Quantum partner-dance in the ^{12}C + ^{12}C system yields sub-Coulomb fusion resonances

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Abstract. A preliminary study of the ^{12}C + ^{12}C sub-Coulomb fusion reaction using the time-dependent wave-packet method is presented. The theoretical sub-Coulomb fusion resonances seem to correspond well with observations. The present method might be a more suitable tool for expanding the cross-section predictions towards lower energies than the commonly used potential-model approximation.

1. Introduction

The ^{12}C + ^{12}C fusion reaction plays a key role in the chemical evolution of the Universe [1].The fusion cross section at energies near the Gamow peak ($\sim$ 1.5 MeV) is critical for the stellar carbon-burning rates, which is usually obtained by extrapolating high-energy fusion data [2, 3, 4], as direct measurements are very difficult to carry out at very low energies ($\leq$ 3 MeV). The observed fusion resonances make the extrapolation very uncertain [5, 6, 7].

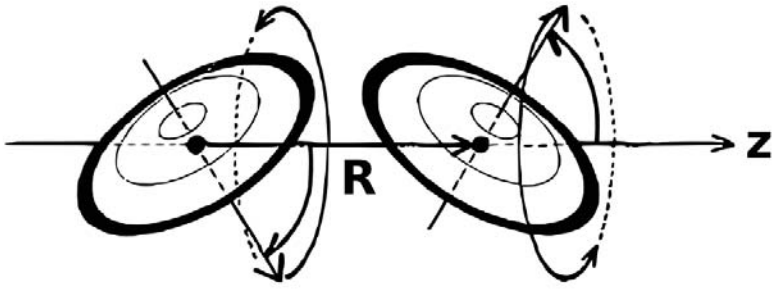

Figure 1. The ^{12}C + ^{12}C nuclear molecule, exhibiting collective excitation modes.

The resonances may be mainly related to collective excitation modes in the dinuclear system, when the two ^{12}C nuclei come into contact (Fig. 1). The ^{12}C intrinsic symmetry axis vibrates and rotates with respect to the internuclear axis. The single-particle molecular shell structure

at small internuclear distances is very sensitive to the alignment of the ^{12}C nuclei [8]. Non-axial symmetric configurations preserve the individuality of the overlapping nuclei, while this is not the case for the axial symmetric configuration. The former favors re-separation, and the latter fusion. The competition among these configurations, as a function of the incident energy and orbital angular momentum, should result in molecular resonance structures in the fusion excitation curve.

2. Method

This picture is here quantified using the time-dependent wave-packet (TDWP) method which has not been much exploited in nuclear physics [9] unlike chemical physics [10]. This method involves three steps [10]:

(1) the definition of the initial wave function $\Psi(t = 0)$,

(2) the propagation $\Psi(0) \rightarrow \Psi(t)$, dictated by the time evolution operator, $\exp(-i\hat{H}t/\hbar)$, where $\hat{H}$ is the total Hamiltonian,

(3) the calculation of molecular collective states and the fusion cross section from the time-dependent wave function, $\Psi(t)$.

The wave function and the Hamiltonian are represented in a multi-dimensional numerical grid. In this work, these are considered a function of five collective coordinates (Fig. 1): the internuclear distance R, and the (θ_1, ϕ_1) and (θ_2, ϕ_2) spherical angles of the ^{12}C nuclei symmetry axis, thus reducing the complexity of the quantum many-body reaction problem. Moreover, the wave function is *not* expanded in any intrinsic basis (e.g., rotational or vibrational states of the individual nuclei), but it is calculated directly. The outgoing-wave-boundary condition at large internuclear distances as well as the *irreversible* process of fusion at small internuclear distances (usually described with an ingoing-wave-boundary condition) are here treated with the absorbing-boundary-condition [11]. The low-energy collision is described in the rotating center-of-mass frame within the nuclear molecular picture [12].

A simple case is the ^{16}O + ^{16}O collision, as the nuclei are spherical and inelastic coupled-channels effects are rather small, e.g, see Fig. 1 in Ref. [13]. It is a one-dimensional problem that involves only the internuclear-distance coordinate, R, whose solution within the present method can be compared to that of the time-independent Schrödinger equation with the usual boundary conditions [14, 15].

Figure 2 shows the transmission-coefficient excitation function for the total angular momentum $J = 0\hbar$, which is determined by two methods: solving the time-independent Schrödinger equation [15] (solid line) and employing the present method (squares). The agreement between the two methods is very good, demonstrating both the reliability of the time-dependent wave-packet method and how a single wave-packet propagation can provide accurate transmission coefficients over a range of collision energies [10].

3. Results and Discussion

Figure 3 shows cuts of the collective potential energy surface (PES) which is calculated using the finite-range liquid drop model [16] with nuclear shapes of the two-center shell model [8]. Fusion is here determined by two processes treated simultaneously: (i) the tunneling through the many Coulomb barriers in Fig. 3, and (ii) the ^{12}C nuclei re-alignment in the potential pockets. The 90-90 alignment (dashed line) dominates the Coulomb barriers penetrability, but the 0-0 alignment (solid line) is critical for fusion. Non-zero J partial waves substantially contribute to the fusion cross section, as the centrifugal contribution to the 90-90 Coulomb barrier is small for low $J \leq 6\hbar$ and the J-dependent Coriolis interaction, whose strength increases with J and the system compactness, strongly drives the non-axial symmetric configurations towards the potential pocket of the axial symmetric one (solid line), where fusion occurs.

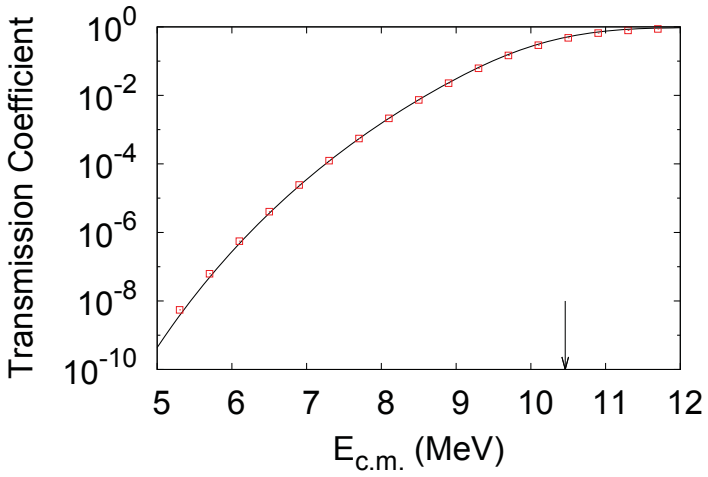

Figure 2. Excitation function of the transmission coefficient through the Coulomb barrier (the Broglia-Winther potential in Ref. [15]) for the ^{16}O + ^{16}O central collision, calculated with two methods: (1) solving the time-independent Schrödinger equation (solid line), and (2) the time-dependent wave-packet method (squares). The agreement between the two methods is very good. The arrow indicates the Coulomb-barrier energy.

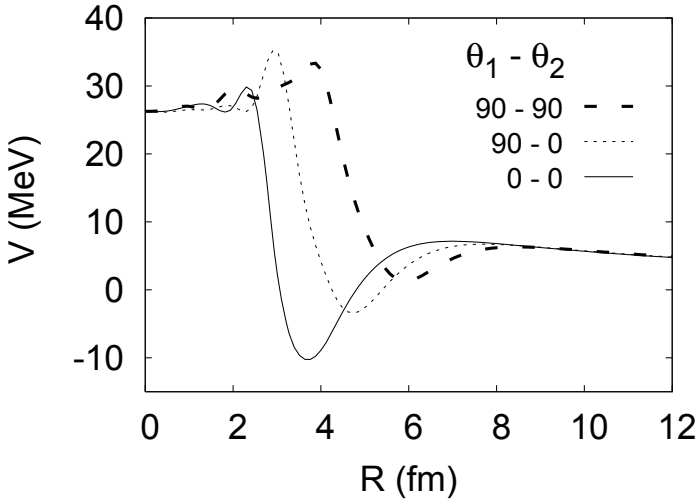

Figure 3. Cuts of the ^{12}C + ^{12}C collective PES, $V(R, \theta_1, \phi_1 = 0, \theta_2, \phi_2 = 0)$, as a function of the nuclei separation and alignment. The 90-90 alignment (dashed line) facilitates the access by tunneling to the potential pockets ($3 - 6$ fm). These are explored by the system, guided by the kinetic-energy operator [17]. The Coriolis force drives non-axial symmetric configurations towards the potential pocket of the axial symmetric configuration (solid line), where fusion occurs by a strong absorption.

The molecular resonance states with a total width of $\sim 100 - 180$ keV are presented in Fig. 4, which represent doorway states that irreversibly decay into more complex compound-nucleus states (simulated by the strong absorption providing the *inclusive* fusion probability) and the

^{12}C + ^{12}C continuum scattering states.

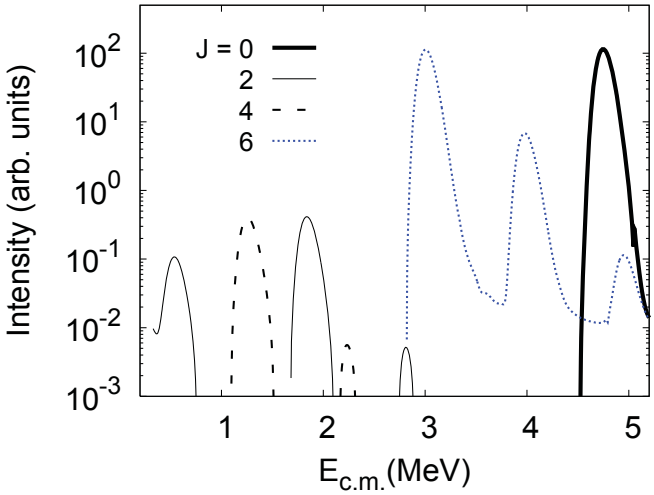

Figure 4. Energy spectrum of the ^{12}C + ^{12}C system in the potential pockets of Fig. 3. The peaks are molecular resonances with a given spin and total width of $\sim 100-180$ keV. The widths relate to the total time interval spent by the dinuclear system in the potential pockets. These resonances are doorway states for fusion, which decay into more complex compound-nucleus states and the ^{12}C + ^{12}C scattering states. Various resonances exist near the Gamow peak energy (1.5 MeV).

These molecular collective states are reflected in the total fusion excitation curve, as presented in Fig. 5 through the S-factor that includes $J \leq 8\hbar$. The fusion cross section, $\sigma_{fus}(E)$, is calculated taken into account the identity of the interacting nuclei and the parity of the radial wave function (only even partial waves J are included), i.e., $\sigma_{fus}(E) = \pi\hbar^2/(2\mu E)\sum_J(2J+1)(1+\delta_{1,2})P_J(E)$, where μ is the reduced mass, E is the incident energy in the center-of-mass frame and P_J is the partial fusion probability. The S-factor is $S(E) = \sigma_{fus}(E)E\exp(2\pi\eta)$, where the Sommerfeld parameter $\eta = (\mu/2)^{1/2}Z_1Z_2e^2/(\hbar E^{1/2})$ and Z_i is the charge number of the ^{12}C nuclei. The features resulting from the preliminary TDWP calculations (solid line) are consistent with those observed in the experimental data [2, 4] (squares). While the widths of the resonances show good agreement, the amplitudes are noticeably different: the theoretical predictions are too low in the higher energy range and too high in the lower energy range. This issue could be clarified using a bigger numerical grid (for checking convergence) and incorporating shell and pairing corrections into the collective mass and PES [15].

4. Summary

Using time-dependent wave-packet dynamics within a nuclear molecular picture, a quatitative study of the ^{12}C + ^{12}C sub-Coulomb fusion has been presented. Preliminary calculations are very promising, indicating a close correlation between molecular collective states and fusion. The present method might be a more suitable tool for expanding the cross section predictions towards lower energies than the usually employed potential-model approach.

Acknowledgments

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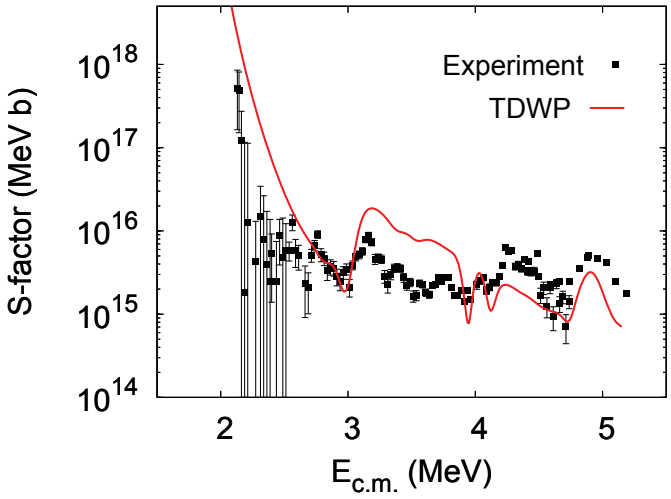

Figure 5. S-factor excitation function. There is a good *qualitative* agreement between measurements [2, 4] and preliminary TDWP calculations, highlighting the symphysis of molecular structure and fusion.

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