Journal of Physics: Conference Series **489** (2014) 012072 doi:10.1088/1742-6596/489/1/012072

Impact of Image Noise on Gamma Index Calculation

M Chen, X Mo, D Parnell, G Olivera, D Galmarini, and W Lu

21st Century Oncology, USA

mchen@rtsx.com

Abstract. Purpose: The Gamma Index defines an *asymmetric* metric between the evaluated image and the reference image. It provides a quantitative comparison that can be used to indicate sample-wised pass/fail on the agreement of the two images. The Gamma passing/failing rate has become an important clinical evaluation tool. However, the presence of noise in the evaluated and/or reference images may change the Gamma Index, hence the passing/failing rate, and further, clinical decisions. In this work, we systematically studied the impact of the image noise on the Gamma Index calculation. Methods: We used both analytic formulation and numerical calculations in our study. The numerical calculations included simulations and clinical images. Three different noise scenarios were studied in simulations: noise in reference images only, in evaluated images only, and in both. Both white and spatially correlated noises of various magnitudes were simulated. For clinical images of various noise levels, the Gamma Index of measurement against calculation, calculation against measurement, and measurement against measurement, were evaluated. Results: Numerical calculations for both the simulation and clinical data agreed with the analytic formulations, and the clinical data agreed with the simulations. For the Gamma Index of measurement against calculation, its distribution has an increased mean and an increased standard deviation as the noise increases. On the contrary, for the Gamma index of calculation against measurement, its distribution has a decreased mean and stabilized standard deviation as the noise increases. White noise has greater impact on the Gamma Index than spatially correlated noise. Conclusions: The noise has significant impact on the Gamma Index calculation and the impact is asymmetric. The Gamma Index should be reported along with the noise levels in both reference and evaluated images. Reporting of the Gamma Index with switched roles of the images as reference and evaluated images or some composite metrics would be a good practice.

1. Introduction

The Gamma Index defines an asymmetric metric between two images for quantitative image comparison [1, 2]. The dosimetric tolerances, e.g. the dose difference (DD) of 3% and the distance-to-agreement (DTA) of 3 mm, are formulated as weighting factors in the metric, so that the Gamma Index can be used as an indicator for dosimetric pass (<=1) or fail (>1). However, image noise can change the Gamma Index value and thus change the passing/failing rate, and further, clinical decision. For example, the flat panel imager (portal imager) has a certain level of electronic noise. By using different image resolutions, it results in different noise statistics and different Gamma passing rates.

The noise effect in the Gamma Index was studied in [3, 4] for simulated cases and Monte Carlo (MC) based dose calculation, respectively. During the conference, we were informed of the study [3], which was similar to ours but with different emphasis: their emphasis was on the MC dose calculation, while ours was more on the portal comparisons. Due to the asymmetry of the Gamma Index, noise in


Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

the two images, the evaluated (searching) image and the reference (searched) image, could cause opposite effects. Here we refer to the image in which the samples are looking for the closest match, as the evaluated (searching) image, and the other image, in which the samples are being searched, as the reference (searched) image. Intuitively, the presence of noise in the reference image makes the search more likely to find a good match between the two compared images, so that the Gamma Index becomes lower and the passing rate higher. On the other hand, assuming the difference of the evaluated and the reference image is caused by noise, the presence of noise in the evaluated image moves the searching samples away from the reference and makes it harder to find a good match, so that the Gamma Index becomes higher and the passing rate lower. For simple cases, such as a constant reference image, we could establish analytic formulation to describe the statistics of the Gamma Index, assuming a certain statistics of noise. For other cases, the analytic formulations become very complicated, so numerical simulations were used instead to obtain the statistics of the Gamma Index. We also look at the effect of noise in portal image comparison.

2. Methods

The Gamma Index of an image g against another image f is defined as

$$\Gamma\left(g(x), f\right) = \min_{y}\left(\sqrt{\left(\frac{f(y)-g(x)}{D}\right)^2 + \left(\frac{y-x}{d}\right)^2}\right) \tag{1}$$

where D and d are DD and DTA criteria, respectively.

For simple distributions, such as:

$$f(x) = a \tag{2}$$

and

$$g(x) = b + n(x) \tag{3}$$

where a and b are constant, and $n(x)$ is the noise at x depicted by a Gaussian distribution, the Gamma Index of $f(x)$ against g can be calculated as

$$\Gamma\left(f(x), g\right) = \min_{y}\left(\sqrt{\left(\frac{b-a+n(y)}{D}\right)^2 + \left(\frac{x-y}{d}\right)^2}\right) \tag{4}$$

and the Gamma Index of $g(x)$ against f can be calculated as

$$\Gamma\left(g(x), f\right) = \min_{y}\left(\sqrt{\left(\frac{b-a+n(x)}{D}\right)^2 + \left(\frac{x-y}{d}\right)^2}\right) = \left|\frac{b-a+n(x)}{D}\right| \tag{5}$$

Note that the difference $b-a$ can be absorbed into the mean μ of noise $n(x)$, i.e. $b-a$ can be added into μ. Therefore, without loss of generality, we can assume $b=a$. If the noise $n(x)$ has a mean μ and standard deviation σ, then it can be derived that $\Gamma(g(x),f)$ has a mean of

$$E\left(\left|\frac{X}{D}\right|\right) = \frac{\mu}{D}\cdot\mathbf{erf}\left(\frac{\mu}{\sqrt{2}\sigma}\right) + \sqrt{\frac{2}{\pi}}\frac{\sigma}{D}e^{\frac{-\mu^2}{2\sigma^2}} \tag{6}$$

where X denotes a Gaussian random variable of mean μ and standard deviation σ and E denotes the expectation. On the other hand, the statistics for $\Gamma\left(f(x), g\right)$ are much more complicated to derive. Therefore, we only simulated its statistics by varying the mean and standard deviation.

For simulations, we defined the image $f(x,y)=100$, for $|x|,|y|\le 5$ cm, and 0 otherwise, in a 20×20 cm^2 field, and the image $g(x,y)=f(x,y)+n(x,y)$, where $n\sim N(\mu,\sigma)$ is Gaussian distributed. We simulated various μ and σ and summarized the results of the Gamma Index in terms of its mean and passing rate. We also simulated the impact of correlated noise, which were generated by convolving the noise n with local averaging kernels of various sizes.

For real data, we compared portal images of the same treatment beam but in different fractions and compared the measured portals with the corresponding calculated portals. Reduction of noise standard deviation (STD) σ was effected by local averaging, and σ was estimated using panel data outside of the treatment field. We then calculated the Gamma Index passing rate with respect to the noise STD. We also calculated DD and plotted its passing rate, based on the 3% criteria, along with the Gamma passing rate.

3. Results

In Figure 1, we show the mean of the Gamma Index calculated using the formula (6) for a noisy evaluated image against a noise free reference image. The mean is displayed as the image intensity and plotted against the noise mean μ (Y-axis) and STD σ (X-axis).

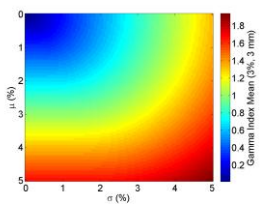
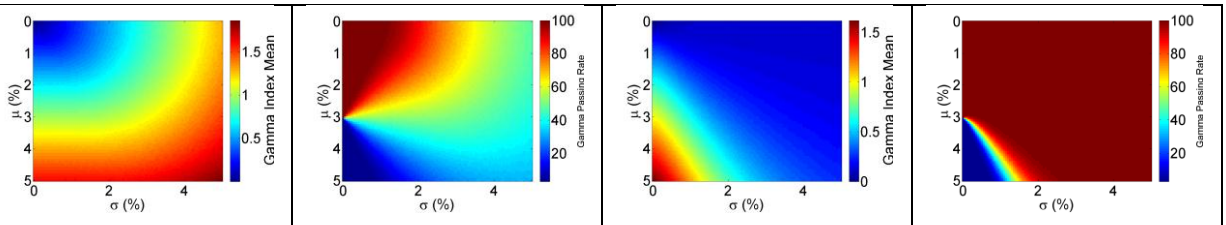


Figure 1 The mean of the Gamma Index calculated using the formula (6).

Figure 2 are the results of numerical simulations for the Gamma Index mean and the passing rate of $\Gamma(f(x),g)$ and $\Gamma(g(x),f)$. Because of the symmetry with respect to $\mu=0$, we only plot the results for $\mu\ge 0$. Note that the result of simulation (Figure 2 (a)) agrees with that of calculation based on the formula (Figure 1). We can see that the mean of the Gamma Index increases and the Gamma passing rate decreases, as the mean and the STD of noise in the searching image increase (Figure 2 (a)), and the mean of the Gamma Index decreases, as the STD of noise in the searched image increases.



(a) mean of $\Gamma(g(x),f)$ (b) pass rate of $\Gamma(g(x),f)$ (c) mean of $\Gamma(f(x),g)$ (d) pass rate of $\Gamma(f(x),g)$

Figure 2 Numerical simulations. The Gamma Index was calculated based on 3%, 3 mm criteria. (a) The mean of $\Gamma(g(x),f)$. (b) The passing rate of $\Gamma(g(x),f)$. (c) The mean of $\Gamma(f(x),g)$. (d) The passing rate of $\Gamma(f(x),g)$.

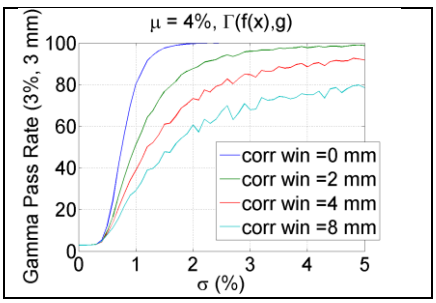
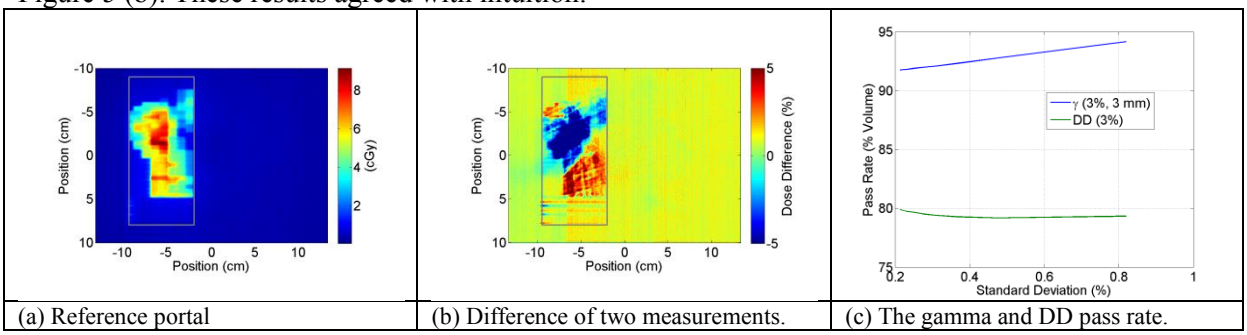


Figure 3 Impact of noise correlation. The legend describes the window sizes of the convolution kernels.

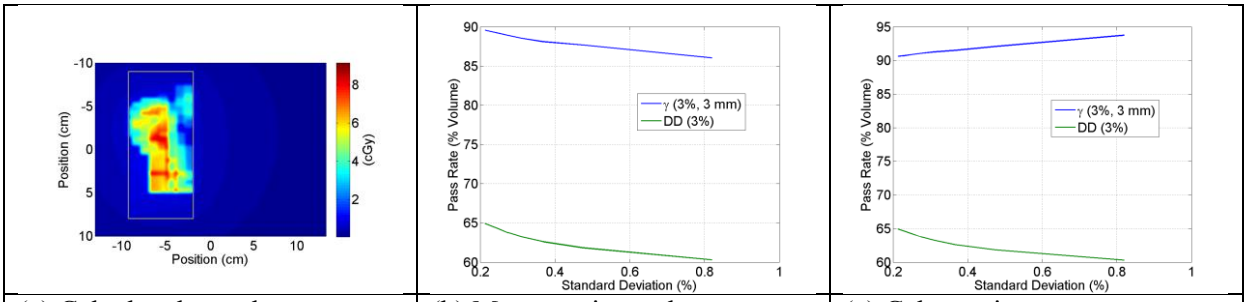
Figure 3 indicates that the noise correlation increase the mean of the Gamma Index and thus decreases the passing rate. The 0 mm correlation window indicates white noise. In Figure 4 and Figure 5, we show the results of portal measurement to measurement and measurement to calculation, respectively. Figure 4 shows (a) the reference portal, (b) the difference of two

portals of the same beam in different fractions, and (c) the Gamma and DD passing rate with respect to the noise STD. Figure 5 (a) shows the calculated portal for the same delivery setup of the measurement in Figure 4 (a). It can be seen that the impact of noise for clinical data is similar to that for simulations: noise in the reference image increases the Gamma passing rate, as shown in Figure 4 (c) and Figure 5 (c), and noise in the evaluated image decreases the Gamma passing rate, as shown in Figure 5 (b). These results agreed with intuition.



(a) Reference portal | (b) Difference of two measurements. | (c) The gamma and DD pass rate.

Figure 4 Clinical data. (a) The reference portal image. (b) The difference image (test-ref). (c) The Gamma passing rate and dose difference passing rate against noise STD σ.



(a) Calculated portal. | (b) Meas. against calc. | (c) Calc. against meas.

Figure 5 Measurement compared with calculation. (a) Calculated portal. The Gamma passing rate and DD passing rate w.r.t. noise STD for (b) calculation against measurement and (c) measurement against calculation.

4. Conclusions

Noise could have significant impact on the Gamma Index calculation. For example, the impact of noise can be seen when using different image resolution: lowering image resolution reduces noise and thus results in an increased Gamma Index and a decreased Gamma Index passing rate. Hence, it is necessary to include the STD of image noise when reporting the Gamma Index, as it can be referenced as uncertainty for the Gamma Index evaluation.

References

[1] Low, D. A., Harms, W. B., Mutic, S. and Purdy, J. A. A technique for the quantitative evaluation of dose distributions 1998 *Med Phys* **25** 5 656-61
[2] Chen, M., Lu, W., Chen, Q., Ruchala, K. and Olivera, G. Efficient gamma index calculation using fast euclidean distance transform 2009 *Phys Med Biol* **54** 7 2037-47
[3] Graves, Y. J., Jia, X. and Jiang, S. B. Effect of statistical fluctuation in monte carlo based photon beam dose calculation on gamma index evaluation 2013 *Phys Med Biol* **58** 6 1839-53
[4] Low, D. A. and Dempsey, J. F. Evaluation of the gamma dose distribution comparison method 2003 *Med Phys* **30** 9 2455-64