

# Astrometric tests of General Relativity in the Solar system

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**Abstract.** Micro-arcsec astronomy is able to verify the predictions of theoretical models of gravitation at a level adequate to constraint relevant parameters and select among different formulations. In particular, this concerns the weak field limit applicable to the Sun neighborhood, where competing models can be expressed in a common framework as the Parametrised Post-Newtonian and Parametrised Post-Post-Newtonian formulations. The mission Gaia is going to provide an unprecedented determination of the  $\gamma$  PPN parameter at the  $10^{-6}$  level. Other recently proposed concepts, as GAME, may improve the precision on  $\gamma$  by one or two orders of magnitude and provide constraints on other crucial phenomenological aspects. We review the key concepts of astrometric tests of General Relativity and discuss a possible development scenario.

**Keywords:** Fundamental astronomy and astrophysics; General Relativity and Gravitation; Experimental tests of gravitational theories

## 1. Introduction

General Relativity (GR) and competing theories of gravitation are often expressed in the weak field conditions applicable to the Solar System through the Parametrized Post-Newtonian (PPN) or Parametrized Post-Post-Newtonian (PPPN) formulation, in order to provide a convenient framework for comparison of their predictions and for validation with respect to experimental results [1]. One of the key effects of a gravitation source is often summarised as a “deformation of space-time”, usually expressed in terms of the most relevant PPN parameters  $\gamma$  and  $\beta$ .

The most recent determinations of the deflection angle value, and/or of  $\gamma$ , are based mostly on time delay on artificial probes, e.g. Cassini [3], reaching in the latter case a precision  $\sigma(\gamma)/\gamma \simeq 10^{-5}$ . The best precision on  $\beta$  is provided by “grand fit” of the Solar System dynamics, also including the ranging data from space probes, and reaching the precision range on  $\sigma(\beta)/\beta$  of  $3 \times 10^{-5}$  [4],  $2 \times 10^{-5}$  [5], and  $8 \times 10^{-5}$  [6]. In the near future, experiments are expected to improve on our knowledge of both  $\beta$  and  $\gamma$  by about one order of magnitude, respectively by the missions Gaia [7, 8] ( $\sigma(\gamma)/\gamma \simeq 10^{-6}$ ) and Solar Orbiter [9] ( $\sigma(\beta)/\beta \simeq 10^{-5}$ ) of the European Space Agency (ESA).

For a recent overview of testing some aspects of relativistic gravity in the Solar System, see, e.g., [10] and references therein.

Beyond this precision level, the reference to PPN parameters may be considered inadequate, and the model for data reduction shall likely be formulated directly in PPPN formalism; this



also implies direct verification of stringent upper bounds on some Lorentz invariance violation parameters [11]. Besides, GR can act as a cosmological attractor for scalar-tensor theories of gravity, with expected deviations from unity of  $\gamma$  in the  $10^{-5}$  to  $10^{-7}$  range [12]. During the last decade, strong observational evidence of accelerated expansion of the Universe has been provided. This is interpreted in the concordance  $\Lambda$ CDM scenario as the effect of a long range perturbation of visible matter gravity, generated by the so-called Dark Energy. Previous data at different scale length are explained with the existence of non-barionic Dark Matter (e.g. galaxy rotation curves), or even with modifications of GR (e.g. flyby anomalies [13]). However, there are claims that these data could also be explained with a modified version of GR, using a curvature invariant  $R$  no longer constant in the Einstein equations ( $f(R)$  gravity theories) [14]. Present experimental data are not sufficiently precise to allow discrimination between these scenarios, which requires e.g. a  $10^{-7}$  level, or better, measurement of  $\gamma$  [15].

## 2. Astrometric Tests

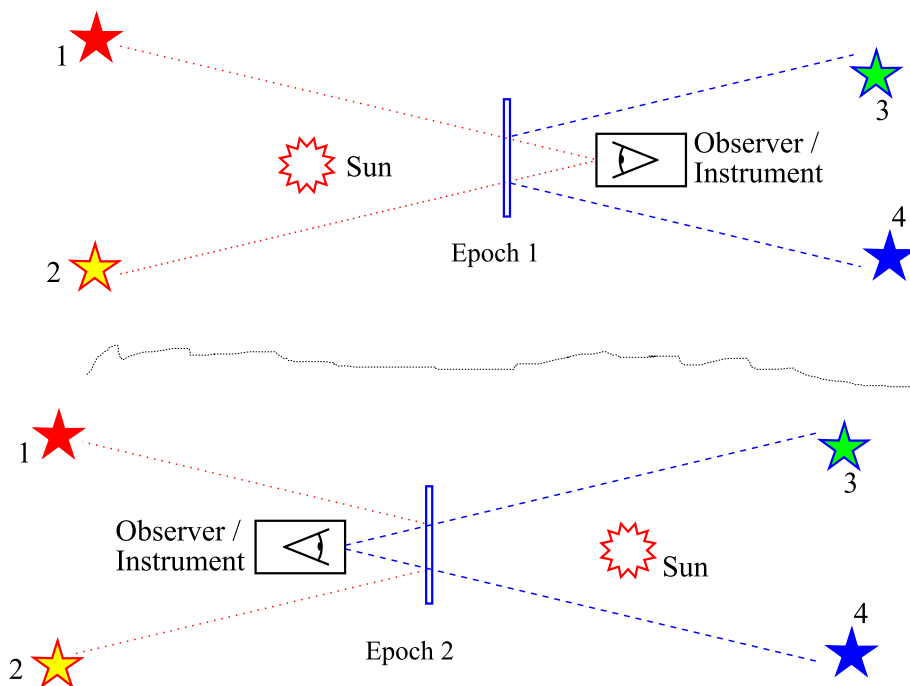
Astrometry is intrinsically suited to be exploited for precision improvement on both  $\gamma$  and  $\beta$ , because the measurement of relative positions over time of natural sources (mostly bright stars) can be used to achieve a better determination of the light deflection angle, on one side, and of planetary ephemerides, on the other. In particular, the differential estimate of  $\gamma$  requires the measurement over large angles with respect to the gravitation source, at least to relate deflected and undeflected objects. Other modern experiments, based on atomic clocks, accelerometers and ranging [16, 17, 18], are also designed for the same science goal of gravitation theories verification, but they exploit the temporal part of the effects. The different approaches are therefore complementary, and the independent results can be used to validate each other.

The classical astrometric parameters are positions, proper motion (tangential) and parallax of objects over the celestial sphere; the latter, corresponding to the apparent target displacement induced by the Earth's orbital motion, may be used to associate the angular measurements to linear coordinates. Complemented with radial velocities, this provides a full kinematical description of the observed sample. Such classical mechanics description requires proper relativistic formulation in order to take correctly into account the motion of the observer and of the sources, as well as the gravitational effects of space-time curvature [19, 20].

Light deflection reaches a peak value of  $1''.74$  at the solar limb, and decreases rapidly at increasing angular distance, and therefore, in order to estimate the  $\gamma$  parameter at the  $10^{-6}$  level and beyond, micro-arcsec ( $\mu$ as) level measurements of relative star positions are required at a few degrees from the Sun. Averaging over many stars, it is possible to improve on the limited individual measurement precision (due to the source magnitude and instrument resolution).

Gaia [<http://sci.esa.int/gaia/>], to be launched in the next few months, is based on twin telescopes observing stellar fields around two lines of sight (LOS) on the equatorial plane of a spinning satellite; the combination of rotation, precession and orbital motion will provide repeated coverage of the celestial sphere over the five year mission lifetime. Simultaneous observation is expected to provide large benefits in the mitigation of systematic errors, in particular removing as common mode the satellite attitude disturbances in the differential measurement of angular separation among stars in either field. The two fields of view are also folded by a beam combiner onto the same CCD mosaic detector; introducing a common mode instrument sub-system, other systematic error contributions from hardware and operation are expected to be factored out. The set of measurements is then processed to determine, in a global solution, the astrometric parameters of about one billion stars, and the instrument calibration parameters. This process also includes, as a global parameter, the value of  $\gamma$  providing the space-time curvature value best matching the whole data set.

The Gravitation Astrometric Measurement Experiment (GAME) [21] is a scalable concept, based on proven technology, for a space mission aimed at determination of the key parameters  $\gamma$



**Figure 1.** Differential measurement by simultaneous imaging of high and low deflection fields

and  $\beta$ , in the PPN formulation of gravitation theories. The expected precision for the medium class mission version is in the range  $\sigma(\gamma)/\gamma \simeq 10^{-8}$  and  $\sigma(\beta)/\beta \simeq 10^{-6}$ , corresponding to an improvement of one to two orders of magnitude with respect to the best experimental results foreseen for the next future.

GAME relies heavily on differential measurement, observing simultaneously field pairs close to the Sun, i.e. in high deflection conditions (fields 1 and 2 in figure, top panel), and away from the Sun, affected by low deflection (resp. fields 3 and 4). The measurement of the same fields is repeated six months later, when the Sun, due to Earth's orbital motion, is between fields 3 and 4, which become the high deflection fields. The low deflection measurement provides the instrument calibration and monitoring of its relevant parameters. This approach provides a convenient first-order factorisation of the  $\gamma$  parameter in the measurement combination, i.e. a much simpler model with reduced correlation among the unknowns, thus ensuring a much better control of systematic errors.

By observing regions close to the Ecliptic plane, GAME is also suited to measure repeatedly the position of the main Solar System planets with respect to background stars, thus supporting a high precision “grand fit” solution, hence a better estimate of the  $\beta$  parameter. Also, a significant fraction of the mission lifetime may be devoted to astrometric characterisation of already detected exo-planetary systems [22], in order to provide fundamental data on their multiplicity and structure. Simpler versions of GAME may be implemented on a comparably small scale, with proven technology, still retaining a good performance level ( $\sigma(\gamma)/\gamma \simeq 10^{-7}$  and  $\sigma(\beta)/\beta \simeq 6 \times 10^{-6}$ ), thus paving the way for a future full-fledged design.

### 3. Conclusions

A future micro-arcsec astrometry mission may benefit from the ongoing experience in Gaia to devise a dedicated experiment for high precision determination of the parameters relevant to gravitation theories, aiming at verification of General Relativity. The GAME concept exploits

multiple differential measurement scheme to control systematic errors, and addresses both  $\gamma$  and  $\beta$  parameters of the PPN formulation, with a potential improvement by at least one order of magnitude on each of them with respect to current and forthcoming experiments, thus constraining by two orders of magnitude the theory “phase space”.

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