

Vibro-acoustic analysis of composite plates

A S Sarigül and E Karagözü

Dokuz Eylül University, Faculty of Engineering, Department of Mechanical Engineering, Tınaztepe-Buca 35937, İzmir/TURKEY

E-mail: saide.sarigul@deu.edu.tr and erkankaragozlu@hotmail.com

Abstract. Vibro-acoustic analysis plays a vital role on the design of aircrafts, spacecrafts, land vehicles and ships produced from thin plates backed by closed cavities, with regard to human health and living comfort. For this type of structures, it is required a coupled solution that takes into account structural-acoustic interaction which is crucial for sensitive solutions. In this study, coupled vibro-acoustic analyses of plates produced from composite materials have been performed by using finite element analysis software. The study has been carried out for E-glass/Epoxy, Kevlar/Epoxy and Carbon/Epoxy plates with different ply angles and numbers of ply. The effects of composite material, ply orientation and number of layer on coupled vibro-acoustic characteristics of plates have been analysed for various combinations. The analysis results have been statistically examined and assessed.

1. Introduction

Vibro-acoustic behaviour of flexible plate-cavity systems is an important research topic in the design of aircrafts, spacecrafts, land vehicles and ships. To design a competitive product meeting consumers' demands, the price must be kept as low as possible by reducing part thicknesses which is much more important in vehicle design that effects the fuel consumption. Since 1960s researchers have tried to construct mathematical models to predict the behaviour of solid-fluid interaction. Lyon [1] and Pretlove [2] have made first analytical studies for a flexible plate backed by a fluid containing cavity under weak coupling assumption. Fahy [3] has examined a similar system by the modal coupling approach. The improvements in numerical solutions and rapid increase in software technology have made the solution of complex shaped systems possible. Zienkiewicz et al. [4] and Craggs [5] have used finite element method to model a coupled structure-fluid system. Niyogi et al. [6] have made coupled vibro-acoustic analysis of composite structures. In that work, finite element/boundary element formulation has been used and effects of number of ply, wall thickness and damping ratio on the variation of pressure level have been examined.

In the present study, coupled vibro-acoustic analyses of thin composite plates backed by air containing cavity were performed by using finite element method (FEM). The effects of type of composite material, ply angle and number of ply on the coupled system's natural frequencies were examined. A regression analysis was carried out in order to put forward the mathematical relation between these variables; and to estimate the coupled natural frequencies of the system from the uncoupled natural frequencies of the composite plate.



2. Finite element modelling and analysis of the system

In order to examine the system and obtain reliable results both flexible plate and acoustic cavity should be modelled correctly. Both models were constructed by using finite elements.

2.1. Finite element modelling of the flexible plate and acoustic cavity

Different finite element alternatives for modelling of structural and acoustical parts of the system are shown in figures 1 and 2, respectively. Shell elements are used for thin/moderately thick plate or shell structures. Linear shell element includes 4 nodes whereas quadratic shell element includes 8 nodes. Each node has 6 degrees of freedom, as 3 translations and 3 rotations. Linear and quadratic brick elements given in figure 2 are used for modelling 3-D acoustical volumes and include 8 and 20 nodes, respectively. Each node has 4 degrees of freedom, as 3 translations and 1 pressure.

It is a general procedure to use the linear shell element for thin plates. Therefore, this type of element was used in the first benchmarking studies for the uncoupled vibrations of composite plates. This solution was performed for a square three-ply composite plate with physical parameters; $E_1/E_2 = 2.45$, $G_{12} = 0.48E_2$, $\nu_{12} = 0.23$, $\nu_{21} = 0.0939$, $\rho = 8000 \text{ kg m}^{-3}$, $h = 0.06 \text{ m}$, $h/a = 0.006$. Here, E_1 and E_2 are longitudinal and transverse modulus of elasticity, respectively; G_{12} , in-plane shear modulus; ν , Poisson's ratio; ρ , density of the plate material; h , plate thickness and a is side length of the plate. Computed natural frequency parameters ($\beta_i = \omega_i a^2 \sqrt{\rho h / D_{01}}$; ω_i , natural frequency; $D_{01} = E_1 h^3 / (1 - \nu_{12} \nu_{21})$, rigidity of the plate) are presented in table 1 and compared with solutions from the literature. Finite element results were computed by using linear and quadratic shell elements and different discretizations. The results for linear 30x30 elements are not so coarse however linear 40x40 elements give better solutions. The results obtained by quadratic elements are more sensitive and the effect of mesh size is immaterial for this case. That is, quadratic shell elements are more appropriate for modal deflections of composite plates. On the other hand, it is known that coupled vibro-acoustic solutions require sensitive models for both isotropic plates and acoustical volumes at their back. For these reasons, in the present study for the coupled system in figure 3, composite plate and acoustic cavity were modelled by quadratic shell (30x30) and brick (30x30x6) elements, respectively. For the interface between the acoustic cavity and the flexible plate an interaction was defined and nodes on this surface were left free to move. The nodes on the other surfaces of the cavity were restricted to move but not to rotate. Plate edges were assumed to be clamped.

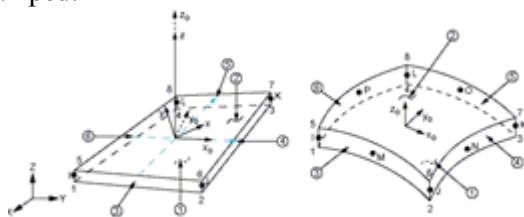


Figure 1. Linear and quadratic shell elements.

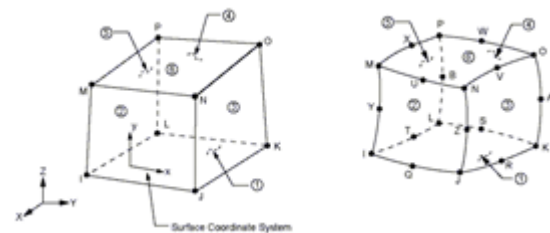


Figure 2. Linear and quadratic brick elements.

2.2. Analysis of the coupled system

Equation of motion of a freely vibrating vibro-acoustic system without damping, may be written as,

$$\left(-\omega_n^2 \begin{bmatrix} M_s & 0 \\ M_{fs} & M_a \end{bmatrix} + \begin{bmatrix} K_s & K_{fs} \\ 0 & K_a \end{bmatrix} \right) \begin{Bmatrix} u \\ p \end{Bmatrix} = 0 \quad (1)$$

Here, ω_n is natural frequency, u is displacement, p is pressure. M_s, K_s, M_a, K_a are mass and stiffness matrices of plate and acoustic cavity, respectively. M_{fs} and K_{fs} are coupled system matrices for the flexible plate. Solution of the characteristic equation obtained from equation 1 gives eigenvalues and eigenvectors of the system which yield natural frequencies and mode shapes. Uncoupled solution of

the plate has symmetric K and M matrices and it is solved by Block Lanczos Method. However, in the coupled solution these matrices are not symmetric; therefore unsymmetrical solution method is used. On the other hand, FEM solution is performed on the basis of Mindlin Plate Theory (First Order Shear Deformation Theory).

Finite element analyses of the system were carried out for three composite plate materials (E-glass/Epoxy, Kevlar/Epoxy, Carbon/Epoxy), four ply angles (0° , 15° , 30° , 45°) and three different numbers of ply (three, four, five). Physical parameters of the composites are given in table 2. In table 3, the first uncoupled and coupled natural frequencies of the plate and also their ratios are presented. Fundamental frequency is the most important natural frequency in mechanics and also the most affected frequency from the coupling. Fundamental frequencies increase when number of ply increases and decrease when ply angle increases in the considered interval (0° - 45°). Coupling is more effective on E-Glass/Epoxy plate which is more flexible than the others. Plates with three-ply and 45° fibre angle seem to be more affected from coupling.

Table 1. Natural frequency parameters (β_i) of fully clamped, square, three-ply composite plates.

Ply Angle		Mode Sequence Number (i)					
		1	2	3	4	5	6
0,0,0	DSC [7]	29.087	50.792	67.279	85.629	87.112	118.500
	FEM (30x30) ^L	29.154	51.080	67.745	86.166	88.269	119.688
	FEM (40x40) ^L	29.116	50.925	67.497	85.860	87.678	119.027
	FEM (30x30) ^Q	29.066	50.728	67.179	85.466	86.935	118.188
	FEM (40x40) ^Q	29.060	50.728	67.179	85.460	86.935	118.182
45,-45,45	DSC [7]	28.337	54.623	60.430	83.658	101.940	105.600
	FEM (30x30) ^L	28.391	54.957	60.761	84.214	103.308	106.855
	FEM (40x40) ^L	28.355	54.771	60.566	83.877	102.584	106.169
	FEM (30x30) ^Q	28.308	54.535	60.318	83.451	101.668	105.304
	FEM (40x40) ^Q	28.308	54.535	60.317	83.445	101.668	105.298

^L: Linear elements,

^Q: Quadratic elements

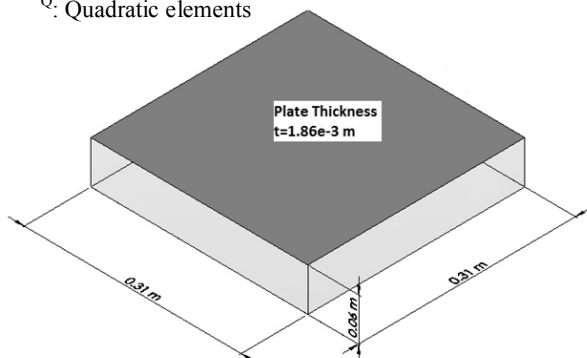


Figure 3. Flexible composite plate backed by air-filled cavity.

Table 2. Physical parameters of the composites.

Physical Parameters [7]	E-Gl/ Ep	Kev/ Ep	Car/ Ep
Fibre Volume Ratio	0.55	0.60	0.63
Density (ρ , kg/m ³)	2100	1380	1580
Longit. Mod. (E_1 , GPa)	39	87	142
Transv. Mod. (E_2 , GPa)	8.6	5.5	10.3
In-pl. Sh. Mod. (G_{12} , GPa)	3.8	2.2	7.2
Major Poisson Ratio (ν_{12})	0.28	0.34	0.27

Table 3. Fundamental frequencies of composite plates (Hz)

Mat.	No Ply	Uncoupled Frequency				Coupled Frequency				Ratio (Coupled/Uncoupled)			
		0°	15°	30°	45°	0°	15°	30°	45°	0°	15°	30°	45°
E-Gl	3	101.15	99.49	96.25	94.70	130.53	129.21	126.66	125.43	1.29	1.30	1.32	1.32
/	4	101.15	99.91	97.51	96.36	130.53	129.66	127.96	127.14	1.29	1.30	1.31	1.32
Ep	5	101.15	100.23	98.42	97.53	130.53	129.98	128.87	128.31	1.29	1.30	1.31	1.32
Kev	3	166.15	159.94	147.69	141.89	193.37	188.01	177.35	172.26	1.16	1.18	1.20	1.21
/	4	166.15	161.73	153.56	149.92	193.37	190.04	183.51	180.46	1.16	1.18	1.20	1.20
Ep	5	166.15	163.06	157.32	154.77	193.37	191.49	187.28	185.2	1.16	1.17	1.19	1.20
Car	3	201.88	195.22	181.96	175.54	222.36	216.19	203.9	197.94	1.10	1.11	1.12	1.13
/	4	201.88	197.16	188.29	184.25	222.36	218.20	210.28	206.61	1.10	1.11	1.12	1.12
Ep	5	201.88	198.56	192.36	189.56	222.36	219.64	214.30	211.80	1.10	1.11	1.11	1.12

3. Statistical studies on finite element results

The effects of uncoupled plate frequency, ply angle and number of ply on coupled frequency were examined statistically for each plate material. For this purpose, three linear models in which coupled frequency is dependent variable were constructed by using regression analysis.

The results of regression analysis are presented in Table 4. Here, “B” represents the coefficients of the regression equation for predicting the dependent variable from the independent variables. “Sig.” indicates the significance level of each variable for the model. Zero value indicates the highest significance. “R”, “R Square” and “Adjusted R Square” show the correlation between the observed and predicted values of the dependent variable. In table 4, on the basis of “Adjusted R Square” values, the models for E-Glass/Epoxy, Kevlar/Epoxy and Carbon/Epoxy plates represent the coupled system by 99.7%, 99.8% and 100% accuracies, respectively. Among the three models, the one for Carbon/Epoxy plate is the least sensitive system to acoustic pressure. That is, the least effective coupling behaviour is present in Carbon/Epoxy plate. This result is compatible with table 1 that shows minimum Coupled/Uncoupled frequency ratios for Carbon/Epoxy material.

Table 4. Model summary and coefficients of linear models.

Model		Unstandardized Coefficients		Sig.	95.0% Confidence Interval for B		Adjusted R Square
		B	Std. Error		Lower Bound	Upper Bound	
1 (E-Gl/Ep)	(Constant)	35.230	4.652	.000	24.503	45.958	.997
	Uncoup. Freq.	.940	.047	.000	.831	1.049	
	Ply Angle	.025	.006	.002	.012	.038	
	No of Ply	.060	.046	.233	-.047	.166	
2 (Kev/Ep)	(Constant)	29.209	6.458	.002	14.316	44.101	.998
	Uncoup. Freq.	.985	.042	.000	.888	1.081	
	Ply Angle	.074	.018	.003	.032	.116	
	No of Ply	.139	.179	.460	-.274	.553	
3 (Car/Ep)	(Constant)	25.481	2.591	.000	19.506	31.456	1.000
	Uncoup. Freq.	.974	.014	.000	.942	1.005	
	Ply Angle	.032	.006	.001	.017	.046	
	No of Ply	.085	.064	.219	-.062	.233	

Using these models, coupled frequencies may be estimated from the uncoupled frequencies. For example, the estimation for E-Glass-Epoxy plate with 30° ply angle and 4 layers can be made by using Model 1 in table 4 as,

$$\text{Coupled Frequency} = 35.230 + 0.94 * 97,51 + 0.025 * 30 + 0.06 * 4 = 127.88 \text{ Hz}$$

The coupled system solution for this plate is presented in table 3 as 127.96 Hz. Consequently, the estimation is very sensitive; and proves that this procedure may be used safely leading to great savings in both modelling effort and computation time for the considered flexible plate-cavity systems.

References

- [1] Lyon R H 1963 *J. Acoust. Soc. Am.* **35** 1791-7
- [2] Pretlove A J 1965 *J. Sound Vib.* **2** 197-209
- [3] Fahy F 1969 *J. Sound Vib.* **10** 490-512
- [4] Zienkiewicz O C and Newton R E 1969 *Proc. Int. Symp. on Finite Element Techniques*, (Stuttgart) 1-9
- [5] Craggs A 1971 *J. Sound Vib.* **15** 509-28
- [6] Niyogi A G, Laha M K and Sinha P K 2000 *Aircr. Eng. Aerosp. Tech.* **72** 345-57
- [7] Seçgin A and Sarıgül A S 2008 *J. Sound Vib.* **315** 197-211