Journal of Physics: Conference Series **490** (2014) 012078 doi:10.1088/1742-6596/490/1/012078

Numerical modelling of electromagnetic loads on fusion device structures

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Abstract. In magnetic confinement fusion devices, during abnormal operations (disruptions) the plasma begins to move rapidly towards the vessel wall in a vertical displacement event (VDE), producing plasma current asymmetries, vessel eddy currents and open field line halo currents, each of which can exert potentially damaging forces upon the vessel and in-vessel components. This paper presents a methodology to estimate electromagnetic loads, on three-dimensional conductive structures surrounding the plasma, which arise from the interaction of halo-currents associated to VDEs with a magnetic field of the order of some Tesla needed for plasma confinement. Lorentz forces, calculated by complementary formulations, are used as constraining loads in a linear static structural analysis carried out on a detailed model of the mechanical structures of a representative machine.

1. Introduction

Disruptions represent one of the main concerns for tokamak operations, especially in view of fusion reactors or experimental test reactors under design [1]. In elongated plasmas, vertical control can be lost generating what are called *Vertical Displacement Events* (VDEs), that induce plasma current asymmetries, eddy currents and halo currents, each of which can exert potentially damaging forces upon the tokamak components. In particular, when the plasma hits the first wall (FW), enormous currents are injected in the passive conductive structures of the machine, in a region submerged in the magnetic field needed for plasma confinement giving rise to appalling forces. Aim of this paper is to describe a methodology used for approaching the problem from the engineering point of view. The proposed procedure, based on multiple physical models, is summarized in the following steps.

(i) Development of a detailed 3D model of the fusion device under study (including all conductive structures surrounding the plasma) and identification of the mesh elements facing the plasma;

(ii) Evaluation of the currents exchanged between plasma and FW (BCs for step *iii*) by means of a suitable plasma model (full MHD or simplified);

(iii) Evaluation of the resistive distribution of the current density $\mathbf{J}$ in each mesh element by means of a 3D electromagnetic code (see Sec. 2);

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(iv) Evaluation of Lorentz force ($\mathbf{f} = \mathbf{J}\times\mathbf{B}$) in each mesh element, assuming a known distribution of magnetic flux density $\mathbf{B}$, to provide the electromagnetic loads (nodal forces) for step v;

(v) Solution of a linear static structural problem (see Sec. 3).

2. Stationary current conduction problem

An efficient numerical code (CAFE[1]) is used to solve the stationary current conduction problem in a three-dimensional, non-trivial domain. In fusion devices, the geometry is so complicated that complementarity is used to provide a robust error estimator for mesh refinements and to have a reliable control on the accuracy of the solution [2]. An irrotational electric field is obtained by a geometric formulation based on the electric scalar potential $\mathbf{V}$, whereas a formulation based on the electric vector potential $\mathbf{T}$ is used to obtain a solenoidal current density.

Let us consider a conductive region K immersed in a perfectly insulating medium. K is covered by a hexahedral mesh, whose incidences are encoded in the *cell complex* $\mathcal{K}$ represented by the standard incidence matrices $\mathbf{G}$, $\mathbf{C}$ and $\mathbf{D}$ [3], [4], [5]. A dual barycentric complex $\tilde{\mathcal{K}}$ is obtained from $\mathcal{K}$ by using the *barycentric subdivision*: $\tilde{\mathbf{G}} = \mathbf{D}^T$, $\tilde{\mathbf{C}} = \mathbf{C}^T$ and $\tilde{\mathbf{D}} = -\mathbf{G}^T$ represent the incidence matrices of $\tilde{\mathcal{K}}$.

In the first formulation, based on the electric scalar potential $\mathbf{V}$, fixing the potential on one arbitrary node to zero, the other potentials are found by solving the symmetric and positive definite linear system $\mathbf{G}^T\mathbf{S}\mathbf{G}\mathbf{V} = \tilde{\mathbf{I}}_s$, where $\mathbf{S}$ is the Ohm's constitutive matrix and $\tilde{\mathbf{I}}_s$ is the current through the boundary dual faces.

The second formulation, based on a electric vector potential $\mathbf{T}$, requires a complicated topological pre-processing due to the fact that the domain under study is topologically non-trivial[2] [6]. The *electric current* $\mathbf{I}$ can be expressed in terms of $\mathbf{T}$ as

$$\mathbf{I} = \mathbf{I}_s + \mathbf{C}\mathbf{T} + \sum_{i=1}^{N} I_g^i \mathbf{\Pi}^i, \tag{1}$$

where the 2-cocycles $\mathbf{\Pi}^i$ are the *thick links* [7] and I_g^i are the *independent currents*. By combining the discrete Ohm's law $\tilde{\mathbf{U}} = \mathbf{R}\mathbf{I}$ and Faraday's law $\mathbf{C}^T\tilde{\mathbf{U}} = \mathbf{0}$ with (1) and taking into account a set of *non-local Faraday's laws*, the final algebraic linear system of equations is obtained [8]

$$\begin{aligned} \mathbf{C}^T\mathbf{R}\mathbf{C}\mathbf{T} + \textstyle\sum_{j=1}^{N}\left(\mathbf{C}^T\mathbf{R}\mathbf{\Pi}^j\right) I_g^j &= -\mathbf{C}^T\mathbf{R}\mathbf{I}_s \\ \left(\mathbf{\Pi}^{iT}\mathbf{R}\mathbf{C}\right)\mathbf{T} + \textstyle\sum_{j=1}^{N}\left(\mathbf{\Pi}^{iT}\mathbf{R}\mathbf{\Pi}^j\right) I_g^j &= -\mathbf{\Pi}^{iT}\mathbf{R}\mathbf{I}_s. \end{aligned} \tag{2}$$

The coefficients of the source current $\mathbf{I}_s$ on mesh faces belonging to $\partial\mathcal{K}$ are set to the plasma halo current imposed through them. The sparse and symmetric linear system (2) can be solved using an *ungauged* technique with iterative linear solvers, whereas the standard *tree-cotree gauge* is applied for direct solvers.

3. Static Structural Analysis

The components involved in the computational domain represent the conductive structures surrounding the plasma. For each of them an accurate geometric model is carried out by a 3D CAD software and a multiblock structured hexahedral mesh is generated by a commercial software. A reference machine with an ITER-like model (including vessel, port extensions, blanket modules and divertor) is considered, discretized with more than 1 million elements (all hexahedra, see Fig. 1). First components facing the plasma are the Blanket Modules (BMs).

[1] CAFE (Computer Aided Fusion Engineering) is a research code developed by first and third authors.

[2] K is a compact connected subset of the 3D Euclidean space which possesses topological handles and cavities.

Lower area of the domain is occupied by the Divertor. The conductive structure surrounding the BMs and the Divertor represents the Vacuum Vessel (VV) made of two shells welded by means of the stiffening ribs. All these structural elements have been assumed as made of stainless steel (AISI 316LN, at a reference temperature $t = 150C$).

In order to set up the FE-model we have considered each contact area welded to one another, so non linear contacts or gaps are not investigated. No symmetries are taken into account: we developed this procedure considering the most general application, since most fusion reactors present no symmetries at all. Besides the structure is constrained with Single Point Constraints (SPC), created on the lower nodes of the VV lower port extensions by fixing them on the ground for preventing the six degrees of freedom.

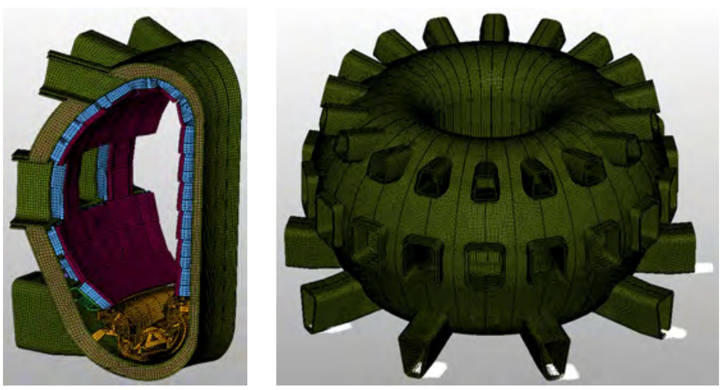
Finally, we have set up a load step where constraints, the gravitational force and nodal electromagnetic forces computed by CAFE, are applied in order to perform a linear static analysis for computing static elastic stress. The problem, in the linear form $\mathbf{F} = \mathbf{KU}$, where $\mathbf{F}$ is the nodal forces vector, $\mathbf{K}$ is the stiffness matrix, and $\mathbf{U}$ the vector representing the structure degrees of freedom, is solved by Altair RadiOss Bulk© code using a direct method for sparse matrices.

4. Numerical results

The proposed procedure has been applied to two test cases. First, axisymmetric BCs are assumed (step i) to asses the procedure in the presence of almost symmetric electromagnetic loads, corresponding to an upward symmetric VDE: the results exhibit the expected symmetry in terms of current density flowing in the conductive structures J (Fig. 2(a)), force density f (Fig. 2(c)) and equivalent (*Von Mises*) stress (Fig. 3(a)). Then, a representative asymmetric case, corresponding to an upward asymmetric VDE is considered: the currents exchanged between plasma and FW (BCs calculated at step i) are provided by a simplified plasma model, coherent with experimental observation associated with the so called *kink mode* of the plasma [1]. The results exhibit the expected asymmetry for current density J flowing in the structures (Fig. 2(b)), force density f (Fig. 2(d)) and equivalent (*Von Mises*) stress (Fig. 3(b)).

5. Conclusions

A procedure, based on multiple physical models, to calculate the electromagnetic loads induced by halo currents on fusion devices has been proposed and applied to representative test cases.



(a) (b)

Figure 1. Representative fusion device, discretized with 1,088,055 hexahedra. (a) Detail of the 3D model, including vessel, ports, blanket modules and divertor. (b) Whole model, with Single Point Constraints (SPC) created on the lower nodes of the VV lower port extensions.

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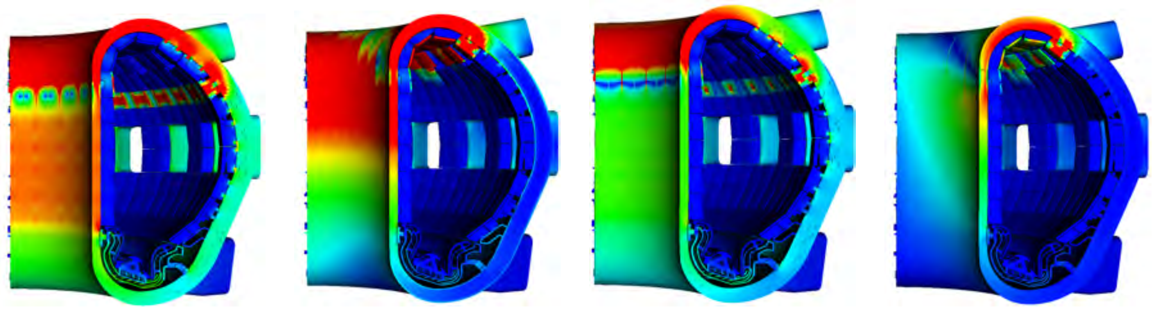
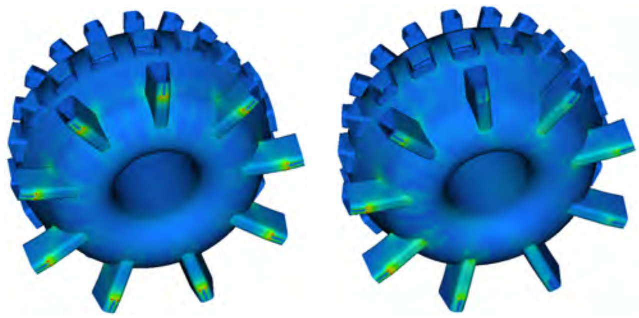


(a) (b) (c) (d)

Figure 2. Current density J (color map saturation (red): $0.1A/mm^2$): (a) symmetric, (b) asymmetric. Force density f (saturation: $10^6 N/m^3$): (c) symmetric, (d) asymmetric.



(a) (b)

Figure 3. Equivalent (*Von Mises*) stress (order of magnitude $10MPa$) for upward VDES: (a) symmetric, (b) asymmetric.

6. Acknowledgment
This work was partially supported by the European Communities under the Contract of Association between EURATOM and ENEA/Consorzio RFX, and by the Italian MIUR under PRIN grant 2010SPS9B3. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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