Spatial planning via extremal optimization enhanced by cell-based local search

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Abstract. A new treatment is presented for land use planning problems by means of extremal optimization in conjunction to cell-based neighborhood local search. Extremal optimization, inspired by self-organized critical models of evolution has been applied mainly to the solution of classical combinatorial optimization problems. Cell-based local search has been employed by the author elsewhere in problems of spatial resource allocation in combination with genetic algorithms and simulated annealing. In this paper it complements extremal optimization in order to enhance its capacity for a spatial optimization problem. The hybrid method thus formed is compared to methods of the literature on a specific characteristic problem. It yields better results both in terms of objective function values and in terms of compactness. The latter is an important quantity for spatial planning. The present treatment yields significant compactness values as emergent results.

1.Introduction

Land use planning and spatial resource allocation are important and challenging management activities, leading to mathematical optimization problems of high complexity, often solved by heuristic methods. Spatial optimization problems of this type have been treated in the context of forest planning ([1] and [2]) and land use design ([3], [4]) by utilizing concepts of simulated annealing, genetic algorithms and cellular automata; also, in the context of groundwater management ([5], [6]) and for spatial resource allocation ([7], [8], [9]), along with multi-objective treatments ([10], [11]).

A land use planning problem with a nonlinear objective function and by the use of simulated annealing has been treated in [12]. The present paper deals with a similar problem, which is typical of its kind and may be considered to possess generic characteristics. The same instance of the problem has been presented in the literature ([3], [13]) and in this way direct comparisons will be possible. The basic method followed here will be extremal optimization. It will be accompanied by a cell-based local search, thus forming a new hybrid.

Extremal optimization (EO) is a stochastic method originating in self- organized critical models of co-evolution, such as the Bak–Sneppen model [14]. The method has been employed for the solution of classical combinatorial optimization problems, such as the graph bi-section [15] and the traveling salesman problem [16]. EO has also been extended to continuous optimization problems [17] and to multi-objective ones [18]. No application of EO to a land use or spatial optimization problem is noted in the literature, at least to the present author's knowledge. This paper presents such an application, along with comparisons to alternative methods of the literature. For the specific trial problem the EO cell-based hybrid of this paper exhibits a better performance in comparison to results of the literature.

Moreover, compactness of the resulting configurations is significant, as amply documented in the literature (e.g. [19]). In this work compactness comes out as an emergent result.

2. Problem description and formulation

A fictive two-dimensional grid is considered divided into cells, each one of which represents a land block. Each land block will be assigned a specific land use taken from a finite set of possible land uses. Each one of the land uses entails a certain development cost depending on the location to which it is applied. The problem consists in determining an optimal distribution of land uses over the blocks of the area under study, so as to minimize the total development cost.

Following [7], let (i, j) be the coordinates of the center of the typical cell with i= 1,2,…a and j=1,2,..,b, where a and b are the lengths of the two sides of the orthogonal grid. The blocks can be numbered consecutively, so that the cell number k runs as k = 1,2,…,a•b.

Let L = {1,2,…, a•b } be the set of cells as numbered according to the above numbering scheme and let W={1,2,..,m} be the set of land uses, numbered from 1 to m:

Let p: L $\longrightarrow$ W be a function that assigns a land use to each cell. This function gives rise to a set

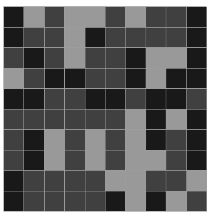
$$C = p(L) = \{p(1),., p(a \bullet b)\} = \{w_1,.., w_{a \bullet b}\} \qquad (1)$$

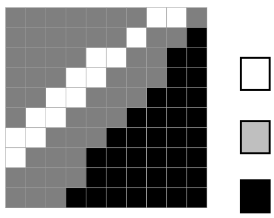
where $$w_k = p(k) \in W, \quad k = 1,2,...,a \bullet b$$

The set C of Equation (1) will be called the configuration of the land use mosaic as depicted in figure 1. The objective function F is a functional that maps such possible configurations to the set of positive real numbers:

$$F : C \to \mathbb{R}^{+}$$

The objective functional represents the development cost associated with the distribution of land uses. figure 2 shows the development cost as a function of location. Each one of the cell-blocks carries one of three different price lists for the various possible land uses.





Lu1	Lu2	Lu3
750	400	1500
2700	1450	6000
5550	3000	13500

Fig. 1. Typical configuration Fig. 2. Land use development cost in terms of location

Let PL be a price list containing the m respective development costs, such that PL(w) is development cost for land use w. Also, let PL_k be the price list attached to cell k, according to figure 2. Then $PL_k(w_k)$ will be the development cost for the land use w_k corresponding to cell k, according to Equation 1. Thus the local development cost will be

$$f_k = PL_k(w_k) \qquad (2)$$

The total cost will be

$$F = \sum_{k=1}^{a \bullet b} f_k \qquad (3)$$

To summarize, F depends on the configuration C as defined in Equation (1) and it forms the functional to be minimized. A constraint will be added to the formulation of the problem that prescribed fixed proportions for the numbers of the cells occupied by the corresponding land uses, namely let
$L_i = \{k\in L|w_k=i\}$, i = 1,2,.., m be the subset of L consisting of cells with land use i and let $M_i = |L_i|$ be the cardinality of L_i.
Then the constraint requires the M_i's to be constant and, of course,

$$\sum_{i=1}^{m} M_i = a \bullet b \tag{4}$$

The above constraint was not included in [12]. It is taken into account in References [3] and [13]. It will also be included in the present problem.

3. Method of solution

3.1 Extremal optimization

The general idea of the EO method is to eliminate undesirable or low-quality parts of possible solutions, in contrast to other optimization metaheuristics, such as genetic algorithms, that work by favoring high-quality solutions. According to the EO method, the constituent parts are evaluated and each one of them is assigned a fitness that represents its state. In the present problem, these parts are the cells numbered as k= 1,2,.,a•b and the corresponding fitness for cell k is characterized by Equation (2).

The cells are ranked according to their fitness. The cell with the worst fitness is selected and the following perturbation is performed: The selected cell is kept aside and another cell is picked at random from the rest of the configuration. Then the attributes of the two cells are exchanged (figure3). Thus the proportion of land uses is preserved and the constraint (Equation 4) is not violated.

A general pseudo-code for the EO method is the following:

a. Initialize the configuration C (Equation 1). Set $C_{best} = C$, $F_{best} = F(C)$.
b. Evaluate the fitness f_i of cell i, i=1, 2,…n, where n = a•b is the number of cells.
c. Find the index j such that $f_j \geq f_i \quad \forall i$
d. Perturb C and find $C' \in N(C)$ such that cell j will be modified as above.
e. Accept C′ unconditionally.
f. If $F(C') < F_{best}$, then set $C_{best} = C'$, $F_{best} = F(C')$.
g. Repeat until a maximum number of iterations is reached.

In order to relax the stringent condition of always having to pick the worst element in step c, a probability structure can be defined over the ranks of the cells m, m= 1,2,.,a•b. If m denotes a rank (m=1,2,..,n), then m=1 will be assigned to the cell with the worst fitness and the probability is equal to

$$P(m) : m^{-\tau}, \ 1 \leq m \leq a \bullet b \tag{5}$$

where τ is a parameter of the algorithm, taken as τ = 2.5.

The element to be picked will be chosen according to the probability of Equation (5), thus modifying step c. Therefore, the worst element will not necessarily be chosen. It will only have a high chance to be selected, since the probability distribution of Equation (5) favors the lower ranks.

The EO method enhanced with the stochastic selection is known as the τ-EO method [14].

It will be shown that the τ-EO method alone will not yield satisfactory results and for that reason it is complemented by a local search to be described in the next section.

3.2 Local neighborhood rule

In this section perturbations will be considered at the cell level. Before that, a neighborhood structure will be defined for each cell and then a rule will determine state changes of the cells within the neighborhood, utilizing concepts of cellular automata. More specifically, a von Neumann neighborhood [5] will be defined for each cell, as shown below, in figure 4. Obviously, this definition will be suitably adjusted for the boundary cells.

A local search will be conducted following a neighborhood rule, in the sense originally defined by Fotakis [20] and further explored in [10] and [11]. The neighborhood rule aims at improving the configuration on a local scale. For the present problem the local rule will be defined as follows:

Let $f_k = PL_k(w_k)$ as in Equation 2.

Let N_k denote the set of cells constituting the neighborhood of cell k and let l_k denote the cardinality of N_k.

Then for each $l = 1,2,..,l_k$ compute

$f_l = PL_l(w_l)$ again according to equation (2).

Set $g_k = f_k + f_l$.

Exchange attributes of cells k and l : $f_{k\leftarrow l} = PL_k(w_l) \quad f_{l\leftarrow k} = PL_l(w_k)$

Set $g_l = f_{k\leftarrow l} + f_{l\leftarrow k}$ and find index l_{min} such that $g_{l\,min} = \underset{l\le l\le l_k}{\mathrm{Min}}\{g_l\}$

If $g_{l_{min}} < g_k$ then exchange mutually the attributes of cells k and l_{min} (figure 4)

Otherwise no exchange takes place.

The above process is repeated for all k = 1,2,.., ab.

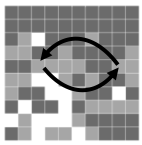


Fig. 3 Perturbation of step d

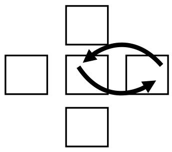


Fig. 4 Neighborhood rule

The neighborhood rule preserves by definition the proportion constraint (equation 4). The rule is applied to every generated configuration of step d in the above pseudo-code.

4. Results and discussion

The τ-EO method was applied first without the local neighborhood rule (NR) and then in conjunction with the rule. Three possible land uses were considered and their respective development costs are depicted in figure 2, where a 10x10 grid is shown. In both cases the initial mosaic contained 57 cells for land use 1, 29 cells for land use 2 and 14 cells for land use 3. Figure 5 shows the resulting best configuration after 200 iterations. The objective function value is 316350 and the compactness amounts to 274.

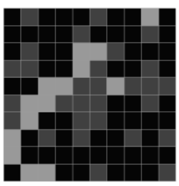


Fig. 5. Application of τ-EO alone.
F = 316350, compactness = 274

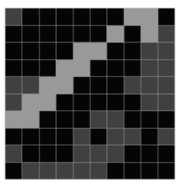


Fig. 6. Application of τ-EO with NR
F = 293500, compactness = 318

Better results were obtained by the conjunctive use of τ-EO with the neighborhood rule. After 200 iterations the objective function gave a value of 293500 with a compactness of 318 (figure 6).

These results compare directly with those obtained by Eldrandaly [13] by the use of expression programming. For exactly the same problem the corresponding results, after 1400 generations, gave a function value of 299850 with a compactness of 192. Therefore, the results of figure 6 point to a superior performance of τ-EO enhanced by the neighborhood rule.

Compactness is measured by counting, for each cell, the number of cells in its neighborhood that have the same attribute. References ([3] and [14]) give precise definitions. In contrast to general practice, compactness is not included here as part of the objective function or as a separate constraint. Significant compactness values come out as emergent results. The same feature has been demonstrated in other related works of the author [6]. Apparently, the neighborhood rule contributes positively to a greater compactness, as shown in those publications.

The issue of compactness is important for spatial optimization because compactness facilitates considerably the planning activity. For this reason, further investigation of compactness from the point of view of the present paper would be beneficial to the study of this aspect.

Also, the method presented here lends itself to the treatment of more involved spatial nonlinear functions. This can be realized, if, in addition to land use planning, the allocation of commodities over the land blocks is considered, along with the respective transportation cost. Finally, some more difficulty may be added, if the objective function is non-separable with respect to individual cell contributions, such as in problems of groundwater resource allocation, as it has been presented elsewhere in the context of genetic algorithms [12].

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