A heterogeneous fleet vehicle routing model for solving the LPG distribution problem: A case study

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Abstract: Vehicle Routing Problem (VRP) is an important management problem in the field of distribution and logistics. In VRPs, routes from a distribution point to geographically distributed points are designed with minimum cost and considering customer demands. All points should be visited only once and by one vehicle in one route. Total demand in one route should not exceed the capacity of the vehicle that assigned to that route. VRPs are varied due to real life constraints related to vehicle types, number of depots, transportation conditions and time periods, etc. Heterogeneous fleet vehicle routing problem is a kind of VRP that vehicles have different capacity and costs. There are two types of vehicles in our problem. In this study, it is used the real world data and obtained from a company that operates in LPG sector in Turkey. An optimization model is established for planning daily routes and assigned vehicles. The model is solved by GAMS and optimal solution is found in a reasonable time.

Keywords: Vehicle routing; Heterogeneous fleet; Distribution; LPG sector

1. Introduction

Tremendous advance of production, transportation and communication means eased to access knowledge, technology and resulted in a rough competition among companies. In order to survive in this competition, companies should carry out their activities more efficiently, faster and less costly. They examine their processes and try to eliminate unnecessary costs. Distribution of goods is a kind of such activities. Companies make studies and investments in this field to get the processes under control.

Increased globalization and offshore sourcing has created a massive increase in the complexity of supply lines and transportation networks. This has led firms to give more importance to the transportation and distribution function and its associated long term design decisions. Essentially transportation is tried to consider as a whole process to benefit from the all modes of the transportation. Transportation decisions include transportation mode selection, shipment size, vehicle routing, and scheduling, all of which are directly related to the location of transshipment points, warehouses, customer and factories [1].

According to the time horizon for the problems considered in the transportation decisions literature, the studies can be gathered under the strategic, tactical and operational decision levels. In this study we concentrated on the operational decision level. Operational level consists of daily and real time

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decisions and requires transactional data. Operational (short-term) decisions are performed in a highly dynamic environment where the time factor plays an important role. It includes the routing and dispatching of vehicles and crews; the allocation of scarce resources.

In a vehicle routing problem (VRP) a fleet of m vehicles is available to visit the customers. Vehicle k has a capacity of Q_k and the duration of its route cannot exceed D_k. The VRP consists in designing m vehicle routes of minimum total cost such that (i) each route starts and ends at the depot; (ii) each customer is visited by exactly one route; and (iii) the total demand of the customers visited by route k does not exceed Q_k and its duration does not exceed D_k. The VRP is a hard combinatorial optimization problem. [2]

In the VRP, while most solution methods have assumed a single depot and a homogeneous fleet, real-world problems usually include multiple terminals and a finite set of vehicles with non-uniform capacity. Moreover, the best solution for the VRP that minimizes the total distance traveled by the vehicles to visit all the customers often includes undesirable high waiting times. A more suitable VRP cost function should be a combination of fixed vehicle utilization costs and variable operational costs with the latter ones including distance and travel times, waiting time and service time costs [3].

The heterogeneous fleet vehicle routing problem (HVRP) is a problem of simultaneously determining the composition and routing of a heterogeneous fleet of vehicles in order to service a pre-specified set of customers with known delivery demands from a central depot. The number of vehicles of each type is assumed to be unlimited. The HVRP consists of designing a set of vehicle routes, each starting and ending at the depot, and such that each customer is visited exactly once, the total demand of a route does not exceed the capacity of the vehicle assigned to it, and the total cost is minimized [4].

The HVRPs have been studied since mid 80s. Some papers published in this field are [5], [6], [7], [8].

The paper is organized as follows. Section 2 presents the heterogeneous fleet vehicle routing model for a LPG distribution company. The results of the proposed model are reported in section 3 and also Section 3 concludes the paper.

2. A heterogeneous fleet vehicle routing model for a LPG distribution company

Distribution of materials is a challenging problem faced by major industrial and construction companies. In the same manner, in our study it is used the real world data for this kind of distribution or routing problem and obtained from a company (we called as ABC Company) that operates in Liquefied Petroleum Gas (LPG) sector in Turkey. ABC Company is one of the biggest distributors of LPG sector in Turkey. The logistics department of the company can see the current gas levels of the stations by software, and calculates the daily demands. The distribution routes are determined daily, heuristically. The purpose of this study is finding optimal routes for the daily demands of 13 stations in city X. The optimization model is constructed considering customer demands, station locations, vehicle types and vehicle capacities. Optimization model is solved by GAMS for the real data for the demands in the 2nd of April 2012 obtained from the company.

Tanks are used for distributing LPG to the stations in the company. There are two types of tanks, namely T1 and T2. T1 and T2 have a capacity of 10.5 and 15 tones of LPG, respectively. There are 9 T2 tanks and one Type-1 tank in the fleet of the company.

The real data of daily routes and their costs for the previous days are taken from the company first. Then the variable costs of the roads between stations are determined. Also, the coordinates of the stations are taken from the company and are used as inputs, and the distances between stations are calculated by using the website [9]

The main assumptions of the model are as follows. It is assumed that there is no time restriction. Traffic disruptions are ignored. The same unit costs are determined for both directions between stations. A station may be visited by more than one vehicle. The transportation cost per kilometer for vehicle T1 is 70 cents while it is 100 cents for vehicles T2. The company owns the tanks, so there is no difference in fixed cost whether the vehicles are assigned to any route or not.

The depot is assigned to the first node, and each customer is also assigned to a node. The number of customers visited is limited to 3 because the company aims to distribute more gas to any station at one visit.

The notations used and the mathematical model of the problem is as follows:

Notations:

- c_{ijk} : the variable cost of the arc between the node i and node j
- m_k : the unit cost of transportation for the kth vehicle (cent/km)
- u_{ij} : the distance between the node i and node j
- x_{ijk} : 0-1 variable for the usage of the arc between the nodes i and j by kth vehicle
- Q_k : the capacity of kth vehicle
- d_j : demand of the node j
- e_i : demand of the node i

Objectives:

$$min \sum_{i=1}^{14}\sum_{j=1}^{14}\sum_{k=1}^{10} x_{ijk} * c_{ijk} \qquad (1)$$

Constraints:

$$\sum_{k=1}^{10}\sum_{j=1}^{14} \mathbf{x_{ijk}} = \mathbf{1}, \qquad \mathbf{for\ i} = \{\mathbf{2,3,4,\ldots,14}\} \qquad \mathbf{(2)}$$

$$\sum_{k=1}^{10}\sum_{i=1}^{14} \mathbf{x_{ijk}} = \mathbf{1}, \qquad \mathbf{for\ i} = \{\mathbf{2,3,4,\ldots,14}\} \qquad \mathbf{(3)}$$

$$\sum_{j=1}^{14} x_{1jk} = 1, \qquad \mathbf{for}\ \forall k \qquad \mathbf{(4)}$$

$$\sum_{i=1}^{14} x_{i1k} = 1, \qquad \mathbf{for}\ \forall k \qquad \mathbf{(5)}$$

$$\sum_{i=1}^{14}\sum_{j=1}^{14} \mathbf{x_{ijk}} * \mathbf{d_j} \leq \mathbf{Q_k}, \qquad \textbf{for } \forall k \tag{6}$$

$$\sum_{i=1}^{14}\sum_{j=1}^{14} \mathbf{x_{ijk}} * \mathbf{d_j} = \sum_{i=1}^{14}\sum_{j=1}^{14} \mathbf{x_{ijk}} * \mathbf{e_i}, \qquad \textbf{for } \forall k \tag{7}$$

$$x_{ijk} + x_{jik} \leq 1 \;\; for\; i = \{2,3,4, \dots, 14\}, j = \{2,3,4, \dots, 14\} \; and \; \forall k \tag{8}$$

$$\sum_{i=2}^{14}\sum_{j=2}^{14} \mathbf{x_{ijk}} \leq \mathbf{2}, \qquad \textbf{for } \forall k \tag{9}$$

$$x \in \{0,1\} \tag{10}$$

$$c_{ijk} = m_k * u_{ij}, \qquad \textbf{for} \quad \forall i, j \; ve \; k \tag{11}$$

Equation (1) is the objective function. The objective is to minimize the total transportation cost. Equation (2) and equation (3) state that each customer should be visited once by any vehicle if the customer assigned to route of that vehicle. Equation (4) and equation (5) ensure that each vehicle assigned to any route should leave and arrive at depot once. Equation (6) is the capacity constraint for each vehicle. Equation (7) forbids the allocation of the same node by different vehicles. Equation (8) prevents using any assigned route in reverse direction. Equation (9) states that at most 3 customers should be visited by any vehicle. Equation (10) says that x is a binary variable. Equation (11) states that the total transportation cost between the node i and the node j for vehicle k is equal to multiplication of the unit transportation cost of the vehicle and the distance of between the nodes.

3. Results and conclusions

The model of the problem is solved optimally by using GAMS software in about 15 minutes. The total cost is calculated as 47791.4 cents. One type-1 vehicle and six type-2 vehicles are assigned to the routes. The routes for the vehicles are as follows:

Vehicle 1 (type 1): 1-11-9-1
Vehicle 2 (type 2): 1-14-7-1
Vehicle 3 (type 2): 1-6-13-1
Vehicle 4 (type 2): 1-12-4-2-1
Vehicle 5 (type 2): 1-3-8-1
Vehicle 7 (type 2): 1-10-1
Vehicle 9 (type 2): 1-5-1

The model decided not to use vehicles 6, 8, and 10. However, considering the fact that, 9 vehicles are identical, there are alternative solutions.

In this paper we presented the heterogeneous vehicle routing problem which is capable of handling a variety of real-life requirements. In our study, an optimization model for determining the daily routes for real customers is constructed. The optimal solution is found. Because the company has been setting the routes heuristically, the optimal solution found by the model is absolutely better than or equal to a heuristic solution. In the real world operational systems are always customer specific. For this reason our problem and proposed model has proven to be an interesting research topic with many possibilities for future research

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