Solution of the Ginzburg-Landau equations with different interfaces extended to multiply connected domains

J. Barba-Ortega, Sindy J. Higuera, Miryam R. Joya

Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia

E-mail: `jjbarbao@unal.edu.co`

Abstract. The thermodynamical properties of a thin superconducting film with a central square hole are found theoretically. It is assumed that two opposite outer edges of the sample are in contact with a thin layer of metallic material while the other two edges are in contact with a thin layer of superconducting material, its configuration allows to control the vortex entry in the sample. In this work, we solve numerically the Ginzburg Landau equations with general boundary conditions using the Link variable method extended to multiply connected domains. It is shown that the value of the magnetization, the first vortex entry field, and the free energy are sensitive to the material in contact with the inner edge of the hollow.

1. Introduction

The properties of a mesoscopic superconductor are influenced by the geometry of the sample, structural defects and interfaces, several experimental and theoretical studies were published, where the vortex state of superconducting clusters made of different kinds of superconducting material were investigated. The authors found that the superconducting/normal state H-T phase boundary of these structures reveals an oscillatory behavior caused by the formation of different stable vortex configurations in these small clusters of defects [1, 2, 3, 4]. In previous works we studied the effect of weak structural defects and general boundary conditions on the vortex configurations in several geometries. We found that the critical fields are independent of the geometry of defect with weak pinning force and depend strongly on the boundary conditions [5, 6, 7]. In this paper we analyze the superconducting state in a mesoscopic film with a central square hole in presence of an external applied magnetic field H_e. The two opposite outer edges of the film are in contact with a metallic material while the other two edges are in contact with a superconducting material. We consider three cases for the thin layer in contact with the inner edges of the hole 1) metallic material 2) dielectric material and 3) superconducting at higher critical temperature material. We calculate magnetization, free energy and density of superconducting electrons. Our calculations show that the first and second vortex penetration field depend on the nature of the defects. For case (3) there is a higher surface energy barrier and are not vortices in the defect.
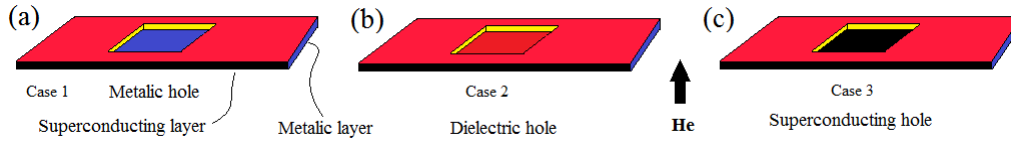
Figure 1. Layout of the studied samples. Superconducting film with lateral surfaces in contact with a thin layer of metallic and superconducting material with a central a) metallic, b) dielectric and c) superconducting hole.

2. Theoretical Formalism

Superconducting state properties are described in the Ginzburg-Landau Theory by the order parameter ψ, and the potential vector $\mathbf{A}$. The TDGL equations in absence of external currents are given by [8, 9, 10]:

$$
\begin{aligned}
\frac{\partial \psi}{\partial t} &= -(i\nabla + \mathbf{A})^2 \psi + (1-T)\psi(1-|\psi|^2)\,, \\
\frac{\partial \mathbf{A}}{\partial t} &= (1-T)\mathrm{Re}\left[\bar{\psi}(-i\nabla - \mathbf{A})\psi\right] - \kappa^2 \nabla \times \mathbf{h}\,,
\end{aligned}
\tag{1}
$$

T is temperature in critical temperature units T_C, order parameter in ψ_∞ units, length in units of coherence length at zero temperature $\xi(0)$, fields in $H_{c2}(0)$ units and free energy G in units of $G_0 = H_{c2}^2 V/8\pi$. $\kappa = \lambda(0)/\xi(0)$ is the material-dependent Ginzburg-Landau parameter. The domain is multiply connected, for the external interface and the internal surface of the hole, we use a general boundary conditions via deGennes extrapolation length b. The magnetic field inside the hole is uniform and only depends on time, since there are no currents in the hole. The boundary conditions for the external and internal surfaces were as follows.

$$
(-i\nabla - \mathbf{A})\psi \cdot \mathbf{n} = \frac{i\hbar}{b}\psi|_n \Rightarrow
\begin{cases}
b \to \infty \Rightarrow \text{Superconducting/Dielectric interface} \\
b > 0 \Rightarrow \text{Superconducting/Superconducting interface} \\
b < 0 \Rightarrow \text{Superconducting/Metallic interface}
\end{cases}
\tag{2}
$$

Let $\mathbf{n}$ be a unit vector normal to the interface [11, 12].

3. Results and Discussion

The parameters used in our numerical simulations were: $\kappa = 2.17$, which is a value for to a thin film of Nb with thickness d (assuming $\xi(0) = 380$ nm, $T_c = 3.7K$, $d \approx 60$ nm), $T = 0$, the dimensions of the sample and internal hole were $S_h = 20\xi(0) \times 6\xi(0)$ and $S_h = 4\xi(0) \times 4\xi(0)$ respectively. The unit cell length was 0.25×0.25. We use $b = 0.2\xi(0)$ for the external metallic interface and $b = -2.75\xi(0)$ for the external superconducting interface. Also we simulates a metallic, dielectric and superconducting hole with $b = 2\xi(0)$, $b = -2\xi(0)$ and $b \to \infty$ respectively. In Fig. 1 we plot a layout of the studied samples. A superconducting film with lateral surfaces in contact with a thin layer of metallic and superconducting material with a central a) metallic hole, b) dielectric hole, and c) superconducting hole. In Fig. 3 we plot the contour plot of the supercurrent for a film for $b = -2\xi(0)$, $b \to \infty$ and $b = 2\xi(0)$ in the defect at $H_e = 0.42H_{c2}(0)$ for a same vorticity $N = 4$. We can observe that for $b = -2\xi(0)$ there are not vortices in the defect due to the repulsion force ofthe another superconducting at higher critical temperature material. In Fig. 3 we plot the contour plot of the magnetic induction for the film for $b = -2\xi(0)$, $b \to \infty$ and $b = 2\xi(0)$ in the hole. We can observe that the magnetic induction decrease when decrease b for the same magnetic field. In Fig. 4 we depict the magnetization $-4\pi M$ (left), free energy G (right) and the contour plot of the order parameter $|\psi|$ (insects, left) as a function of
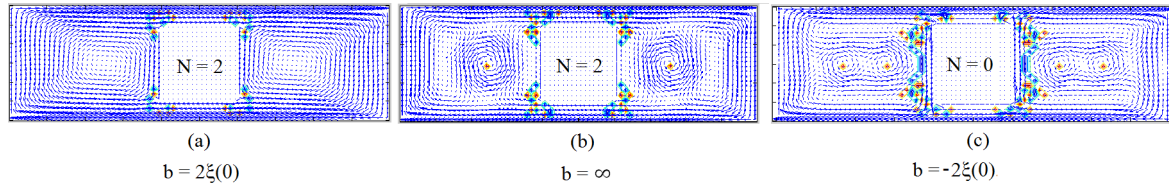
Figure 2. (Color online) Contour plot of the supercurrent for a film for $b = 2\xi(0)$, $b \to \infty$ and $b = -2\xi(0)$ in the defect at $H_e = 0.42H_{c2}(0)$ for $N = 4$.
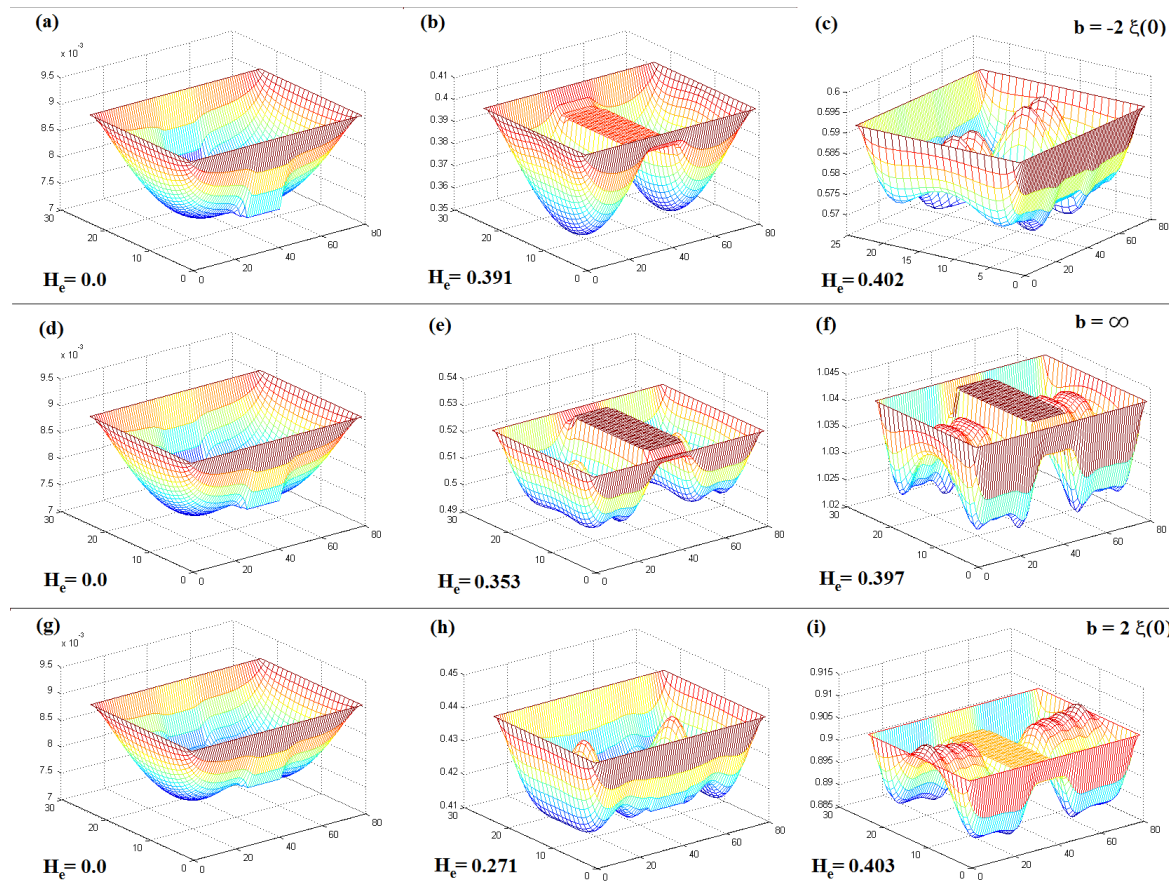
Figure 3. (Color online) Contour plot of the magnetic induction for a film with a square defect.

the magnetic field for $b = -2\xi(0)$, $b \to \infty$ and $b = 2\xi(0)$. We can see from these figures that the first H_1 and second H_2 vortex penetration fields dependents on the internal boundary condition. We have $H_1 = 0.391$ and $H_2 = 0.402$ when we use $b = -2\xi(0)$, $H_1 = 0.353$ and $H_2 = 0.397$ for $b \to \infty$ and $H_1 = 0.271$ and $H_2 = 0.403$ for $b = 2\xi(0)$. In the insects of Fig. 4 (left) we show the contour plot of the order parameter for (a) $N = 2$ at $H_1 = 272, 0.354, 0.392$ and (b) $N = 4$ at $H_2 = 404, 0.398, 0.403$ for $b = 2\xi(0)$, $b \to \infty$ and $b = -2\xi(0)$, respectively, (N indicates the vorticity). Due to the boundary conditions the two first vortices enter in the sample although the edge with the lower energy surface barrier and are attracted quickly towards to the dielectric or metallic defect forming a giant vortex with vorticity $N = 2$. Is interesting to note that the presence of the superconducting layer into the defect acts like a repulsion force preventing the entry of vortices.
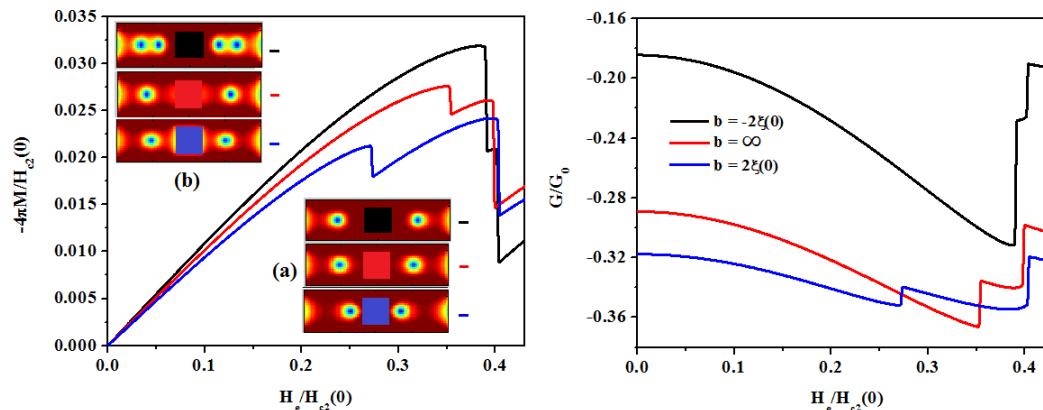
Figure 4. (Color online) Magnetization $-4\pi M$ (Right) and Gibbs free energy G (Left) as a function of the magnetic field for the film with a square defect

4. Conclusions

We studied the effect of the nature of a central defect on the thermodynamical properties of a mesoscopic superconducting film by solving the time dependent Ginzburg-Landau equations. We took three values for the deGennes parameter: $b = -2\xi(0)$ and $b = 2\xi(0)$ simulates a defect with its internal surface in contact with a thin superconducting and metallic layer, respectively, and $b \to \infty$ simulates a dielectric material. Our results have shown that the first and second vortex entries fields, H_1 and H_2, depend strongly on the nature of the defect. Additionally, due to the high repulsive force of the superconducting defect, there are no vortices into the superconducting hole at low magnetic fields.

References

[1] V.V. Moshchalkov, L. Gielen, C. Strunk, R. Jonckheere, X. Qiu, C. Van Haesendonck, and Y. Bruynseraede, Nature (London) 373, 319 (1995).
[2] G. R. Berdiyorov, S. H. Yu, Z. L. Xiao, F. M. Peeters, J. Hua, A. Imre, and W. K. Kwok. Phys. Rev. **B** 80, 064511 (2009).
[3] Guo-Qiao Zha, M. V. Milosevic, Shi-Ping Zhou, and F. M. Peeters. Phys. Rev. **B** 84, 132501 (2011).
[4] W. Gillijns, M. V. Milosevic, A. V. Silhanek, V. V. Moshchalkov, and F. M. Peeters. Phys. Rev. **B** 76, 184516 (2007).
[5] J. Barba-Ortega, Edson sardella, J. Albino Aguiar, Supercond. Sci. Technol.24, 015001 (2011).
[6] J. Barba-Ortega, Edson sardella, J. Albino Aguiar, E. H. Brandt, Physica C, **479**, 49 (2012).
[7] J. Barba-Ortega, Edson sardella, J. Albino Aguiar, F. M. Peeters, Physica C, **487**, 47 (2013).
[8] W. D. Gropp, H. G. Kaper, G. K. Leaf, D. M. Levine, M. Palumbo, and V. M. Vinokur, J. Comput. Phys. **123**, 254 (1996).
[9] G. C. Buscaglia, C. Bolech and A. López,*Connectivity and Superconductivity*, J. Berger, and J. Rubinstein (Eds.), Springer,(2000).
[10] V. V. Moshchalkov *et al.* in *Connectivity and Superconductivity*, edit by J. Berger and J. Rubinstein (Springer, Heidelberg, 2000).
[11] P. G. de Gennes *Superconductivity of Metals and Alloys*, edit by Westview Press, (1999).
[12] M. Thinkham *Introduction to Superconductivity*, edit by McGraw Hill, New York, (1996).