

# Magnetic flux quantum in a Superconducting concave/convex disk

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**Abstract.** The vortex state in a thin mesoscopic superconducting disk with a concave/convex surface is found theoretically. It is assumed that the outer edge of the sample is in contact with a metallic material. This configuration decreases the Bean-Livingston surface barrier energy, that is, it allows the vortex entry into the sample at lower magnetic field. In this work, we solve numerically the Ginzburg Landau equations with the metal/superconducting boundary condition using the link variable method in polar coordinates. It is shown that for a determined value for the deGennes parameter the superconductor becomes a type I superconductor and the irregular surface allows the vortex giant formation. The value of the thermodynamical properties decreases with this boundary condition and kind of surface.

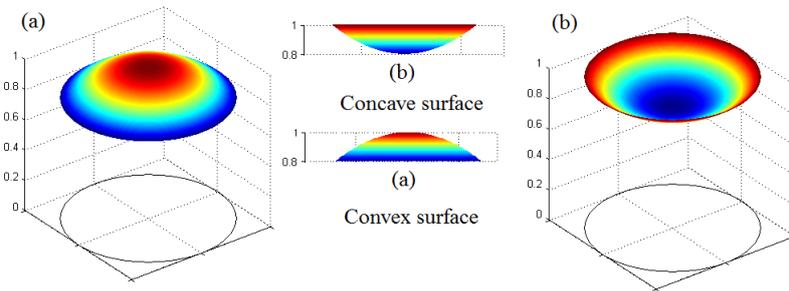
## 1. Introduction

In the last years, mesoscopic superconducting heterostructures have been studied from both experimental and theoretical point of view. The critical superconducting parameters can be controlled for mesoscopic samples and the vortex matter can be influenced by their geometry, boundary conditions and structural defects. In addition, a giant vortex state can occur in very confined geometries [1, 2]. In superconducting disks the irregularity of the surface can be present and they can act as pinning or antipinning centers. In previous works, we studied the effect of weak defects on the vortex configurations in a circular geometry. We found that the vortex configurations are strongly influenced by the geometry of the defects on the sample [3, 4]. In this paper we will study the effects of a concave/convex surface with a metallic interface on the formation of vortices and how it influences the induced superconducting current for a thin mesoscopic disk in the presence of a perpendicular external magnetic field. To this end, we use the time dependent Ginzburg-Landau (TDGL) equations.

## 2. Theoretical Formalism

Many systems in superconductivity are described by the TDGL equations. These are a set of two equations which couples the order parameter  $\psi$  and the vector potential  $\mathbf{A}$ , which are the two fundamental quantities describing the superconducting state. The non-dimensional version





**Figure 1.** Layout of the studied samples. Superconducting disk with (a) concave and (b) convex surface.

of these equations is given by [5, 6, 7]:

$$\frac{\partial\psi}{\partial t} = -(i\nabla + \mathbf{A})^2\psi + (1 - T)\psi(1 - |\psi|^2) \quad (1)$$

$$\frac{\partial\mathbf{A}}{\partial t} = \mathbf{J}_s - \kappa^2\nabla \times \mathbf{h} \quad (2)$$

$$(3)$$

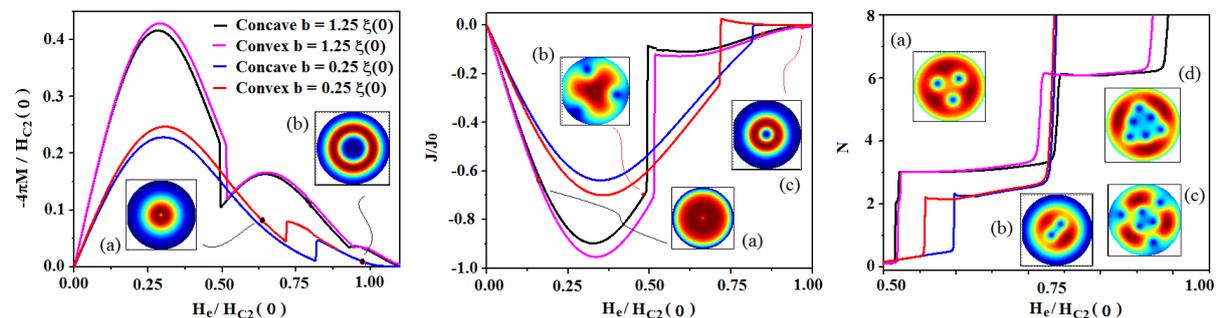
$\mathbf{J}_s = (1 - T)\text{Re}[\bar{\psi}(-i\nabla - \mathbf{A})\psi]$  is the supercurrent density measured in units of  $J_0 = \hbar c^2/8\pi e\xi$ ,  $T$  is temperature in units of the critical temperature  $T_C$ , order parameter is in units of  $\psi_\infty$  of the Meissner state, length is in units of coherence length at zero temperature  $\xi(0)$  and fields in units the bulk upper critical field  $H_{c2}(0)$ ;  $\kappa = \lambda(0)/\xi(0)$  is the material dependent Ginzburg-Landau parameter. In the limit of very thin disk, the first equation 1 for the variable surface of the sample can be rewritten as:

$$\frac{\partial\psi}{\partial t} = -\frac{1}{\tau}(i\nabla + \mathbf{A}_e) \cdot \tau(i\nabla + \mathbf{A}_e)\psi + (1 - T)\psi(1 - |\psi|^2).$$

$\tau$  is just a function which describes the variability of the surface of the sample. The magnetic field is considered nearly uniform inside the superconductor  $\mathbf{H}_e = \nabla \times \mathbf{A}_e$ , where  $\mathbf{H}_e$  is the external applied field; in polar coordinates  $\tau(r, \theta) = 1 - 0.2(r/R)^2$  for the concave surface and  $\tau(r, \theta) = 0.8 + 0.2(r/R)^2$  for the convex surface, where  $R$  is the radius of the disk. We have taken a disk of radius  $R = 5\xi(0)$  (see Fig. 1). We will assume a metallic/superconducting interface, that is,  $(-i\nabla - \mathbf{A})\psi \cdot \mathbf{n} = (i/b)\psi|_n$  [7].

### 3. Results and Discussion

The parameters used in our numerical simulations were:  $\kappa = 2.17$ , which is a value for to a thin film of Nb with thickness  $d$  (assuming  $\xi(0) = 380$  nm,  $T_c = 3.7K$ ,  $d \approx 60$  nm),  $T = 0$ . The largest unit cell size at the edge of disk was  $0.25 \times 0.25$ . We used the values  $b = 1.25\xi(0)$  and  $b = 0.25\xi(0)$  for the external metallic interfaces. In Fig. 1 we plot the layout of the studied samples, a superconducting disk with (panel (a)) concave and (panel (b)) convex surface. In Fig. 2 we depict the magnetization  $-4\pi M$  (left), supercurrent density  $J$  (middle) and vorticity  $N$  (right) and the contour plot of the order parameter  $|\psi|$  (insects) as a function of the magnetic field for a disk with concave/convex surface and metallic interface simulated by two values of  $b$  previously given, as a function of the magnetic field  $H_e$ . We can see from these figures that the first critical field  $H_{c1}$  for the nucleation of vortices depends strongly on the boundary conditions and slightly on the kind of surface. On the other hand, the second critical field  $H_{c2}$



**Figure 2.** (Color online) Contour plot of magnetization, supercurrent and vorticity for a disk with concave and convex surface and metallic interface  $b = 1.25\xi(0)$  and  $b = 0.25\xi(0)$ , as a function of the magnetic field  $H_e$ .

depends just of the boundary condition. We have  $H_{c1} = 0.489, 0.513$  and  $H_{c2} = 1.086$  for a concave and convex surface, respectively, when we use  $b = 1.25\xi(0)$ , and  $H_{c1} = 0.715, 0.815$  and  $H_{c2} = 1.00$  for a convex and concave surface respectively when we use  $b = 0.25\xi(0)$ . In the insets of Fig. 2 (left) we show the contour plot of the order parameter for (a)  $N = 0$  at  $H_e = 0.625$  and (b)  $N = 2$  at  $H_e = 0.98$  for  $b = 0.25\xi(0)$ , considering the stationary state both surface geometry. In the insets of Fig. 2 (middle) we show the contour plot of the order parameter for (a)  $N = 0$  at  $H_e = 0.135$  in a stationary state, (b)  $N = 3$  at  $H_e = 0.488$  for  $b = 1.25\xi(0)$  in a non-stationary state and (c)  $N = 8$  at  $H_e = 0.9$  for both surfaces and  $b$  values. Finally in the insets of Fig. 2 (right) we show the contour plot of the order parameter for (a)  $N = 3$  and (c,d)  $N = 6$  for a convex surface and (b)  $N = 2$  for a concave surface for both values of  $b$ . Due to the boundary conditions the first three vortices enter in the sample at lower magnetic field for  $b = 1.25\xi(0)$  case. By increasing the magnetic field three more vortices enter into the sample one by one forming a giant vortex with vorticity  $N = 6$ . For the case of  $b = 0.25\xi(0)$  the first two vortices enter into the sample and are attracted quickly towards to the center of the disk forming a giant vortex with vorticity  $N = 2$ . Upon increasing the magnetic field we have vortex transitions from  $N$  to  $N + 1$ . It is interesting to note that the presence of the metallic interface changes the values of the first and second critical fields. Furthermore for samples in contact with metallic material represented by  $b < 0.138\xi(0)$ , no vortex can be formed for any magnetic field and a continuous entrance of magnetic flux was observed.

#### 4. Conclusions

We studied the effect of an irregular surface on the thermodynamical properties of a mesoscopic superconducting disk in the presence of an external applied magnetic field by solving the time dependent Ginzburg-Landau equations. We have taken two values for the deGennes parameter  $b = 1.25$  and  $b = 0.25$  which simulate a metallic interface in a concave/convex surface. Our results have shown that the first critical field dependent strongly on the values of  $b$  and slightly on the kind of surface. In addition, the second critical field depends just on the values of  $b$ . We found that for  $b = 0.138$ , the superconductor becomes a type I superconductor, since only allows a continuous magnetic field penetration without any nucleation of vortices, even a single one.

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