

Optimal Cycle Time and Preservation Technology Investment for Deteriorating Items with Price-sensitive Stock-dependent Demand Under Inflation

Nita H. Shah and Arpan D. Shah

Department of Mathematics, Gujarat University,
Ahmedabad, Gujarat, India

E-mail: nitahshah@gmail.com, arpan.math@gmail.com

Abstract. The article analyzes economic order quantity for the retailer who has to handle imperfect quality of the product and the units are subject to deteriorate at a constant rate. To control deterioration of the units in inventory, the retailer has to deploy advanced preservation technology. Another challenge for the retailer is to have perfect quality product. This requires mandatory inspection during the production process. This model is developed with the condition of random fraction of defective items. It is assumed that after inspection, the screened defective items are sold at a discounted rate instantly. Demand is considered to be price-sensitive stock-dependent. The model is incorporating effect of inflation which is critical factor globally. The objective is to maximize profit of the retailer with respect to preservation technology investment, order quantity and cycle time. The numerical example is given to validate the proposed model. Sensitivity analysis is carried out to work out managerial issues.

1. Introduction

In any firm, deterioration of items is an important concern. There are various factors that affect a manufacturing process. So the deterioration of items is inevitable and plays an important role as utility of an item does not remain constant. To decrease deterioration, manufacturers have started investing in preservation technology. Many researchers have worked on controlling deterioration and derived models on the same. Ghare and Schrader (1963) developed the first EOQ model for inventory following exponential decay. Hariga (1995) Developed EOQ model for deteriorating items with shortages and time varying demand. Jaggi and Mittal (2007) developed EOQ model for deteriorating items with time dependent demand under inflationary conditions. Jaggi and Mittal (2011) developed an EOQ model for deteriorating items with imperfect quality. Jaggi et al. (2012) developed a model for effect of inspection for deteriorating items with time dependent demand under inflationary conditions. Other articles by Raafat et al. (1991), Widyadana et al. (2011), Yong and Huang (2013), Chen and Dye (2013), Hsieh (2013) and Hsieh and Dye (2013) are available in the same direction.

During a production process, it is impossible to achieve 100% quality products. Therefore, it is assumed that random fraction of defective items is produced at the end of production cycle and such items are sold at discounted rate instantly. On this concept, many researchers have developed various mathematical models. Porteus (1986) suggested model for optimal lot sizing, process quality improvement and setup cost reduction. Rosenblatt and Lee (1986) provided model for economic production cycles with imperfect production processes. Cardenas-Barron (2000) developed an EPQ model for items with imperfect quality. Goyal and Cardenas-Barron (2002) provided an EPQ model for items with imperfect quality. Other interesting articles are by Schwaller (1988), Zhang and Gerchak (1990), Salamah and Jaber (2000), Cardenas-Barron (2009-a, 2009-b), Cardenas-Barron et al. (2010).



Many countries are affected by inflation on a large scale and most of the models have not taken inflation rate into account. Inflation rate play an important role in deciding inventory policy for any firm. So, for determining optimum inventory policy, inflation rate can not be ignored. Also, there are many researchers who developed variants of inventory models with different constraints under inflation. Misra (1975) provided analysis of inflationary effects on inventory systems. Buzacott (1975) provided EOQ model with inflation. Datta and Pal (1991) suggested inventory model with linear time dependent demand rate and shortages under the effect of inflation and time value of money. Hariga and Ben-Daya (1996) derived optimal time varying lot sizing models under inflationary conditions. Ray and Chaudhari (1997) derived an EOQ model with stock-dependent demand, shortages, inflation and time discounting. Other interesting articles by Sarkar and Pan (1994) and Sana and Chaudhari (2003) are available in the same direction.

Inspired by this aspect, this paper proposes a model for deteriorating item with price-sensitive stock-dependent demand under inflation. Defective items are sold at a discounted rate after screening process. Demand is considered as price-sensitive and taken as a power function of selling price under inflation. Model does consider preservation technology investment to reduce the proportion of deteriorating items. This study doesn't allow any shortages during the cycle time. The objective is to maximize the profit of the firm with respect to optimal selling price, optimal length of replenishment cycle and optimal preservation technology investment.

The rest of the paper is organized as follows. In section 2, the notations and assumptions are presented. In section 3, mathematical model to maximize the total profit per unit time is developed. In section 4, solution procedure is given. Numerical examples are presented in section 5 and conclusion is drawn in section 6.

2. Notations and Assumptions:

| | |
|----------------|--|
| A | Replenishment cost per order |
| C | Purchase cost per unit |
| P | Selling price per unit, $P > C$ (a decision variable) |
| h | Unit inventory holding cost per unit time |
| T | Cycle length (a decision variable) |
| $I(t)$ | Inventory level at time t |
| Q | Order quantity per cycle (a decision variable) |
| $D(P, I(t))$ | Demand rate |
| η | Price elasticity Markup cost, $\eta > 1$ |
| C_s | Screening cost per unit |
| P_s | Selling price of defective unit |
| θ | Deterioration rate ($0 \leq \theta \leq 1$) |
| u | Preservation technology investment per unit to reduce deterioration rate (a decision variable) |
| $f(u)$ | Proportion reduced deterioration rate, $f(u) = 1 - e^{-mu}$, $0 < f(u) < 1$ |
| r | Discount rate representing the time value of money |
| k | Inflation rate per unit time |
| R | Net inflation rate = $(r - k)$ |
| λ | Proportion decrease in selling price (defective items) |
| ϵ | Proportion of defective items |
| t_1 | ξT , where $0 < \xi < 1$ |
| $\pi(P, T, u)$ | Profit function |

Assumptions

1. The inventory system involves a single deteriorating item over an infinite planning horizon.
2. Existence of deterioration items assumed with the initiation of production and the proportion reduced because of preservation technology are taken to be uniform.
3. There is no repair or replacement of deteriorating items during the period under consideration.
4. Shortages are not allowed.
5. Defective items are sold at a discounted rate after screening process.
6. Screening process ends at time t_1 which is pre-specified.
7. The demand rate,

$$D(P, I(t)) = \int_0^T (\alpha + \beta I(t)) P^{-\eta} dt, \alpha > 0, \beta > 0, \eta > 1$$
is a non-negative power function of selling price under inflation.
8. Replenishment occurs instantly.
9. Effect of inflation is incorporated.

3. Mathematical model:

Consider a firm running a single item with a price-sensitive stock-dependent demand $D(P, I(t))$ and deterioration rate (θ). In order to reduce the deterioration rate, the firm is investing amount (u) in the preservation technology. The proportion of reduced deterioration rate is denoted by $f(u)$. Inventory level reaches to zero at the end of each replenishment cycle due to deterioration rate and demand. The differential equation representing inventory status during interval $[0, T]$ is given by,

$$\frac{dI(t)}{dt} = -\theta(1 - f(u))I(t) - D(P), \quad 0 \leq t \leq T \quad (1)$$

with initial boundary condition $I(T) = 0$

Solving the differential equation (1), the inventory level at any instant of time t is given by,

$$I(t) = \frac{\alpha P^{-\eta}}{\theta(1 - f) + \beta P^{-\eta}} \left[e^{(\theta(1-f) + \beta P^{-\eta})(T-t)} - 1 \right], \quad 0 \leq t \leq T \quad (2)$$

The ordering quantity over the replenishment cycle (or the initial inventory on hand), is given by ,

$$Q = I(0) = \frac{\alpha P^{-\eta}}{\theta(1 - f) + \beta P^{-\eta}} \left[e^{(\theta(1-f) + \beta P^{-\eta})T} - 1 \right] \quad (3)$$

Preservation technology investment for a cycle time T is given by uT . For convenience, $f(u)$ will be denoted by f in the model. Other costs involved in profit function are mentioned below.

The replenishment cost is A .

The preservation technology investment under inflation is given by uTe^{-Rt} .

As defective items are sold after screening process ,holding cost cannot be applied to " $\in Q$ " quantity of the items and hence, the inventory holding cost under inflation is given by,

$$HC = \int_0^T hI(t) e^{-Rt} dt - h \in Qe^{-Rt_1}, \quad \text{where } t_1 = \text{time at which screening process ends}$$

The purchase cost of an item is given by,

$$CQ = \frac{C\alpha P^{-\eta}}{\theta(1 - f) + \beta P^{-\eta}} \left[e^{(\theta(1-f) + \beta P^{-\eta})T} - 1 \right]$$

Screening cost : $C_s Q e^{-Rt_1}$

The sales revenue is $P \int_0^T (\alpha + \beta I(t)) P^{-\eta} e^{-Rt} dt + P_s Q \in e^{-Rt_1}$, $P_s = P(1 - \lambda)$

Hence, the profit per unit time is given by,

$$\pi(P, T, u) = \frac{1}{T} \left\{ \begin{array}{l} \text{Sales revenue} - \text{purchase cost} - \text{Ordering cost} \\ - \text{Preservation cost} - \text{Screening cost} - \text{Holding cost} \end{array} \right.$$

$$= \frac{1}{T} \left\{ \begin{array}{l} P \int_0^T (\alpha + \beta I(t)) P^{-\eta} e^{-Rt} dt + P_s Q \in e^{-Rt_1} - \frac{C \alpha P^{-\eta}}{\theta(1-f) + \beta P^{-\eta}} \left[e^{(\theta(1-f) + \beta P^{-\eta})T} - 1 \right] \\ - \left[\int_0^T h I(t) e^{-Rt} dt - h \in Q e^{-Rt_1} \right] - C_s Q e^{-Rt_1} - u T e^{-RT} - A \end{array} \right. \quad (4)$$

4. Solution methodology:

The objective of this study is to maximize the profit with respect to optimal selling price, optimal length of replenishment cycle and optimal preservation technology investment.

By solving $\frac{\partial \pi}{\partial P} = 0, \frac{\partial \pi}{\partial T} = 0, \frac{\partial \pi}{\partial u} = 0$, we obtain the optimum value of (P, T, u) . Generated equations are solved by mathematical software.

5. Numerical example:

To illustrate the solution procedure, in this section we give the following example:

Example: In this example we consider the reduced deterioration rate $f(u) = 1 - e^{-mu}$, $m > 0$ and parametric values are as follows:

$\theta = 30\%$, $\alpha = 10000$ units/year, $\beta = 20\%$, $h = \$3$ /unit/year, $m = 5\%$, $C = \$10$ /unit/year, $A = \$50$ /order, $\eta = 1.25$, $C_s = \$3$ /unit/year, $\in = 10\%$, $\lambda = 30\%$, $r = 12\%$, $k = 6\%$, $\xi = 0.1$. We obtain optimal cycle time $T^* = 0.4521$ years, optimal selling price $P^* = \$79.07$ per unit and optimal investment $u^* = \$7.92$ per unit to reduce deterioration in the inventory system. Hence, seller's optimal profit is $\pi(P^*, T^*, u^*) = \$3566.2$ with purchase of $Q^* = 24.9$ units. Concavity of profit function with respect to two variables by taking optimum value of third variable are exhibited in Figs. 1 – 3.

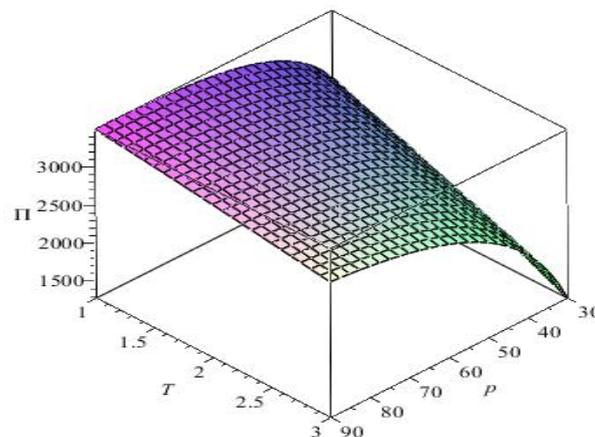


Figure 1: Change in profit with respect to T and P

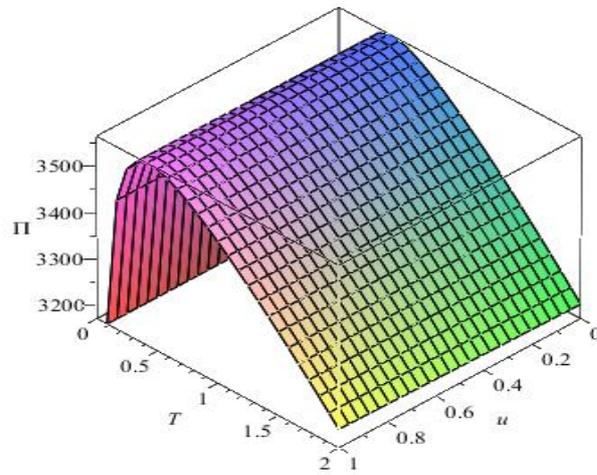


Figure 2: Change in profit with respect to T and u

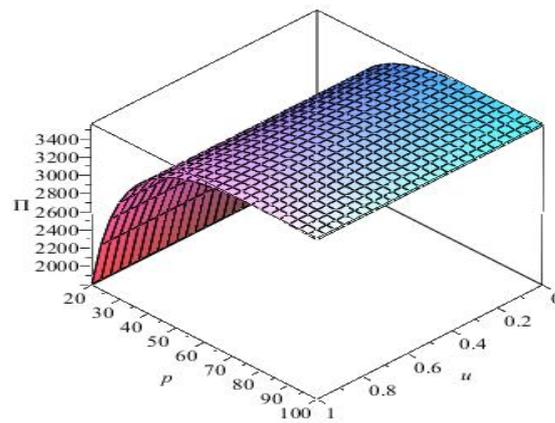


Figure 3: Change in profit with respect to u and P

Effects and results of changing the values of the system parameters on decision variables and profit are shown in Figs 4 to 7 and Table 1.

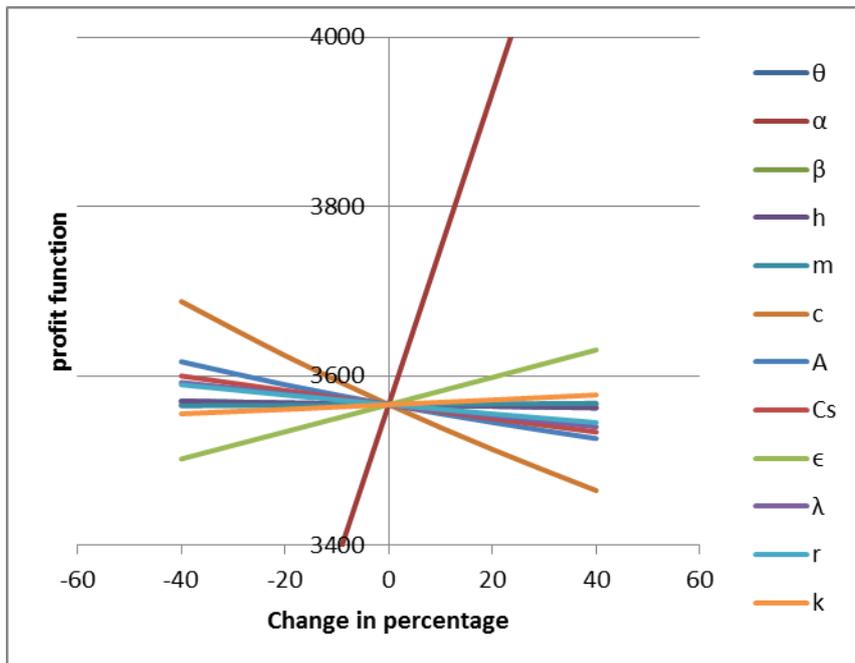


Figure 4: Change in profit function with respect to change in other parameters

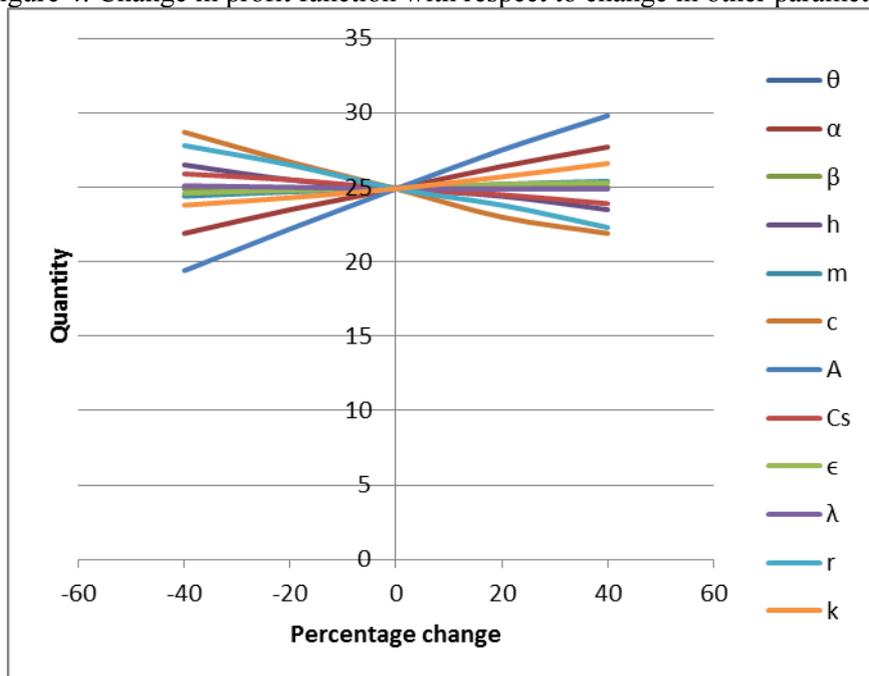


Figure: 5 Change in quantity with respect to change in different parameters

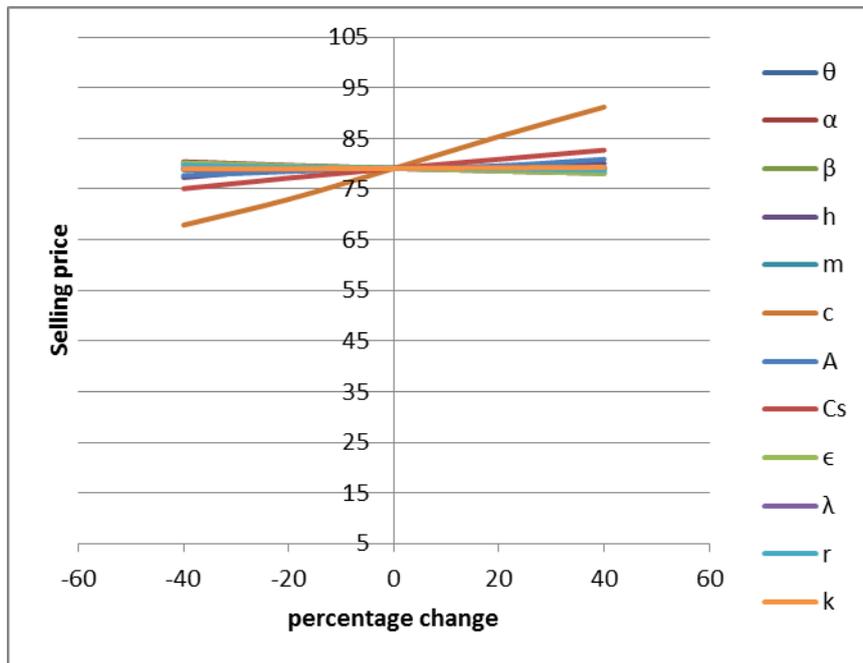


Figure 6: Change in selling price with respect to variation in different parameters.

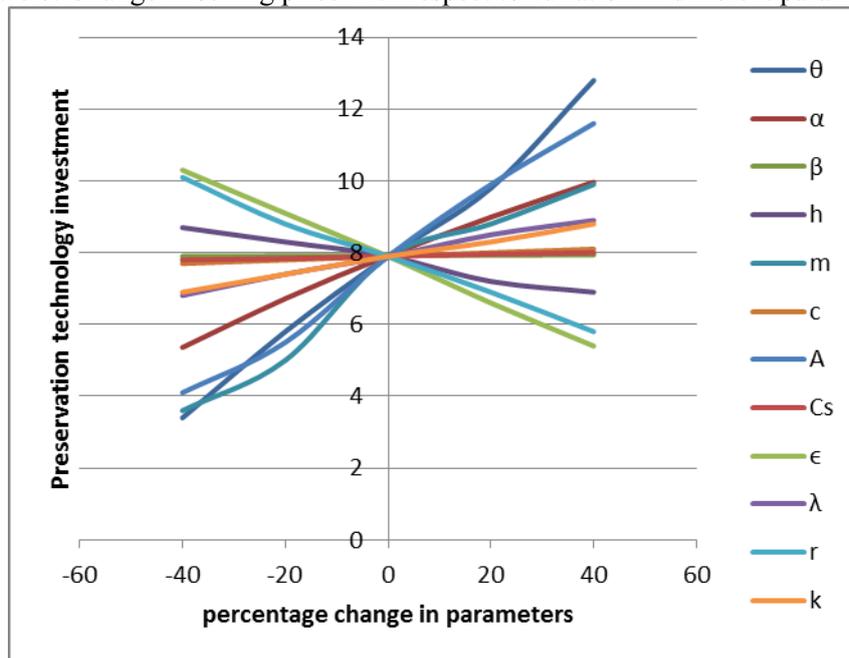


Figure 7: Percentage change in parameters versus preservation technology investment

| Decision variable | Sensitively positive | Sensitively negative | Ineffective | Negligible |
|-------------------|------------------------------------|------------------------------------|-------------|----------------------------------|
| π | α, ϵ, k | $\theta, h, C, A, C_s, \lambda, r$ | NA | β, m |
| Q | $\alpha, m, A, \epsilon, k$ | h, C, C_s, r | θ | β, λ |
| P | h, C, A, C_s | α, m | θ | $\beta, \epsilon, \lambda, r, k$ |
| u | $\theta, \alpha, m, A, \lambda, k$ | h, ϵ, r | - | β, C, C_s |

Table 1: Effectiveness of different parameters on decision variables

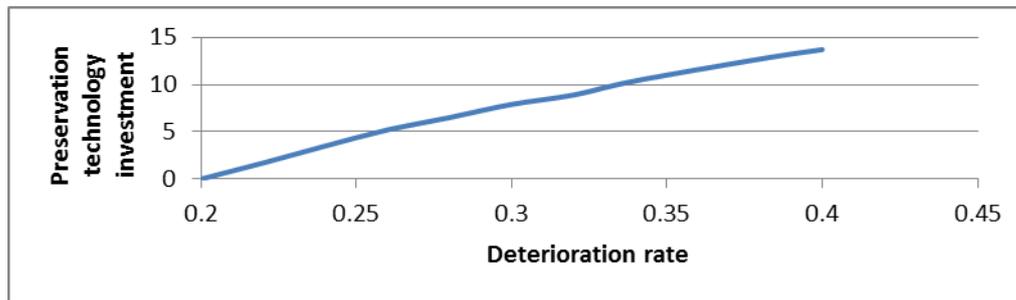


Figure 8: Deterioration rate versus preservation technology investment

Fig.8 shows the relation between deterioration rate and preservation technology investment. The trend suggests that higher investment is required for high deterioration of items in inventory system and investment is not required when the rate is marginal.

6. Conclusion:

The main objective of this paper to optimize the profit function with respect to replenishment cycle time, preservation technology investment and selling price when demand is price-sensitive and stock-dependent with inflationary effects. All the above graphs show optimality obtained in each case. It is observed that the profit function is positively affected by scale demand (α), proportion of defective items (ϵ), inflation rate per unit time (k) and negatively affected by deterioration rate (θ), inventory holding cost (h), purchase cost (C), screening cost (C_s), proportion decrease in selling price for defective items (λ), discounted rate deciding the time value of money (r). Also, positive effect of scale demand (α), proportion of defective items (ϵ), inflation rate per unit time (k) and replenishment cost per order (A) seen on quantity. Quantity is negatively affected by holding cost (h), purchase cost (C), screening cost (C_s) and discounted rate deciding the time value of money (r). Overall summary is given in table 1.

Deterioration versus preservation technology suggests high investment required if product is highly deteriorated in nature. Also, investment is not required if deterioration is less or marginable.

References:

- [1] Buzacott, J.A. (1975), 'Economic Order Quantities with Inflation', *Operational Research Quarterly*, 26, 553-558
- [2] Cardenas-Barron, L.E (2000), 'Observation on Economic Production Quantity Model for Items with Imperfect Quality', *International Journal of Production Economics*, 67, 201.
- [3] Cardenas-Barron, L.E (2009a), 'Optimal Ordering Policies in Response to a Discount Offer: Extensions', *International Journal of Production Economics*, 122, 774-782.
- [4] Cardenas-Barron, L.E (2009b), 'Optimal Ordering Policies in Response to a Discount Offer: Corrections', *International Journal of Production Economics*, 122, 783-789.
- [5] Cardenas-Barron, L.E. (2011), 'The Derivation of EOQ/EPQ Inventory Models with Two Back orders Costs Using Analytic Geometry and Algebra', *Applied Mathematical Modelling*, 35, 2394-2407.
- [6] Cardenas-Barron, L.E., Smith, N.R, and Goyal, S.K. (2010), 'Optimal Order Size to Take Advantage of a One-time Discount Offer with Allowed Backorders', *Applied Mathematical Modeling*, 34, 1642-1652.
- [7] Chen, Y.R. and Dye, C.Y. (2013), 'Application of particle swarm optimization for solving deteriorating inventory model with fluctuating demand and controllable deterioration rate', *International Journal of Systems Science*, 44 (6): 1026-1039.

- [8] Chung-Yuan Dye, Tsu-Pang Hsieh (2013), 'A particle swarm optimization for solving lot-sizing problem with fluctuating demand and preservation technology cost under trade credit', *J. Global Optimization*, 55(3), 655-679.
- [9] Datta, T.K., and Pal, A.K. (1991), 'Effects of Inflation and Time value of Money on an Inventory Model with Linear Time-Dependent Demand Rate and Shortages', *European Journal of Operational Research*, 52,326-333.
- [10] Ghare, P.M., and Schrader, S.K. (1963), 'A Model for Exponentially Decaying Inventory', *Journal of Industrial Engineering*, 14, 38-243.
- [11] Goyal, S.K., and Cardenas-Barron, L.E. (2002), 'Note on: Economic Production Quantity Model for Items with Imperfect Quality- A Practical Approach', *International Journal of Production Economics*, 77, 85-87.
- [12] Hariga, M.A (1995), 'An EOQ Model for Deteriorating Items with Shortages and Time-varying Demand', *Journal of the Operational Research Society*, 46,398-404.
- [13] Hariga, M.A., and Ben-Daya, M. (1996), 'Optimal time varying Lot-sizing Models Under inflationary Conditions', *European Journal of Operational Research*,89,313-325.
- [14] Hsieh and Dye (2013), 'A production-inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time', *Journal of Computational Applied Mathematics* (239), 25-36.
- [15] Jaggi and et.al. (2012), 'Effects of inspection on retailer's ordering policy for deteriorating items with time-dependent demand under inflationary conditions', *International Journal of Systems Science*, 44:9, 1774-1782.
- [16] Jaggi, C.K., and Mittal, M. (2007), 'An EOQ Model for Deteriorating Items with Time-dependent demand Under Inflationary Conditions', *Indian Journal of Mathematics and Mathematical Sciences*,3,139-147.
- [17] Jaggi, C.K., and Mittal, M. (2011), 'Economic Order Quantity Model for Deteriorating Items with Imperfect Quality', *Revista Investigation Operational*, 32,107-113.
- [18] Misra, R.B. (1975), 'A study of Inflationary Effects on Inventory Systems', *Logistic Spectrum*, 9, 260-268.
- [19] Porteus, E.L. (1986), 'Optimal Lot Sizing, Process Quality Improvement and Setup Cost Reduction', *Operations Research*, 34,137-144.
- [20] Raafat, F., Wolfe, P.M., and Eldin, H.K.(1991), 'An Inventory Model for Deteriorating Items', *Computers and Industrial Engineering*,20,89-94.
- [21] Ray, J., and Chaudhuri, K.S. (1997), 'An EOQ Model with Stock-dependent Demand, Shortage, Inflation and Time discounting', *International Journal of Production Economics*, 53,171-180.
- [22] Rosenblatt, M., and Lee, H. (1986), 'Economic Production Cycles with Imperfect Production Processes', *IIE Transactions*, 18, 48-55.
- [23] Salameh, M.K., and Jaber, M.Y. (2000), 'Economic Production Quantity Model for Items with Imperfect Quality', *International Journal of Production Economics*, 64, 59-64.
- [24] Sana, S., and Chaudhuri, K.S. (2003), 'An EOQ Model with time-dependent Demand, Inflation and Money value for a Warehouse Enterpriser', *Advanced Modern Optics*, 5, 135-146.
- [25] Sarkar, B.R., and Pan, H. (1994), 'Effects of Inflation and Time value of Money on Order Quantity and Allowable Shortage', *International Journal of Production Economics*, 34, 65-72.
- [26] Schwaller, R.L. (1988), 'EOQ under Inspection Costs', *Production and Inventory Management*, 29, 22-24.
- [27] Widyadana, G.A., Cardenas-Barron, L.E., and Wee, H.M. (2011), 'Economic Order Quantity Model for deteriorating Items and Planned Backorder Level', *Mathematical and Computer Modeling*, 54, 1569-1575.

- [28] Yong, H. and Huang, H. (2013), 'Optimizing Inventory and Pricing Policy for Seasonal Deteriorating Products with Preservation Technology Investment', *Journal of Industrial Engineering*, vol.2013, Article ID 793568, 7 pages, doi:10.1155/2013/793568
- [29] Zhang, X. and Gerchak, Y. (1990), 'Joint Lot-Sizing and Inspection Policy in an EOQ Model with Random Yield', *IIE Transaction*, 22, 41-47.