

Unsteady compressible flows in channel with varying walls

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Abstract. This study deals with numerical solution of a 2D and 3D unsteady flows of a compressible viscous fluid in 2D and 3D channel for low inlet airflow velocity. The unsteadiness of the flow is caused by a prescribed periodic motion of a part of the channel wall, nearly closing the channel during oscillations. The channels shape is a simplified geometry of the glottal space in the human vocal tract. Goal is numerical simulation of flow in the channels which involves attributes of real flow causing acoustic perturbations. The system of Navier-Stokes equations closed with static pressure expression for ideal gas describes the unsteady laminar flow of compressible viscous fluid. The numerical solution is implemented using the finite volume method and the predictor-corrector MacCormack scheme with artificial viscosity using a grid of quadrilateral cells. The unsteady grid of quadrilateral cells is considered in the form of conservation laws using Arbitrary Lagrangian-Eulerian method. The application of developed method for numerical simulations of flow fields in the 2D and 3D channels, acquired from a developed program, are presented for inlet velocity $u=4.12$ m/s, inlet Reynolds number $Re=4481$ and the wall motion frequency 100 Hz.

1. Introduction

A current challenging question is a mathematical and physical description of the mechanism for transforming the airflow energy in human vocal tract (convergent channel) into the acoustic energy representing the voice source in humans. The voice source signal travels from the glottis to the mouth, exciting the acoustic supraglottal spaces, and becomes modified by acoustic resonance properties of the vocal tract [1]. The airflow coming from the lungs causes self-oscillations of the vocal folds, and the glottis completely closes in normal phonation regimes, generating acoustic pressure fluctuations. In this study, the movement of the boundary channel is known, harmonically opening and nearly closing in the narrowest cross-section of the channel, making the investigation of the airflow field in the glottal region possible.

2. Governing equations

The system of Navier-Stokes equations has been used as mathematical model to describe the unsteady laminar flow of the compressible viscous fluid in a domain. The system is expressed in non-dimensional conservative form [2]:

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = \frac{1}{Re} \left(\frac{\partial \mathbf{R}}{\partial x} + \frac{\partial \mathbf{S}}{\partial y} + \frac{\partial \mathbf{T}}{\partial z} \right). \quad (1)$$



$\mathbf{W} = [\rho, \rho u, \rho v, \rho w, e]^T$ is the vector of conservative variables where ρ denotes density, (u, v, w) is velocity vector and e is the total energy per unit volume. \mathbf{F} , \mathbf{G} , \mathbf{H} are the vectors of inviscid fluxes and \mathbf{R} , \mathbf{S} , \mathbf{T} are the vectors of viscous fluxes. The static pressure p in inviscid fluxes is expressed by the state equation in the form

$$p = (\kappa - 1) \left[e - \frac{1}{2} \rho (u^2 + v^2 + w^2) \right], \quad (2)$$

where $\kappa = 1.4$ is the ratio of specific heats.

The transformation to the non-dimensional form uses inflow parameters (marked with the infinity subscript) as reference variables (dimensional variables are marked with the hat): the speed of sound $\hat{c}_\infty = 343 \text{ ms}^{-1}$, density $\hat{\rho}_\infty = 1.225 \text{ kg m}^{-3}$, temperature $\hat{T}_\infty = 293.15 \text{ K}$, dynamic viscosity $\hat{\eta}_\infty = 18 \cdot 10^{-6} \text{ Pa} \cdot \text{s}$ and a reference length $\hat{L}_r = 0.02 \text{ m}$.

General Reynolds number in (1) is computed from reference variables $\text{Re} = \hat{\rho}_\infty \hat{c}_\infty \hat{L}_r / \hat{\eta}_\infty$. The non-dimensional dynamic viscosity in the dissipative terms is a function of temperature in the form $\eta = (T/T_\infty)^{3/4}$.

2.1. Computational domains and boundary conditions

The bounded computational domain D_1 used for the numerical solution of flow field in the 2D channel is shown in Figure 1. The domain is symmetric channel, the shape of which is inspired by the shape of the trachea (inlet part), vocal folds, false vocal folds and supraglottal spaces (outlet part) in human vocal tract. The upper and the lower boundaries are the channel walls. A part of the walls changes its shape between the points A and B according to given harmonic function of time and axial coordinate. The gap width is the narrowest part of the channel (in point C) and is oscillating between the minimum $g_{min} = 0.4 \text{ mm}$ and maximum $g_{max} = 2.8 \text{ mm}$. The computational domain D_2 for the numerical solution of airflow in 3D is square channel $L \times H \times H = 8 \times 0.8 \times 0.8$ (160 mm \times 16 mm \times 16 mm).

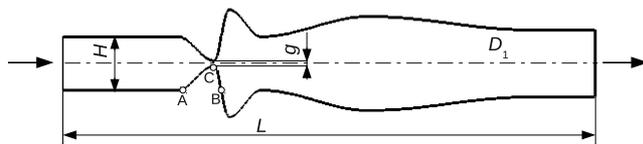


Figure 1. The 2D computational domain D_1 . $L = 8$ (160 mm), $H = 0.8$ (16 mm), $g = 0.08$ (1.6 mm) - middle position.

The boundary conditions are considered in the following formulation:

- (i) Upstream conditions: $u_\infty = \frac{\hat{u}_\infty}{\hat{c}_\infty}$; flow rate at the inlet is constant $H^2 \cdot u_\infty$; $\rho_\infty = 1$; p_∞ is extrapolated from domain.
- (ii) Downstream conditions: $p_2 = 1/\kappa$; $(\rho, \rho u, \rho v, \rho w)$ are extrapolated from domain.
- (iii) Flow on the wall: $(u, v, w) = (u_{wall}, v_{wall}, w_{wall})$ - velocity of the channel walls and for temperature $T = \kappa p / \rho$ is $\frac{\partial T}{\partial \vec{n}} = 0$.

The general Reynolds number in (1) is multiply with non-dimensional value $\frac{\hat{u}_\infty}{\hat{c}_\infty} H$ represents kinematic viscosity scale and for computation of the real problem inlet Reynolds number $\text{Re}_\infty = \hat{\rho}_\infty \hat{c}_\infty \frac{\hat{u}_\infty}{\hat{c}_\infty} H \hat{L}_r / \hat{\eta}_\infty$ is used.

3. Numerical solution

The numerical solution uses finite volume method (FVM) in cell centered form on the grid of quadrilateral cells, see e.g. [2]. In the time-changing domain, the integral form of FVM is derived using Arbitrary Lagrangian-Eulerian (ALE) formulation. The ALE method defines

homomorphic mapping of the reference domain $D_{t=0}$ at initial time $t = 0$ to a domain D_t at $t > 0$ [3]. The explicit predictor-corrector MacCormack (MC) scheme in the domain with a moving grid of quadrilateral cells is used. The scheme is 2nd order accurate in time and space [2]. The higher partial derivatives of velocity and temperature in viscous terms are approximated using dual volumes V'_q on each face q (on edge in 2D) as shown in Figure 2. The last term used in the MC scheme is the Jameson artificial dissipation $AD(W_{i,j,k})^n$ [4] for stability of computation.

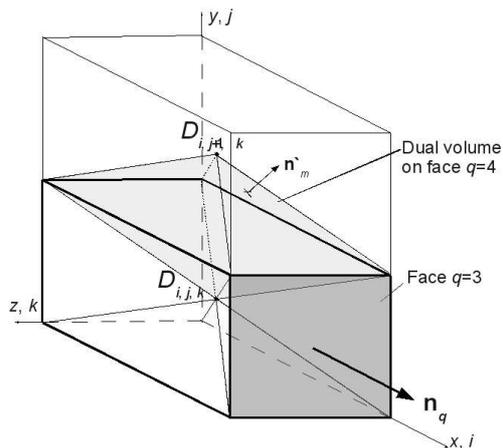


Figure 2. Finite volume $D_{i,j,k}$ and the dual volume V'_q in 3D domain.

The grid of the 2D channel have successive refinement cells near the wall. The minimum cell size in y - direction is $\Delta y_{min} \approx 1/\sqrt{\text{Re}_\infty}$ to resolve capture boundary layer effects.

4. Numerical results

The numerical results were obtained (using a specifically developed program) for the following input data: uniform inflow ratio velocity $\frac{\hat{u}_\infty}{c_\infty} = 0.012$ ($\hat{u}_\infty = 4.116 \text{ ms}^{-1}$), Reynolds number $\text{Re}_\infty = 4481$ and atmospheric pressure $p_2 = 1/\kappa$ ($\hat{p}_2 = 102942 \text{ Pa}$) at the outlet.

The computation in 2D channel has been carried out in two stages. First, a numerical solution is obtained, when the channel between points A and B has a rigid wall fixed in the middle position of the gap width (see Fig. 1). Then this solution is used as the initial condition for the unsteady simulation.

Figure 3 shows the unsteady flow fields computed in domain D_1 . Simulation is captured in four time instants during one vibration period (in the fourth cycle of the wall oscillation). The highest absolute maximum velocity ratio during one vibration period is $\frac{\hat{u}_{max}}{c_\infty} = 0.535$ ($\hat{u}_{max} = 183.5 \text{ ms}^{-1}$) at $g=1.002 \text{ mm}$ (opening phase). The application of the method for low Mach number in 3D domain D_2 is shown in Figure 4 with isolines of the velocity ratio.

5. Discussion and conclusions

The governing system (1) for flow of viscous compressible fluid based on Navier-Stokes equations for laminar flow is tested in 2D and 3D domains. In unsteady simulations (2D domain D_1) was possible to detect a ‘‘Coandă phenomenon’’ and large-scale vortices in the flow field patterns. The direction of the jet is independent on the coarseness of mesh but depends on the geometry of the channel, on the type of mesh in the domain, on the computational scheme [5] and on the governing system of flow. A similar generation of large-scale vortices, vortex convection and diffusion, jet flapping, and general flow patterns were experimentally obtained in physical models of the vocal folds by using Particle Image Velocimetry method in [6]. The method will be used for 3D simulation of unsteady flow in human vocal tract.

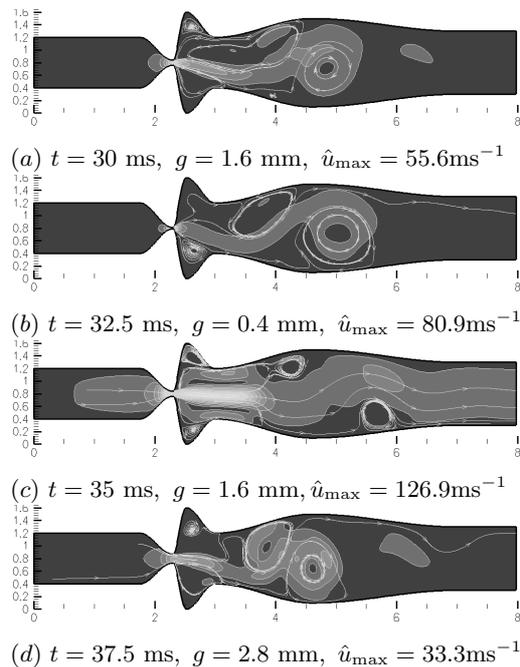


Figure 3. The unsteady numerical solution of the airflow in D_1 - $\hat{f} = 100$ Hz, $\frac{\hat{u}_\infty}{\hat{c}_\infty} = 0.012$, $\text{Re}_\infty = 4481$, $p_2 = 1/\kappa$, 450×100 cells. Data computed during the fourth oscillation cycle. Results are mapped by iso-lines of velocity ratio and by streamlines.

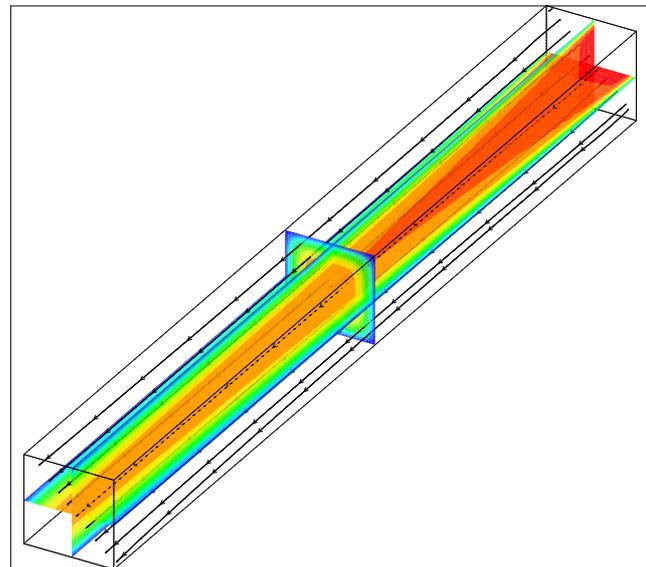


Figure 4. The steady numerical solution of the airflow in D_2 - rigid walls, $\frac{\hat{u}_\infty}{\hat{c}_\infty} = 0.012$, $\text{Re}_\infty = 4481$, $p_2 = 1/\kappa$, $100 \times 60 \times 60$ cells. Results are mapped by iso-lines of velocity ratio and by streamlines.

Acknowledgments

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