 0.448g_{N\sigma}$, fixed by the NSC potential and $g_{\Lambda\sigma} = 0.52g_{N\sigma}$.

present model that are consistent with the observational lower bound on the maximum mass of a compact star. Fig. 4 shows the EoS of finite temperature hypernuclear matter with trapped neutrinos; the lepton fraction is varied in the range $0.1 \leq Y_L \leq 0.4$ (shaded area in Fig. 4). This corresponds to the shaded area. The left and right panels correspond to the softest and the stiffest hypernuclear EoS described above. This compared to the neutrino-less case ($Y_\nu \neq 0$). Clearly neutrinos stiffen the EoS matter - the larger is the fraction of neutrinos, i.e. Y_L , the stiffer is the EoS. The stiffening of the EoS can be attributed to the fact that the thermal population of neutrinos adds its contribution to the pressure of matter.

The obtained EoS can be conveniently represented by piecewise polytropic EoS of the form [3]¹

$$P = \sum_{i=1}^4 K_i (\rho/\rho_0)^{\Gamma_i} \theta(\rho - a_i \rho_0) \theta(b_i \rho_0 - \rho), \quad (5)$$

where Γ_i is the polytropic index, K_i is a dimensionful constant, $[K_i] = \text{MeV fm}^{-3}$, and ρ_0 is the saturation density; the values of the fit parameters can be found in [3].

¹ We use the occasion to correct a misprint in the original formula (Eq. (32) of [3]), where the density normalization by ρ_0 is missing.

4. Conclusions

Because the information on the properties of hypernuclear matter is far less extensive than for nucleons it is currently impossible to exclude hyperons as constituents of densest regions of compact stars. Our study [3] confirms this within a specific relativistic density functional approach to hypernuclear matter with tuned hyperon–scalar-meson couplings. We find that hyperonization in massive stars is favored for small ratios of the hypernuclear-to-nuclear couplings; in particular, hyperons need to be coupled to scalar mesons weaker than predicted by the SU(6) quark model. For certain values of the hyperon–scalar meson couplings hypernuclear EoS can still produce stellar equilibrium configurations of compact stars compatible with the two-solar-mass pulsar observations.

Neutrino trapping leads to strong modification in the population of hyperons and to a shift in the threshold density at which they first appear. As a consequence, a stiffening of the EoS in the early stage of the neutron star formation is observed. Instead of deleptonization with increasing density, seen in neutrino-less matter, the abundances of charged leptons remain constant, which among other things leads to inversion of the density thresholds for appearance of charged Σ 's.

Acknowledgments

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