

# Influence of phase-transition scenarios on the abrupt changes in the characteristics of compact stars

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**Abstract.** We study the abrupt changes in the characteristics of compact stars due to the quark deconfinement phase transition. The hadronic phase is described within the relativistic mean-field theory, including a scalar-isovector  $\delta$ -meson effective field. To describe the quark phase, we use the MIT bag model, in which the interactions between  $u$ ,  $d$  and  $s$  quarks inside the bag are taken into account in the one-gluon exchange approximation. We analyze catastrophic changes of the parameters of the near-critical configuration of compact star and compute the amount of the energy released by a corequake for the two extreme cases of the deconfinement phase transition scenarios. The first one corresponds to the ordinary first-order phase transition (Maxwell scenario) and the second one corresponds to the phase transition calculated using the bulk Gibbs equilibrium conditions and global charge neutrality (Glendenning scenario).

## 1. Introduction

Clarification of the structure and internal composition of compact stars is one of the main problems in modern physics. The circumstance that the central density of neutron stars is several times larger than the saturation density of nuclear matter makes neutron stars a specific natural laboratory for studying the characteristics of exotic states of matter at extremely high densities. Because of the very large densities achieved in the centers of compact stars various exotic particle species and phases of matter such as, for example, hyperons, deconfined quark plasma of  $u$ ,  $d$ ,  $s$  quarks,  $\pi$  and  $K$  meson condensates can arise. Over the past few decades many researchers have intensively studied various aspects related to the formation of exotic degrees of freedom in neutron stars and proposed observational tests that can confirm the existence in the interiors of compact stars such constituents (for review see, e.g., Refs. [1, 2] and references therein). The authors of several papers attempted to draw a conclusion about the existence of quark matter in compact stars through investigations of the mass-radius relation for such stars relying on the fact that compact stars made of quark matter have smaller radii than their hadronic counterparts of the same mass [3]. The studies of the thermal evolution the quark stars can also provide information about the existence of a quark component in the star [4]. Furthermore, the studies of gravitational radiation from compact stars can provide additional channel of information on existence of the quark matter inside compact stars [5].

Phase transitions accompanied by discontinuities of the thermodynamic potentials are of special interest, because they lead to a dynamical rearrangement of neutron stars. Depending on the value of surface tension  $\sigma_s$ , the phase transition from nuclear matter to quark matter can occur within two scenarios [6, 7]: one corresponds to the ordinary first order phase transition



at constant pressure with a density jump (Maxwell construction), the other to formation of mixed hadron-quark matter with a continuous variation of pressure and density. The problem of the formation of a mixed phase taking into account the finite dimensions of the quark structures inside nuclear matter, the Coulomb interaction, as well as the surface energy, has been examined in Refs. [7, 8, 9]. It was shown that the mixed phase is energetically favorable for small values of the surface tension between the quark matter and the nuclear matter. The uncertainty in the value of the surface tension makes it impossible to determine the actual phase transition scenario that will take place. The quark-hadron phase transition can be triggered, for example, by the dynamical process of accretion of matter onto the surface of a hadronic neutron stars. Such accretion can lead to an increase of the central density of the stars and to formation of a new phase containing deconfined quarks in the center of the star. The process of catastrophic rearrangement with the formation of a quark core of finite radius at the star's center will be accompanied by a release of a colossal amount of energy, which is comparable to the energy release during a supernova explosion. Note that a similar process of both restructuring and energy release takes place also in the case of pion condensation in the cores of neutron stars [10, 11, 12, 13, 14].

The recent series of our articles [15, 16, 17] were devoted to a detailed investigation of quark deconfinement phase transition in neutron star matter. The nuclear matter was described within the relativistic mean-field (RMF) theory with the scalar-isovector  $\delta$ -meson effective field. The bulk calculation results of the mixed phase structure (Glendenning construction) [18] were compared with the results of a usual first-order phase transition (Maxwell construction). Here we investigate the energy release and the change in the integral parameters of compact stars due to a phase transition from hadronic to quark matter in these two extreme scenarios and identify how these depend on the chosen scenario of the phase transition.

## 2. Equation of state of neutron star matter

### 2.1. Equation of state of hadronic phase

We use the equation of state (EoS) of Baym, Bethe, and Pethick (BBP) [19] for the description of hadronic phase in the lower density region corresponding to the outer and inner crust of the star. In nuclear and supranuclear density region ( $n \geq 0.1 \text{ fm}^{-3}$ ) we use the relativistic Lagrangian density of many-particle system consisting of nucleons,  $p$ ,  $n$ , electrons and isoscalar-scalar ( $\sigma$ ), isoscalar-vector ( $\omega$ ), isovector-scalar ( $\delta$ ), and isovector-vector ( $\rho$ ) - exchanged mesons. The Lagrangian of the theory is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N [\gamma^\mu (i\partial_\mu - g_\omega \omega_\mu - \frac{1}{2} g_\rho \vec{\tau}_N \vec{\rho}_\mu) - (m_N - g_\sigma \sigma - g_\delta \vec{\tau}_N \vec{\delta})] \psi_N \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma \sigma^2) - \frac{b}{3} m_N (g_\sigma \sigma)^3 - \frac{c}{4} (g_\sigma \sigma)^4 \\ & + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \vec{\rho}_\mu - \frac{1}{4} \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} \\ & + \frac{1}{2} (\partial_\mu \vec{\delta} \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) + \bar{\psi}_e (i\gamma^\mu \partial_\mu - m_e) \psi_e, \end{aligned} \quad (1)$$

where  $\sigma$ ,  $\omega_\mu$ ,  $\vec{\delta}$ , and  $\vec{\rho}^\mu$  are the fields of the  $\sigma$ ,  $\omega$ ,  $\delta$ , and  $\rho$  exchange mesons, respectively,  $m_N$ ,  $m_e$ ,  $m_\sigma$ ,  $m_\omega$ ,  $m_\delta$ ,  $m_\rho$  are the masses of the free particles,  $\psi_N = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$  is the isospin doublet for nucleonic bispinors, and  $\vec{\tau}$  are the isospin  $2 \times 2$  Pauli matrices. Antisymmetric tensors of the vector fields  $\omega_\mu$  and  $\vec{\rho}_\mu$  are given by

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad \mathcal{R}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu. \quad (2)$$

In our calculations we take  $a_\delta = (g_\delta/m_\delta)^2 = 2.5 \text{ fm}^2$  for the  $\delta$  coupling constant, as in Ref. [20]. Also we use  $m_N = 938.93 \text{ MeV}$  for the bare nucleon mass,  $m_N^* = 0.78 m_N$  for the nucleon effective mass,  $n_0 = 0.153 \text{ fm}^{-3}$  for the baryon number density at saturation,  $f_0 = -16.3 \text{ MeV}$  for the binding energy per baryon,  $K = 300 \text{ MeV}$  for the incompressibility modulus, and  $E_{sym}^{(0)} = 32.5 \text{ MeV}$  for the asymmetry energy. Then the five other constants,  $a_i = (g_i/m_i)^2$  ( $i = \sigma, \omega, \rho$ ),  $b$  and  $c$ , can be determined numerically:  $a_\sigma = (g_\sigma/m_\sigma)^2 = 9.154 \text{ fm}^2$ ,  $a_\omega = (g_\omega/m_\omega)^2 = 4.828 \text{ fm}^2$ ,  $a_\rho = (g_\rho/m_\rho)^2 = 13.621 \text{ fm}^2$ ,  $b = 1.654 \cdot 10^{-2} \text{ fm}^{-1}$ ,  $c = 1.319 \cdot 10^{-2}$ . The knowledge of the model parameters allows us to solve the set of four equations in a self-consistent way and to determine the (renamed) mean-fields,  $\sigma \equiv g_\sigma \bar{\sigma}$ ,  $\omega \equiv g_\omega \bar{\omega}_0$ ,  $\delta \equiv g_\delta \bar{\delta}^{(3)}$ , and  $\rho \equiv g_\rho \bar{\rho}_0^{(3)}$ , which depend on baryon number density  $n$  and asymmetry parameter  $\alpha = (n_n - n_p)/n$ . The standard quantum-hadrodynamics (QHD) procedure allows us to obtain the expressions for energy density  $\varepsilon(n, \alpha)$  and pressure  $P(n, \alpha)$  (for details see Ref. [15]).

## 2.2. Equation of state of strange quark phase

To describe the quark phase an improved version of the MIT bag model was used, in which the interactions between  $u$ ,  $d$ ,  $s$  quarks inside the bag are taken in a one-gluon exchange approximation [21]. We choose  $m_u = 5 \text{ MeV}$ ,  $m_d = 7 \text{ MeV}$  and  $m_s = 150 \text{ MeV}$  for quark masses,  $B = 60 \text{ MeV/fm}^3$  for bag parameter and  $\alpha_s = 0.5$  for the strong interaction constant.

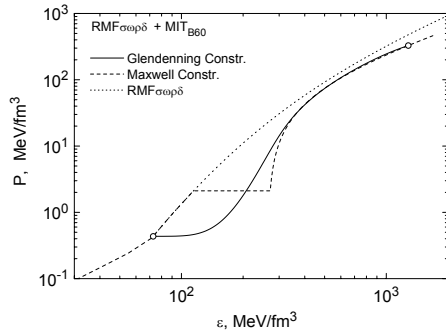
## 2.3. Maxwell and Glendenning constructions

Using these EoS for the nucleonic and the quark phases we first calculate the physical parameters of the phase transition in the case of the Glendenning construction (where these phases satisfy the Gibbs condition and are separately electrically charged, but the global electrical neutrality of the system is maintained). We then examine the second case where both phases are separately neutral and the transition is the usual first-order phase transition corresponding to the well-known Maxwell construction. Model EoSs of neutron star matter for Glendenning and Maxwell construction cases are presented in Fig. 1. In case of the Maxwell construction the phase transition occurs at constant pressure  $P_0 = 2.11 \text{ MeV/fm}^3$  and nucleonic matter with the energy density  $\varepsilon_N = 114.5 \text{ MeV/fm}^3$  coexists with the quark matter whose energy density is  $\varepsilon_Q = 271.4 \text{ MeV/fm}^3$ . In the case of Glendenning construction the deconfinement phase transition proceed through formation of a mixed hadron-quark phase. The boundaries of the mixed phase are  $\varepsilon_N = 72.79 \text{ MeV/fm}^3$ ,  $P_N = 0.43 \text{ MeV/fm}^3$  and  $\varepsilon_Q = 1280.88 \text{ MeV/fm}^3$ ,  $P_Q = 327.75 \text{ MeV/fm}^3$ .

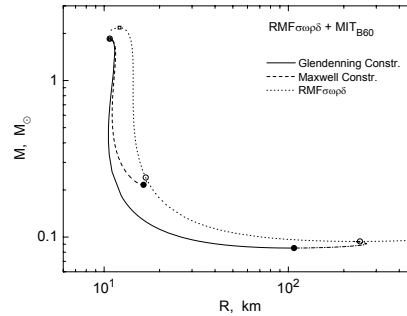
It was shown in Ref. [22] that for an ordinary first order phase transition, the density discontinuity parameter  $\lambda = \varepsilon_S/(\varepsilon_N + P_0)$  plays a decisive role in the stability of neutron stars with arbitrarily small cores made of the denser phase of matter. Here  $P_0$  is coexistence pressure of the two phases, and  $\varepsilon_N$ ,  $\varepsilon_S$  are the energy densities of the normal and superdense phases, respectively. Paraphrasing the conclusions of Ref. [22], we can state that the stability criterions for the first-order hadron-quark phase transition: if  $\lambda < 3/2$ , then a neutron star with an arbitrarily small core of strange quark matter is stable, whereas if  $\lambda > 3/2$ , neutron stars with small quark cores are unstable. In the latter case, for a stable star there is a nonzero minimum value for the radius of the quark core. In case of the Maxwell construction of deconfinement phase transition discussed in this article, the value of jump parameter is  $\lambda = 2.327$ .

## 3. Changes in the Stellar Parameters triggered by the formation of quark phase

Using the neutron star matter EoSs obtained in previous section, we have integrated the Tolman-Oppenheimer-Volkoff (TOV) equations [24, 25] and obtained the gravitational mass  $M$ , radius  $R$ , baryonic mass  $M_0 = m_N N_B$  ( $m_N$  is the nucleon mass and  $N_B$  the total number of baryons) and moment of inertia  $I$  of compact stars for the different values of central pressure  $P_c$ .



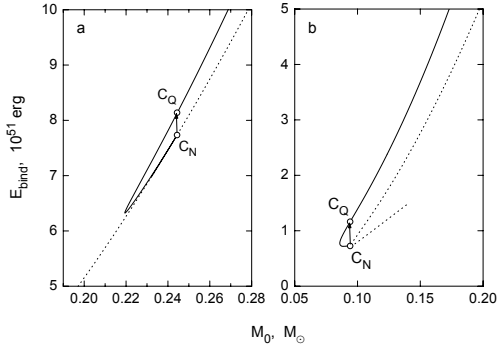
**Figure 1.** EoS of neutron star matter for the two different hadron-quark phase transition constructions. Solid and dashed lines correspond to the Glendenning and Maxwell constructions, respectively, whereas the dotted line to the pure *npe* matter. Open circles represent the mixed phase boundaries.



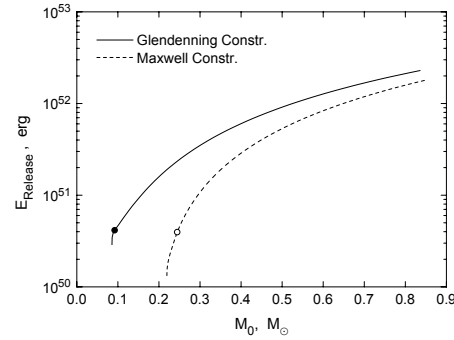
**Figure 2.** Mass-radius relations for model EoSs presented in Fig. 1. The labeling of the curves is the same as in Fig. 1. Open circles mark the critical configurations, solid circles mark the stable hybrid stars with minimal mass. The dash-dotted line between circles corresponds to the branch of unstable stars.

In Fig. 2 we show the  $M(R)$  dependence of compact stars for different quark deconfinement phase transition scenarios. Open circles denote the critical configurations while solid circles denote the stable stars with minimal mass in the center of which there are deconfined quarks. Moreover, in the case of Maxwell construction, this core is composed of a quark-electron plasma, while in the Glendenning construction case it is composed of a mixed quark-hadron matter. We can see that for both phase transition scenarios considered here there are unstable star branches between critical and minimum-mass configurations (dot-dashed line segments). The fact that in the case of Maxwell construction, the  $M - R$  relation exhibits such a behavior, is not surprising, since according to the Seidov criterium [22, 23], the infinitesimal core of denser matter in ordinary first order phase transition is unstable when  $\lambda > 3/2$ . The corresponding EoS of neutron star matter considered here satisfies this condition. The appearance of the unstable branch of compact stars with infinitesimal core consisting of a mixed hadron-quark matter in the case of Glendenning construction is not standard. In fact, in this scenario the energy density is a continuous function of pressure and in most cases leads to a monotonic increase of stars' mass in the domain which corresponds to the lower boundary of the mixed phase. This implies that the configurations with an infinitesimally small core containing the mixed phase are in many cases stable. In the case of Glendenning construction considered here, the branch of unstable compact stars appears near the lower threshold of the mixed phase. Consider the situation where matter is accreted onto the surface of an ordinary neutron star located below the critical configuration. The baryonic mass of the star will increase, the star will reach the mass of the critical configuration at which instance a transition to the configuration with deconfined quark phase will take place.

It is worthwhile to note that the maximum masses of the stars containing deconfined quarks are  $M_{max} = 1.853M_{\odot}$  and  $M_{max} = 1.828M_{\odot}$  for the Glendenning and the Maxwell constructions, respectively. Since the transition of ordinary neutron star to a star containing quark matter occurs at a constant baryon number, it is convenient to consider the star characteristics as functions of baryonic mass  $M_0$ . The binding energies  $E_{bind} = (M - M_0)c^2$  of compact stars as a function of baryonic mass  $M_0$  for the Maxwell and the Glendenning hadron-quark phase



**Figure 3.** Binding energy of compact stars as a function of baryonic mass  $M_0$  for Maxwell (left panel) and Glendenning (right panel) constructions. Dotted line corresponds to the pure nuclear matter.  $C_N$  marks the critical configuration of hadronic star, and  $C_Q$  marks the neutron star with same baryonic mass containing quark phase.



**Figure 4.** The total energy release due to the hadron-quark phase transition as a function of baryonic mass of neutron star  $M_0$  for the Maxwell and the Glendenning constructions. Circles correspond to the  $C_N \rightarrow C_Q$  transition shown in Fig. 3.

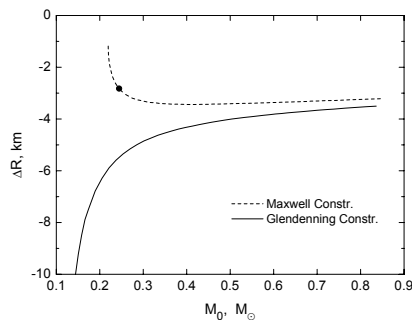
transition scenarios are shown in Fig. 3. Accretion of matter onto the surface of the critical configuration  $C_N$  will lead to a jump-like transition to the configuration  $C_Q$ , which will have a finite-size core containing deconfined quark matter. This transition will be accompanied by an enormous release of energy determined by the difference in binding energies of these configurations:

$$E_{\text{release}} = E_{\text{bind}}(C_Q) - E_{\text{bind}}(C_N) = (M(C_N) - M(C_Q))c^2. \quad (3)$$

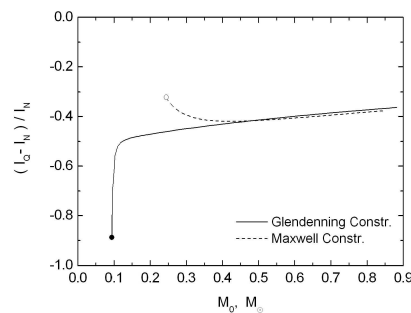
In Fig. 4 we plot the total energy release as a function of baryonic mass of star. One can see that in both scenarios of the phase transition the released energy increases with increasing baryonic mass  $M_0$ . For fixed values of  $M_0$  the energy that is converted in the phase transition is larger in the case of Glendenning construction than in the case of Maxwell construction. In addition, the minimum required baryonic mass for the catastrophic rearrangement of the neutron star, accompanied by the formation of a quark core in the center of the star, is greater in the Maxwell scenario. In the case considered here, the quark deconfinement phase transition in the neutron star interior leads to the energy release of order  $10^{50} \div 10^{52}$  erg. Fig. 5 shows the changes in stellar radius  $\Delta R = R_Q - R_N$  due to the quark deconfinement phase transition as a function of the baryonic mass  $M_0$  in both the Glendenning and the Maxwell scenarios. It is seen that in both cases the compact stars radii decrease. Fig. 6 shows the fractional changes in the moment of inertia of compact stars,  $(I_Q - I_N)/I_N$ , for the two types of phase transition. It is seen that the dependence of the quantities  $\Delta R$  and  $\Delta I/I_N$  on the baryonic mass of the stars are clearly distinct in the two alternative scenarios of phase-transition.

#### 4. Conclusion

Starting from relativistic mean-field description of hadronic phase and improved MIT model description of the quark phase we obtained the EoS of compact stars with a quark-deconfinement phase transitions, which was treated using either the Maxwell or the Glendenning construction. We found the dependence of conversion energy on the baryonic mass of neutron stars and analyzed the changes in stellar radii and moments of inertia due to the deconfinement phase



**Figure 5.** Changes in the stellar radius due to the hadron-quark phase transition as a function of baryonic mass  $M_0$  of a neutron star for both Maxwell and Glendenning scenarios.



**Figure 6.** Fractional changes in the stellar moment of inertia  $\Delta I/I_N$  as a function of baryonic mass  $M_0$  of a neutron star for both Maxwell and Glendenning scenarios.

transitions. We demonstrated that for a fixed value of the baryonic mass of the star, the energy conversion in the case of the Glendenning construction is greater than in the case of the Maxwell construction. The minimum required baryonic mass for the catastrophic rearrangement of the neutron star and the formation of a quark core in the center of the star is greater in the case of the Maxwell construction as compared to the Glendenning construction. In the cases considered here, the quark deconfinement phase transition in the neutron star interior leads to the energy release of order  $10^{50} \div 10^{52}$  erg.

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