A discrete geometric formulation for eddy-current problems in fusion devices

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Abstract.
All thermonuclear controlled fusion devices under construction or design have such high performances to require a special care in the dimensioning of various components, specifically from the electromagnetic point of view. To this purpose, it is fundamental to develop models which are both accurate (i.e. able to describe the physical phenomena) and predictive (i.e. useful not only to explain what happens in running experiments, but also to reliably extrapolate to other range of parameters).

The dynamics of fusion plasmas is often conveniently described by Magneto-Hydro-Dynamics (MHD) equations, which predict that some unstable evolution modes may exist. On the other hand, the complexity of the intrinsically 3D model of the interactions between a realistic unstable plasma, the surrounding passive structures (important to guarantee a good MHD stability) and the active conductors (coils) require the numerical solution of challenging electromagnetic problems.

In this work a discrete geometric formulation for eddy-current problems in the frequency domain is developed; the magnetic fields produced by a typical active coil system is calculated in the presence of 3D conductive structures.

1. Introduction

The dynamics of fusion plasmas is often conveniently described by Magneto-Hydro-Dynamics (MHD) equations, which predict that some unstable evolution modes may exist [1], [2]. The MHD modes are usually categorised using the toroidal mode number n, i.e. the harmonic index of a Fourier decomposition of plasma perturbations along the toroidal direction.

While some MHD instabilities develop on very fast timescales (few milliseconds) and are only marginally influenced by the interaction with the structures surrounding the plasma, one particular category, known as Resistive Wall Modes (RWMs), critically depends on the detailed characteristics of the conductive wall and in particular on its non-uniformities and its typical penetration time for electro-magnetic perturbations [3]. Close-fitting passive conducting structures are an efficient way to prevent the growth of magnetohydrodynamic (MHD) instabilities, but are not suitable for a steady state fusion reactor, because of the finite diffusion time of any material shell.

Given the relatively slow growth rate of RWMs, feedback control by means of active coils is possible and is at present one of the most active research fields. RWMs can develop in both tokamak and Reversed Field Pinch (RFP) devices and for both they represent one of the most important performance limiting MHD instabilities [4], [5]. In general, given the characteristics of both the target instability and the active control method, successful experiments and modelling take greatly advantage of a careful knowledge of the impact of 3-dimensional effects in the mode dynamics [6]. In fact, when the plasma is in presence of closed loop control actions, eddy currents in 3D conducting structures and plasma perturbations will evolve in a coupled way. The comprehension of their interaction represents a very complex problem both on the experimental side and the modelling one [7]. The controller design itself needs an accurate evaluation of all the effects in the coil actions due to the non uniformities of the passive boundary [8], [9].

In this paper we focus on the problem of carefully computing the interactions between active conductors (coils) and passive structures surrounding the plasma.

2. Discrete Geometric Formulation

A discrete geometric formulation for eddy-current problems in the frequency domain is presented, which is based on the circulation of the magnetic vector potential over hexahedral grids.

The 3-D domain of interest $\mathcal{D}$ is covered by a mesh of generic hexahedra, whose incidences are encoded in the *cell complex* $\mathcal{K}$ represented by the standard incidence matrices $\mathbf{G}$, $\mathbf{C}$ and $\mathbf{D}$ [10]. A dual barycentric complex $\tilde{\mathcal{K}}$ is obtained from $\mathcal{K}$ by using the *barycentric subdivision*; its incidence matrices are $\tilde{\mathbf{G}} = \mathbf{D}^T$, $\tilde{\mathbf{C}} = \mathbf{C}^T$ and $\tilde{\mathbf{D}} = -\mathbf{G}^T$.

Three subdomains of $\mathcal{D}$ are identified: the passive conductive region $\mathcal{D}_c$ (including all conductive structures surrounding the plasma, see Fig.1), the non-conductive region $\mathcal{D}_a$ (vacuum or air –outside the vacuum vessel), and the source region $\mathcal{D}_s$ (active coils used to control the plasma instabilities).

When modeling stranded coils, it is useful to introduce integral sources, which avoid to cover $\mathcal{D}_s$ with a fine mesh. With this aim, the circulations of the magnetic vector potential $\mathbf{A}$ along primal edges $e \in \mathcal{D}$ can be expresses as $\mathbf{A} = \mathbf{A}_s + \mathbf{A}_r$, where $\mathbf{A}_r$ are the circulations of the magnetic vector potential due to eddy currents in $\mathcal{D}_c$ and $\mathbf{A}_s$ are the circulations of the magnetic vector potential produced by the sources in $\mathcal{D}_s$.

By combining the discrete Ampère's law and Faraday's law with the discrete counterpart of the constitutive laws for the flux density $\mathbf{B}$ and the current density $\mathbf{J}$, a symmetric complex linear system of equations is obtained [9],

$$\begin{aligned} &(\mathbf{C}^T \boldsymbol{\nu}\, \mathbf{C})\, \mathbf{A}_r = \mathbf{0}, && \forall e \in \mathcal{D}_a \bigcup \mathcal{D}_s \\ &(\mathbf{C}^T \boldsymbol{\nu}\, \mathbf{C} + i\omega\, \boldsymbol{\sigma})\, \mathbf{A}_r = -i\omega\, \boldsymbol{\sigma}\, \mathbf{A}_s, && \forall e \in \mathcal{D}_c \end{aligned} \tag{1}$$

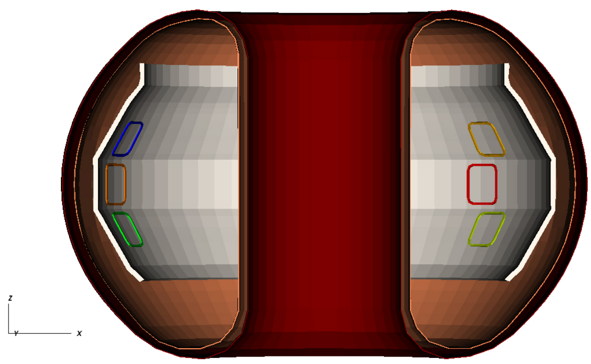


Figure 1. Sketch of a representative machine. The passive conductive region $\mathcal{D}_c$ includes two closed toroidal structures (dark red) and a massive conductive structure (gray) facing the plasma region. A set of typical saddle coils devoted to the active control of RWMs are also shown.

where ω is the angular frequency, $\boldsymbol{\nu}$ and $\boldsymbol{\sigma}$ are square matrices that require metric notions, material properties, and some hypothesis on the fields in order to be computed.

On the RHS, $\mathbf{A}_s$ denotes the circulations of the magnetic vector potential along $e \in \mathcal{D}_c$ produced by the sources in $\mathcal{D}_s$, only; each entry of $\mathbf{A}_s$ can be computed with standard closed formulas.

The sparse and symmetric linear system (1) is solved with a standard *tree-cotree gauge*, using a state-of-the-art direct solver (PARDISO, included in the Intel MKL library).

3. Numerical results

The proposed approach has been applied to calculate the magnetic field produced by a set of saddle coils, individually fed by sinusoidal currents, in the presence of three main conductive structures of a representative machine (see Fig. 1). It must be noted that for this particular geometry it would be possible to model only a part of the geometry due to its toroidal symmetry. Nonetheless, we developed a tool considering the most general application, since most fusion reactors we aim to study present no symmetries at all.

The module of the current density induced on the most relevant structure (facing the plasma) by a sinusoidal current feeding a single saddle coil is shown in Fig. 2. Due to linearity, any combination of currents feeding the RWM coils can be calculated by superposition.

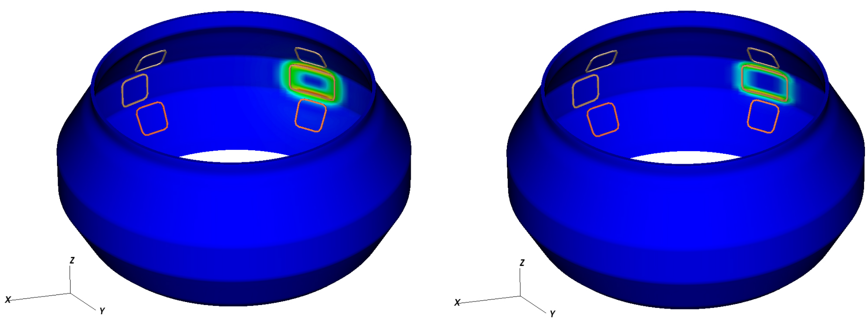
The geometric elements of the mesh are summarized in Table 1, together with degrees of freedom (DOFs) and non zero entries (NNZs) of the system matrix. The numerical solution of the problem by means of a numerical code (CAFE), takes less than 10 minutes[1], including pre-processing (assembling of the system matrix and its RHS) and post-processing (calculation of the magnetic field components by Biot-Savart'law).

The shielding effect of the conductive structure is clearly visible in Fig. 3 where the amplitude of the magnetic flux density is presented (for different frequencies in the range $0 - 100\,Hz$) as a function of the distance along the radial direction from the centre of the coil towards plasma.

4. Conclusions

The development of methods for the active control of MHD instabilities and for the correction of error fields is mandatory in view of fusion reactors. A discrete geometric formulation for eddy-current problems in the frequency domain is introduced to calculate the effects on active control systems of 3D conductive structures surrounding the plasma in a representative machine.

[1] CAFE (Computer Aided Fusion Engineering), research code developed by first and third authors, runs on a workstation equipped with two 8-core processors (Xeon E5-2680 2.7GHz 20MB) and 256GB DDR3-1600 RAM.



(a) (b)

Figure 2. Module of the current density induced on the conductive structure facing the plasma by a sinusoidal current feeding a single saddle coil: (a) $f = 10Hz$, (b) $f = 100Hz$.

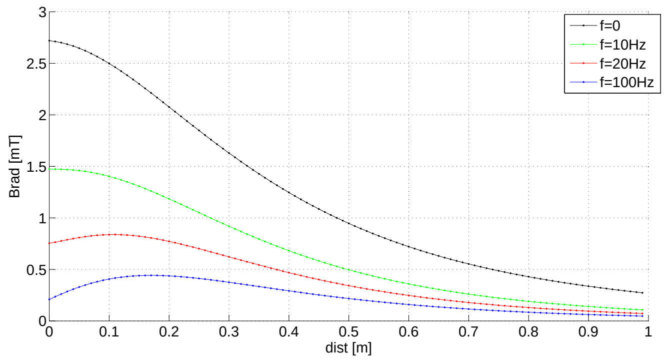


Figure 3. Amplitude of the magnetic flux density as a function of the distance along the radial direction from the centre of the coil towards plasma.

5. Acknowledgment

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6. References

[1] J. P. Freidberg. *Plasma Physics and Fusion Energy.* Cambridge University Press, 2007.
[2] G. Bateman. *G. Bateman, MHD instabilities.* MIT Press, 1978.
[3] M. S. Chu and M. Okabayashi. Stabilization of the external kink and the resistive wall mode. *Plasma Phys. Control. Fusion*, 52:123001, 2010.
[4] F. Troyon, R. Gruber, H. Saurenmann, S. Semenzato, and S. Succi. MHD-limits to plasma confinement. *Plasma Phys. Control. Fusion*, 26:209, 1984.
[5] B. Alper. RFP stability with a resistive shell in HBTX1C. *Plasma Phys. Control. Fusion*, 31:205, 1989.
[6] F. Villone, Y. Q. Liu, R. Paccagnella, T. Bolzonella, and G. Rubinacci. Effects of three-dimensional electromagnetic structures on resistive-wall-mode stability of reversed field pinches. *Phys. Rev. Lett.*, 100:255005, 2008.
[7] M. Baruzzo, T. Bolzonella, S.C. Guo, Y. Q. Liu, G. Marchiori, R. Paccagnella, A. Soppelsa, F. Villone, and Z.R. Wang. 3D effects on RWM physics in RFX-mod. *Nucl. Fusion*, 51:083037, 2011.
[8] G. Marchiori, M. Baruzzo, T. Bolzonella, Y.Q. Liu, A. Soppelsa, and F. Villone. *Plasma Phys. Control. Fusion*, 52:023020, 2012.
[9] P. Bettini, L. Marrelli, and R. Specogna. Calculation of 3D magnetic fields produced by MHD active control systems in fusion devices. *IEEE Trans. on Magnetics*, DOI: 10.1109/TMAG.2013.2279141, in press.
[10] E. Tonti. *The Mathematical Structure of Classical and Relativistic Physics.* Birkhäuser Basel, 2013.

Table 1. Geometric elements (nodes, hexahedra, edges) of the mesh. Corresponding degrees of freedom (DOFs) and non zero entries (NNZs) of the system matrix.

Nodes	819,789
Hexahedra	807,408
Edges	2,447,132
DOFs (with gauge)	1,752,331
NNzs (with gauge)	22,379,984