

# Optimization method for electron beam melting and refining of metals

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**Abstract.** Pure metals and special alloys obtained by electron beam melting and refining (EBMR) in vacuum, using electron beams as a heating source, have a lot of applications in nuclear and aerospace industries, electronics, medicine, etc. An analytical optimization problem for the EBMR process based on mathematical heat model is proposed. The used criterion is integral functional minimization of a partial derivative of the temperature in the metal sample. The investigated technological parameters are the electron beam power, beam radius, the metal casting velocity, etc. The optimization problem is discretized using a non-stationary heat model and corresponding adapted Pismen-Rekford numerical scheme, developed by us and multidimensional trapezoidal rule. Thus a discrete optimization problem is built where the criterion is a function of technological process parameters. The discrete optimization problem is heuristically solved by cluster optimization method. Corresponding software for the optimization task is developed. The proposed optimization scheme can be applied for quality improvement of the pure metals (Ta, Ti, Cu, etc.) produced by the modern and ecological-friendly EBMR process.

## 1. Introduction

Electron Beam Melting and Refining (EBMR) is a method in the special electrometallurgy for production of pure metals and alloys and new materials fabrication by scrap recycling [1-3].

The EBMR process of metals and alloys is accomplished in vacuum chamber using electron beams as a heating source. The raw material is melted, refined and re-solidified in a water-cooled crucible. The electrons fall on the front side of the feeding material and heat it. The molten metal as drops fall into the crucible. The top surface of the molten metal in the crucible is also heated by the e-beam [1-3]. Due to especially difficulties to acquire real time data for the processes in the liquid pool, the successful application and optimization of EBMR depends on the adequate mathematical modeling of the heat transfer processes. This allows to make a study of the influence of the regime parameters and the limiting factors. The most important information that is needed is about the temperature field in the metal ingot during EBMR which can't be precisely evaluated experimentally.

In [4-7] developed stationary and non-stationary heat transfer models are presented and implemented. The non-stationary model, discretized by a modified Pismen-Rekford scheme [6, 8], and the corresponding computer program gives opportunity for simulation of the EBMR process and gaining information about the dynamics of the input and output steams,



temperature fields in vertical and horizontal cross-sections of the ingot, the dynamics of the geometry of the liquid pool, etc.

The knowledge of the geometry of the crystallization front (liquid/solid boundary) is important for studying and optimizing the quality of the obtained pure metal after EBMR. The flatness of the liquid/solid contour (the shape of the molten pools) is directly connected with the quality of the structure of the obtained metal. The flatness is examined and studied through a heuristic approach [9-11] developed by Vutova and Mladenov. Still, by now no analytical criteria for the flatness of the temperature level lines have been proposed.

## 2. An optimization problem for EBMR of metals

### 2.1. Derivation

Recall the non-stationary heat model [6] for EBMR of metals and alloys, the equations describing the heat processes in the cylindrical metal ingot are as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} + \frac{V}{a} \frac{\partial T}{\partial z} = \frac{\rho C_p}{\lambda} \frac{\partial T}{\partial t}, x \in \Omega = \{x = (r, z, t) | 0 \leq r \leq R, 0 \leq z \leq H, 0 < t \leq F\} \quad (1)$$

$$T(x) = T_0(r, z), x \in \Omega_0 = \{(r, z, 0) | 0 \leq r \leq R, 0 \leq z \leq H\} \quad (2)$$

$$\lambda(T(x)) \frac{\partial T}{\partial z}(x) = |P_{surf}(r, t) - \alpha \sigma (T^4(x) - T_{room}^4) - C_p(x) \cdot W_v \cdot T(x)|, x \in \{(r, H, t) | 0 \leq r \leq R, t \geq 0\} \quad (3)$$

$$\lambda(T(x)) \frac{\partial T}{\partial r}(x) = \frac{\partial T^1}{\partial r}(x), x \in \{(R, z, t) | Q \leq z \leq H, t \geq 0\} \quad (4)$$

$$\lambda(T(x)) \frac{\partial T}{\partial r}(x) = -\alpha \sigma (T^4(x) - T_{room}^4), x \in \{(R, z, t) | 0 \leq z < Q, t \geq 0\} \quad (5)$$

$$\lambda(T(x)) \frac{\partial T}{\partial z}(x) = \frac{\partial T^2}{\partial z}(x), x \in \{(r, 0, t) | 0 \leq r \leq R, t \geq 0\}, \quad (6)$$

where  $\rho$ ,  $[g/m^3]$  is the density of the metal;  $C_p$ ,  $[W.s/g.K]$  is the heat capacity;  $\lambda$ ,  $[W/m.K]$  is the thermal conductivity;  $a$ ,  $[m^2/s]$  is the thermal diffusivity and  $a = \frac{\lambda}{\rho C_p} = \frac{1}{k}$ ,  $k = \frac{\rho C_p}{\lambda}$ .  $\frac{V}{a} \frac{\partial T}{\partial z}$  shows the casting, i.e. the heat added by the poured molten metal;  $\alpha$  is the metal's emissivity,  $\sigma = 5.6704 \cdot 10^{-8} [J/s.m^2.K^4]$  is the Stephan-Boltzman constant;  $C_p \cdot W_v \cdot T$  describes the evaporation losses.  $P_{surf}$  is the beam power density function and  $V$  is the casting velocity.

The flatness of the temperature level lines in a vertical cross-section of the cylinder for a fixed moment of time depends on the values of  $\frac{\partial T}{\partial r}$ . For fixed moment of the heating time  $t = t_f$  and height  $z = z_f$ ,  $T(r, z_f, t_f)$  is a strictly decreasing function of the variable  $r$  and  $\frac{\partial T}{\partial r} \leq 0$ .

The problem (1-6) is defined in the domain  $\Omega = [0, R] \times [0, H] \times [0, F]$  and a uniform net is made:

$$W_{h_1, h_2, \tau} = \{(r_i, z_j, t_n) | r_i = ih_1, z_j = jh_2, t_n = n\tau; i = \overline{0, N}, j = \overline{0, M}, n = \overline{0, P}\} \quad (7)$$

Lets consider  $\mathcal{A} = -\iiint_{\Omega_1} \frac{\partial T}{\partial r} dr dz dt$ , where  $\Omega_1 = [R_1, R_2] \times [H_1, H_2] \times [F_1, F_2] \subseteq \Omega$ . Lets

denote  $S = \frac{\partial T}{\partial r}$  and  $S_{i,j}^n = \frac{\partial T}{\partial r}(r_i, z_j, t_n)$ . In terms of  $T$ , the following approximation is held:  $S_{i,j}^n = \frac{T_{i,j}^n - T_{i-1,j}^n}{h_1} + O(h_1)$ .

Discretization of  $\mathcal{A}$  is made via multi-dimensional trapezoidal rule onto  $W_{h_1, h_2, \tau}^1$  - a subset of (7):

$$W_{h_1, h_2, \tau}^1 = \{(r_i, z_j, t_n) | r_i = ih_i, z_j = jh_2, t_n = n\tau; i = \overline{N_1, N_2}, j = \overline{M_1, M_2}, n = \overline{P_1, P_2}\} \quad (8)$$

In (8),  $N_i$  corresponds to  $R_i$ ,  $M_i$  to  $H_i$ , and  $P_i$  to  $F_i$ ,  $i = 1, 2$ . The approximation of  $\mathcal{A}$  is denoted by  $A$  and (9) gives the approximation in terms of  $S$ :

$$\begin{aligned}
-\frac{8}{h_1 h_2 \tau} A &= S_{N_1, M_1}^{P_1} + S_{N_1, M_1}^{P_2} + S_{N_1, M_2}^{P_1} + S_{N_1, M_2}^{P_2} + S_{N_2, M_1}^{P_1} + S_{N_2, M_1}^{P_2} + S_{N_2, M_2}^{P_1} + S_{N_2, M_2}^{P_2} + \\
&+ 2 \left\{ \sum_{i=N_1+1}^{N_2-1} [S_{i, M_1}^{P_1} + S_{i, M_2}^{P_1} + S_{i, M_1}^{P_2} + S_{i, M_2}^{P_2}] + \sum_{j=M_1+1}^{M_2-1} [S_{N_1, j}^{P_1} + S_{N_2, j}^{P_1} + S_{N_1, j}^{P_2} + S_{N_2, j}^{P_2}] + \right. \\
&\quad \left. \sum_{n=P_1+1}^{P_2-1} [S_{N_1, M_1}^n + S_{N_1, M_2}^n + S_{N_2, M_1}^n + S_{N_2, M_2}^n] \right\} \\
&+ 4 \left\{ \sum_{i=N_1+1}^{N_2-1} \sum_{j=M_1+1}^{M_2-1} [S_{i, j}^{P_1} + S_{i, j}^{P_2}] + \sum_{j=M_1+1}^{M_2-1} \sum_{n=P_1+1}^{P_2-1} [S_{N_1, j}^n + S_{N_2, j}^n] + \right. \\
&\quad \left. \sum_{n=P_1+1}^{P_2-1} \sum_{i=N_1+1}^{N_2-1} [S_{i, M_1}^n + S_{i, M_2}^n] \right\} \\
&+ 8 \sum_{i=N_1+1}^{N_2-1} \sum_{j=M_1+1}^{M_2-1} \sum_{n=P_1+1}^{P_2-1} S_{i, j}^n
\end{aligned} \tag{9}$$

## 2.2. Optimization problem formulation

The analytical optimization problem is:

$$\mathcal{A} = - \iiint_{\Omega_1} \frac{\partial T}{\partial r} dr dz dt \rightarrow \min, \tag{10}$$

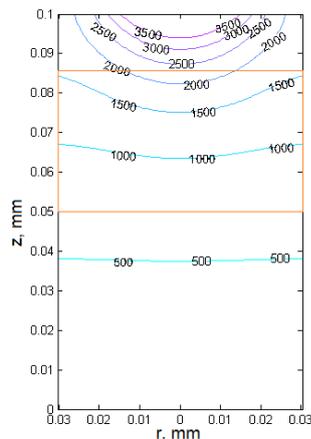
where  $T$  is the temperature field determined by the solution of (1-6). The variables that are controlled in this optimization approach are  $P_{surf}(r, t)$  and  $V(t)$ .

The discretization of the system (1-6) is made on the net (7) by modified Pismen-Rekfort scheme [6]. Thus for input functions  $P_{surf}(r, t)$  and  $V(t)$ , the discrete temperature field  $\{T_{i, j}^n\}_{i=1, N, j=1, M}^{n=1, P}$  is calculated. Hence,  $\mathcal{A}$  can be calculated over this discrete field by (9). In this way the analytical problem (10) is transformed to a discrete one which can be solved by optimization methods. The use of some heuristic methods such as cluster optimization technique is suitable because  $A$  depends on the control variables in a complex and implicit way.

## 3. Results and discussion

Experiments for EBMR of Ti ingots of length  $H = 100mm$  and diameter  $2R = 60mm$  are made for various values of the beam power  $P_b$  and casting velocity  $V$  and experimental data about the concentration of some of the metal's impurities are obtained [12]. Statistical approach, based on experimental data about chemical analysis, is applied and optimal process conditions by minimization of all impurities concentrations are obtained -  $P_b = 11.25kW$ ,  $V = 6mm/min$ ,  $F = 7.37min$  for focused electron beam with  $r_b = 10mm$ .

For these Ti ingots at  $V = 6mm/min$  and total heating time  $7.37min$  one dimensional cluster optimization is made for a beam power  $P_b \in [11kW, 20kW]$  via developed software. In the numerical optimization  $\Omega_1$  is chosen to be  $[0, R] \times [0.5H, 0.85H] \times [0.9F, F]$ . This choice is based on the results obtained by the heat model [6]. In Fig.1, a vertical cross-section of the temperature field is shown for  $P = 11kW$  in the 442<sup>nd</sup> second of the heating. The cluster analysis shows that the criterion (10) is minimized for  $P_b = 11kW$  and confirms the result from the statistical approach [12]. The proposed optimization technique does not need experimental data for chemical analysis of the impurities' concentrations like the approach in [12], which is an important advantage of the optimization method described in this paper.



**Figure 1.** Temperature field (K) in a vertical cross-section of cylindrical Ti ingot during EBM. The dimensions of the ingot are  $H = 100\text{mm}$ ,  $2R = 60\text{mm}$ ,  $V = 6\text{mm/min}$ ,  $r_b = 10\text{mm}$  at the  $442\text{s}$  of the heating process. The orange rectangle is the domain over which the flatness of the radial temperature level lines is optimized.  $T_{melt}$  of Ti is  $1941\text{K}$ .

#### 4. Conclusion

For improvement of the technology for acquiring pure metals with EBM, a new analytical criterion for flatness of the radial temperature level lines is invented. Optimization problem to minimize this criterion over the solutions of non-stationary heat model for different control variables is proposed. The criterion for achieving flatness of the liquid pool is discretized synchronically to the Pismen-Rekford numerical scheme used in the time-dependent heat model. Cluster optimization technique is applied to find heuristically an optimal technological regime for EBM of titanium. The beam power value suggested by the criterion is compared and coincides to the value obtained by experimental data and statistical method. The developed optimization scheme is promising and can be applied to suggest proper technological regimes for EBMR of different metals to improve the quality of the obtained new materials.

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