Skew Divergence-Based Fuzzy Segmentation of Rock Samples

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Abstract. Digital image segmentation is a process in which one assigns distinct labels to different objects in a digital image. The MOFS (Multi Object Fuzzy Segmentation) algorithm has been successfully applied to the segmentation of images from several modalities. However, the traditional MOFS algorithm fails when applied to images whose composing objects are characterized by textures whose patterns cannot be successfully described by simple statistics computed over a very restricted area. Here, we present an extension of the MOFS algorithm that achieves the segmentation of textures by employing adaptive affinity functions that use the Skew Divergence as a measure of distance between two distributions. These affinity functions are called adaptive because their associated area (neighborhood) changes according to the characteristics of the texture being processed. We performed experiments on mosaic images composed by combining rock sample images which show the effectiveness of the adaptive skew divergence based fuzzy affinity functions.

1. Introduction

Digital image segmentation is the process of assigning distinct labels to different objects in a digital image. The task of segmenting out an object from its background in an image becomes particularly hard for a computer when, instead of the intensity values, what distinguishes the object from the background is some textural property.

Textures are generally described as the feel or shape of a surface or substance, and in the areas of imaging they are characterized by statistical distributions of the observed intensities. Textures can be classified as structural or non-structural [1], where structural textures have a repeated pattern associated with a texture periodicity [2], and non-structural or statistical textures do not possess this characteristic and may be described by statistical distributions.

In this article, we are concerned with the segmentation of images formed by a composition of samples of natural non-structural textures, by using a semi-automatic, interactive region-growing segmentation method, called *fuzzy segmentation* [3, 4]. The fuzzy segmentation method is used with adaptive fuzzy affinity functions, whose scales are related to the statistical distribution of the textural patterns they are associated with, and the Skew divergence to compute how close two pixels are in texture space.

2. Related Work

One example of an unsupervised texture segmentation technique is the one proposed by Unser and Eden [5], that extracts local texture properties using local linear transforms, estimates

local statistics at the output of a filter bank and generates a multi-resolution sequence using an iterative Gaussian smoothing algorithm. The texture features are then reduced to a single component that is thresholded to produce the segmentation.

There are also several model-based approaches [6, 7] that deal with the texture segmentation problem by modeling the intensity field of textures as a Markov or a Gauss-Markov random field to represent the local spatial dependencies between pixel intensities. However, these techniques are usually very computationally intensive because they require a large number of iterations to converge. Lehmann [8] proposed an alternative approach that models two-dimensional textured images as the concatenation of two one-dimensional hidden Markov auto-regressive (HMM-AR) models, one for the lines and one for the columns.

3. Fuzzy Segmentation

Because of the general nature of the approach used, we refer to elements of the set V to be segmented as *spels*, which is short for *spatial elements* [9]. The spels can be pixels of an image (as in [4]), but they can also be dots in the plane (as in [10]), or any variety of other things, such as feature vectors (as in [11]). Thus, the theory and algorithm mentioned here can also be applied to data clustering [12] in general.

The goal is to partition V into a specified number of objects, but in a fuzzy way, by assigning a spel to a particular object with a grade of membership (a number between 0 and 1) to that object. In order to do that, we assign, to every ordered pair (c, d) of spels, a real number not less than 0 and not greater than 1, which is referred to as the *fuzzy connectedness* of c to d.

In the original approach [4], fuzzy connectedness is defined in the following general manner. We call a sequence of spels a *chain*, and its *links* are the ordered pairs of consecutive spels in the sequence. The strength of a link is also a fuzzy concept. We say that the *ψ-strength* of a link is the value of a *fuzzy spel affinity* function $\psi : V^2 \to [0, 1]$. A chain is formed by one or more links and the *ψ-strength of a chain* is the ψ-strength of its weakest link; the ψ-strength of a chain with only one spel in it is 1 by definition.

The fuzzy connectedness from c to d is computed using a *fuzzy connectedness* function $\mu_\psi : V^2 \to [0, 1]$ defined by the ψ-strength of the strongest chain from c to d. We can then define the *ψ-connectedness map* f of a set V for a seed spel o as the picture formed by the fuzzy connectedness values of o to c ($f(c) = \mu_\psi(o, c)$), for all $c \in V$.

An *M-semisegmentation* of V is a function σ that maps each $c \in V$ into an $(M + 1)$-dimensional vector $\sigma^c = (\sigma_0^c, \sigma_1^c, \cdots, \sigma_M^c)$, such that σ_0 is non-negative but not greater than 1, for each m ($1 \leq m \leq M$), the value of σ_m^c is either 0 or σ_0^c, and, for at least one m ($1 \leq m \leq M$), $\sigma_m^c = \sigma_0^c$. Finally, an *M-segmentation* of V is an M-semisegmentation such that $\sigma_0^c > 0$ for every spel $c \in V$. There is an unique *M-segmentation* asociated with a set V and sets of seed spels V_m, for $1 \leq m \leq M$, that is n fact the segmentation of that set.

For more information related to the fuzzy segmentation algorithm, the reader should refer to [4], where the FAST-MOFS algorithm [4] is introduced. The FAST-MOFS is a greedy algorithm that computes the grades of membership of all spels to the objects rounding them to three decimal places. This allows us to use an array as a priority queue, instead of a heap, thus lowering the computational complexity of the algorithm [4].

3.1. Fuzzy Segmentation of Textures

The way we specified ψ_m and V_m ($1 \leq m \leq M$) for segmenting an image is the following. We click on some spels in the image to identify them as belonging to the mth object, and V_m is formed by these points and their eight neighbors. We then define two Gaussian density functions with the means and standard deviations computed based on the average values and the absolute differences of pairs of spels in V_m. In [4] the fuzzy affinity function between two spels was then

defined as the average value of these two Gaussians that receive as parameters the average value of the two spels c and d and the absolute difference between them.

The first term of the fuzzy affinity function described above encodes a local degree of homogeneity while the second term captures some inhomogeneity information, that can be construed as a very basic textural information.

In this work, in order to include more information about textural properties in the segmentation process, we use adaptive fuzzy affinity functions whose neighborhood areas are selected according to the distribution of the intensities surrounding the selected seed spels. This selection defines a scale which characterizes the correspondent texture. As mentioned before, the original fuzzy affinity functions used the intensity values of two spels c and d to compute the strength of the link connecting c to d. Here, we collect, opposed to what was done in [4], information of the intensities in the neighborhood area (scale) defined for the texture in question. For more details about that, the reader should refer to [13].

There are several techniques for detecting the window size that characterizes the texture periodicity pattern [2] and even orientation [14] of structural textures. However, such area is not well-defined in the case of non-structural textures, that can exhibit localized distributions of intensities in smaller scales and very different distributions observed at larger scales.

The algorithm used for choosing the appropriate scale works by computing statistics for a neighborhood of size 3×3 (i.e., with diameter equal to 1) surrounding the seed spels, and increasing this neighborhood until the differences of the statistics collected on the neighborhoods of two consecutive sizes are smaller than predefined thresholds. The neighborhood sizes used for each object can also be part of the user input to the segmentation method.

To give the algorithm a notion of distance between the distributions surrounding two pixels, i.e., between the histograms of the pixels belonging to the neighborhoods of these two pixels, we use a measure based on the Kullback-Leibler Divergence [16], namely, the Skew Divergence [15], that is used to compute how far two probability distributions are, and are respectively given by:

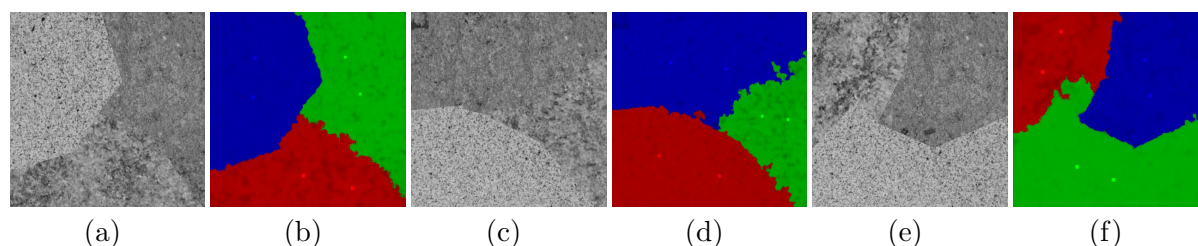$$\mathcal{KL}(p \parallel q) = \sum_x p(x) log \frac{p(x)}{q(x)}, \tag{1}$$

$$\mathcal{S}(p \parallel q) = \mathcal{KL}(q \parallel \alpha p + (1-\alpha)q). \tag{2}$$

The Skew Divergence (2) circumvents an indetermination problem that happens with the Kullback-Leibler Divergence (1) when there is an index x such that $p(x) > 0$ and $q(x) = 0$, and it does that by mixing the two distributions according to the parameter α. In the experiments shown in this paper, this parameter was set to 0.77. Note, from Equations (1) and (2), that both divergences are asymmetric. The general definition of the fuzzy affinity functions employed in MOFS does not limit the usage of asymmetric functions, but one can use the average value of the divergences in both directions as a symmetric measure, if wanted.

4. Results and Discussion

We tested our method on mosaics that were composed by putting together parts of three image samples of different rocks. Each rock sample has a statistical pattern that is recognizable by a human, but poses a challenge for segmentation algorithms. The samples and the mosaics were downloaded from the *The Prague Texture Segmentation Data Generator and Benchmark* [17]. Three of the produced mosaics (with 512×512 pixels each) can be seen in Figure 1, alongside their respective segmentations. In these segmentations, the color of one spel codifies two informations, as the brightness indicates the grade of membership of that spel to the object indicated by the spel's hue.

The three segmentations were performed on a personal computer equipped with 2 Intel Core$^{\text{TM}}$ i7 3.07GHz, with 6GB of RAM running Linux, and took between $1.9s$ and $2.25s$, i.e.,

| (a) | (b) | (c) | (d) | (e) | (f) |

Figure 1. Examples of mosaics produced by combining two Brodatz Textures (a), (c) and (e), and their correspondent connectedness maps (b), (d) and (f).

between $7.25\mu s$ and $8.58\mu s$ per spel. The accuracy reported for the three mosaics shown are 97.58%, 96.43% and 96.01%. It can be seen from the results shown in Figure 1 that the results provide good segmentations for all three textures present in the three mosaics.

5. Conclusion

In this article, we showed how the fuzzy segmentation method can be used to segment images with textural properties by employing adaptive affinity functions and the Skew Divergence. The adaptive fuzzy affinity functions allows the method to use different neighborhood sizes for textures with different characteristics, while the Skew Divergence gives a robust measure of distance between the distributions of the areas surrounding two spels.

As of future work, we are currently working on automatizing the segmentation process for this specific application, thus allowing the method to work without the need of an user indicating the seed spels, as well as on extending this method to color images.

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