Dynamical mass generation – Majorana versus Dirac

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Abstract. We review our new simple dynamical symmetry breaking model for (softly broken) supersymmetric theory, with focus on the rich theoretical structure. In particular, the Holomorphic Supersymmetric Nambu–Jona-Lasinio offers competing symmetry breaking options related to the dynamical generation of Majorana or Dirac type superfield masses. It is relevant to the application of the model to electroweak symmetry breaking related to LHC physics.

1. Introduction to NJL Models with Supersymmetry

Let us introduce a sequence of three model Lagrangians : Firstly, there is the Nambu–Jona-Lasinio (NJL) model [1] of 1961 with the Lagrangian

$$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \ . \tag{1}$$

Then, there is the first supersymmetric version, or the (old) Supersymmetric Nambu–Jona-Lasinio (SNJL) model introduced in 1982 [2, 3];

$$\mathcal{L}_\psi^S = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1 - \tilde{m}^2\theta^2\bar{\theta}^2) + \int d^4\theta \ g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_-(1 - \tilde{m}_c^2\theta^2\bar{\theta}^2) \ , \tag{2}$$

with a dimension six four-superfield interaction. And finally, our focus here of the Holomorphic Supersymmetric Nambu–Jona-Lasinio (HSNJL) model published in 2010 [4];

$$\mathcal{L}_\psi^H = \int d^4\theta \left(\Phi_+^\dagger\Phi_+ + \Phi_-^\dagger\Phi_-\right)(1 - \tilde{m}^2\theta^2\bar{\theta}^2) - \int d^2\theta \ \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-(1 + B\theta^2) \ , \tag{3}$$

which has a dimension five four-superfield interaction.

The theoretical models are applicable to the dynamical breaking of electroweak symmetry. The NJL model for the top quark gives rises to the so-called top mode Standard Model [5] in connection with the idea of infared (quasi-)fixed point. The symmetry breaking vacuum expectation value (VEV) is a top condensate, giving the prediction then of a heavy top mass above 200 GeV. The SNJL model was similarly applied to give the Minimal Supersymmetry Standard Model (MSSM) also around 1990 [6]. Here with possibly two VEVs v and v', there is no direct mass prediction. The NJL kind of mechanism dictates only a large Yukawa coupling, here also for the top quark. The top mass as $m_t = y_t \cdot v$ can admit a lighter top with a small v lower than the full electroweak symmetry breaking value from $\sqrt{v^2 + v'^2}$. The much lighter

bottom mass is from a smaller bottom Yukawa coupling; $m_b = y_b \cdot v'$. However, to the 172.1 GeV top, a small $\tan\beta$ value, < 1.5, as the ratio of the two VEVs is required. The latter has essentially been ruled out. [1] The HSNJL model was constructed with getting around the problem in mind. The practical version has a four-superfield interaction that involves both the top and bottom quarks (superfields). Both scalar and fermion condensates, as part of the superfield condensates, play important roles. It gives an experimentally viable MSSM with $y_t < y_b$ (large $\tan\beta$) with other theoretically nice features [4, 7].

1.1. The Holomorphic Model as an Alternative Supersymmetrization

Supersymmetry is an important theme in modern physics. One of the specially attractive feature, in our opinion, is that scalar fields are now part of the chiral superfields with the chiral fermions. The chirality forbids any gauge invariant mass before breaking any symmetry. Moreover, the full matter (super)field spectrum is now strongly constrained by the gauge symmetry and their anomaly cancelation conditions. Introduction of the vectorlike pair of Higgs superfields with their un-natural gauge invariant mass in the usual formulation of the supersymmetric Standard Model looks particularly unattractive from the theoretical perspective. An NJL mechanism, with the Higgs superfield(s) generated as composite and the electroweak scale generated by strong dynamics is hence very appealing. The HSNJL model construction [4, 8] gives exactly such a scenario that looks compatible with all known experimental constraints. It looks like the only surviving model for electroweak symmetry breaking with a NJL type mechanism.

The NJL is a classic model on dynamical symmetry breaking, characterized by simplicity and beauty. The easy way to understand its physics is to assume that large enough coupling g^2, a bi-fermion scalar composite ϕ will form as the auxiliary field in the Lagrangian

$$
\begin{aligned}
&\mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar\psi_+\bar\psi_-) \\
= \quad & i\bar\psi_+\sigma^\mu\partial_\mu\psi_+ + i\bar\psi_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar\psi_+\bar\psi_- + \phi\psi_+\psi_-) \, .
\end{aligned}
\tag{4}
$$

Equation of motion for ϕ gives $\phi = -g/\mu \, \bar\psi_+\bar\psi_-$, the composite, showing the Lagrangian to be essentially the same as $\mathcal{L}_\psi$. A nontrivial VEV of ϕ, and hence the bi-fermion condensate, gives the symmetry breaking Dirac mass.

To supersymmetrize the NJL model, one may naively put together all the Lagrangian terms replaced by their superfield versions :

$$
i\bar\psi_+\sigma^\mu\partial_\mu\psi_+ \longrightarrow \int d^4\theta \ \Phi_+^\dagger\Phi_+ \, , \qquad\qquad g^2\bar\psi_+\bar\psi_-\psi_+\psi_- \longrightarrow \int d^4\theta \ g^2\Phi_+^\dagger\Phi_-^\dagger\Phi_+\Phi_- \, ,
$$

$$
-\mu g \, \phi\psi_+\psi_- \longrightarrow \int d^2\theta \ \mu g \, \Phi\Phi_+\Phi_- \, , \qquad -\mu^2\phi^*\phi \longrightarrow \int d^2\theta \ \frac{\mu}{2}\Phi\Phi \, .
$$

The above does not work. The problem is that the supersymmetrization of the dimension six four-femion interaction is a Kähler potential term while that of the sort of equivalent Higgs mass and Yukawa terms are superpotential terms. To reconcile the conflict, Ref.[2] suggested the introduction of an extra superfield different from the Higgs to play the composite, namely the modifications

$$
g^2\bar\psi_+\bar\psi_-\psi_+\psi_- \longrightarrow \int d^4\theta \ g^2\Phi'^\dagger\Phi' \qquad\qquad -\mu^2\phi^*\phi \longrightarrow \int d^2\theta \ \mu\Phi\Phi' \, .
\tag{5}
$$

One can see that the equation of motion for Φ gives $\Phi' = -g\,\Phi_+\Phi_-$ as the composite but Φ is the effective Higgs superfield. Soft supersymmetry breaking mass(es) is also needed for a $\langle\Phi\rangle \neq 0$ solution with Dirac mass for $\Phi_+\Phi_-$, and without further modifications, $\langle\Phi'\rangle$ stays zero.

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[1] The model has actually other not very nice features from the theoretical point of view [4, 7].

The HSNJL model is an alternative supersymmetrization of the NJL framework without enforcing the exact NJL Lagrangian terms to be contained in its component field expression. A chiral fermion is supersymmetrized to a chiral superfield. A supersymmetric analog for the four-fermion interaction may be a four-superfield interaction. The latter, besides the dimension six option, has a dimension five option which is holomorphic – a superpotential term like the Higgs mass and Yukawa couplings. Another way to think about it is to look at the supersymmetric form of the last line in the expression of the effective NJL Lagrangian of Eq.(4). It can be written, in the presence of soft supersymmetry breaking mass(es), as

$$\mathcal{L}_\psi^H = \int d^4\theta \left[(\Phi_+^\dagger \Phi_+ + \Phi_-^\dagger \Phi_-)(1 - \tilde{m}^2 \theta^2 \bar{\theta}^2) \right] + \int d^2\theta \left[\frac{\mu}{2} \Phi^2 + \sqrt{\mu G}\Phi\Phi_+\Phi_- \right] + h.c. \ .$$

The structure is exactly what is given by taking $\mathcal{L}_\psi^H$ as given, before the introduction of the auxiliary Higgs superfield, in Eq.(3) with the superpotential $\frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-(1 + B\theta^2)$ ($B = 0$ for the simplest version). Equation of motion for the Higgs superfield Φ gives $\Phi = -\sqrt{G/\mu}\Phi_+\Phi_-$, *i.e.* it is also the composite. Here one can see that by giving up the four-fermion interaction, we have a supersymmetric model that maintains essentially all other features of the NJL model nicely.

1.2. Towards the MSSM

In applying the NJL mechanism to electroweak symmetry breaking and the related low energy phenomenology, all earlier attempt focused only on the top quark. Only one Dirac pair of fermion (super)field is required to implement the mechanism anyway. The heavy mass of the top makes the natural candidate, only that it eventually turned up to be not heavy enough. To apply our holomorphic SNJL model to the MSSM, it is crucial to note that the Yukawa coupling for the bottom quark may be large, provided that we have a large $\tan\beta$. We take both of the third family quarks on the same footing and introduce the holomorphic superpotential term $W = G\,\varepsilon_{\alpha\beta}Q_3^{\alpha a}U_3^{c\,a}Q_3^{\beta b}D_3^{c\,b}(1 + B\theta^2)$. The two Higgs superfields of the MSSM may then be introduced as auxiliary superfields to rewrite W, with the $SU(2)$ and color indices suppressed, as

$$
\begin{aligned}
W &= -\mu\left(H_d - \lambda_t Q_3 U_3^c\right)\left(H_u - \lambda_b Q_3 D_3^c\right)(1 + B\theta^2) + G\, Q_3 U_3^c Q_3 D_3^c (1 + B\theta^2) \\
&= (-\mu H_d H_u + \mu\lambda_t Q_3 H_u U_3^c + \mu\lambda_b H_d Q_3 D_3^c)(1 + B\theta^2) , \qquad (\mu\lambda_t\lambda_b = G)\ .
\end{aligned}
\tag{6}
$$

Equation of motion for H_u gives $H_d = \lambda_t Q_3 U_3^c$ while that of H_d yields $H_u = \lambda_b Q_3 D_3^c$. Note that the supersymmetry breaking parameter B gives both the standard A and B parameters. Contrary to the old SNJL based model [6], we have a symmetric role for H_u and H_d as both are quark superfield composites. The symmetric structure does not apply to the soft SUSY breaking sector. Different soft masses for the different superfields would be the original of the nontrivial $\tan\beta$ value, hence hierarchy between top and bottom masses.

2. Analysis of Dynamical Symmetry Breaking

Dynamical mass generation and symmetry breaking is a very interesting theoretical topic with important phenomenological applications. One of the simplest model of the kind is the NJL model [1]. It is also the first explicit model of spontaneous symmetry breaking. The basic strategy to the non-perturbative analysis is to obtain the gap equation for the self-consistent mass parameter value and look for possible nontrivial solution, typically available under some strong coupling condition (see for example Ref.[9]). For the NJL case with a strong enough (dimension six) four-fermion interaction involving two Weyl fermions, a symmetry breaking Dirac fermion mass would be resulted. The HSNJL model mimics well most basic features of

the NJL model in the setting of softly broken supersymmetry, with however a dimension five four-superfield interaction. The symmetry breaking and Dirac mass generation was established recently in Ref.[8].

Such an analysis for a soft broken supersymmetric theory is somewhat tricky, and published only recently in our paper [8] even for the case of the old SNJL model. The first important step there is to consider the (Dirac) mass parameter as one on the superspace [10]. It is like a constant (chiral) superfield, with also an admissible supersymmetry breaking auxiliary part:

$$\mathcal{M} = m - \theta^2 \eta \,, \tag{7}$$

where m is the supersymmetric(/fermion) mass, while η contributes a supersymmetry breaking scalar left-right mixing mass. This is easy to appreciate, for example, by thinking about the mass as coming from a Yukawa term with the Higgs (superfield) taking a VEV. With the parameter, one can obtain the superfield propagators incorporating the soft breaking [11]. The crucial step then is to consider the generating functional and related two-point correlation functions for the theory on the same level – as superspace entities. We have

$$\Gamma = \int \frac{d^4 p}{2\pi^4} \int d^2\theta \, \Phi_+(-p,\theta) \, \Gamma^{(2)}_{+-}(p,\theta^2) \, \Phi_-(p,\theta) + h.c. + \cdots \tag{8}$$

giving the gap equation formally as

$$-\mathcal{M} = \Sigma^{(loop)}_{+-}(p,\theta^2)\Big|_{\text{on-shell}} \,. \tag{9}$$

With the approach, we went through the supergraph calculation to obtained, for the first time [8], the gap equation for Dirac mass generation as given by

$$
\begin{aligned}
m &= 2mg^2 I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \,, \\
\eta &= -\eta \, g^2 \tilde{m}_C^2 \, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \,,
\end{aligned}
\tag{10}
$$

for the SNJL model (I_1 and I_2 are a couple of loop integrals the details of which we skip here). Considering the m equation along, assuming $\eta = 0$ or equivalently $\tilde{m}_C^2 = 0$, one obtains the known nontrivial solution obtained in Ref.[3] with a effective potential analysis based on the assumption on the formation of the Higgs superfields as composites and otherwise. The gap equation analysis in that sense proves the composite Higgs formation. And the full set of superspace gap equation offers more interesting general solutions.

For the more interesting case of the HSNJL model, we obtained

$$
\begin{aligned}
m &= \frac{\bar{\eta}G}{2} \, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \,, \\
\eta &= \bar{m}G \, I_1(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) + \frac{\bar{\eta}GB}{2} \, I_2(|m|^2, \tilde{m}^2, |\eta|, \Lambda^2) \,,
\end{aligned}
\tag{11}
$$

as a tightly coupled set of equations. One cannot see possible nontrivial solution when neglecting the η part. And at least for some simple limiting cases, we have established the existence of nontrivial solution.

3. Majorana versus Dirac Mass Generations

Unlike the SNJL model, the HSNJL model offers more mass generation options. The possibility of superfield condensate $\langle \Phi_+ \Phi_- \rangle$ is what gives rise to the Dirac mass term $\mathcal{M}\Phi_+\Phi_-$ from the holomorphic four-superfield interaction, explicitly, as $\frac{G}{2} \langle \Phi_+\Phi_- \rangle \Phi_+\Phi_-$. However, there

may be the option of say a condensate of $\langle \Phi_+ \Phi_+ \rangle$ or $\langle \Phi_- \Phi_- \rangle$ giving Majorana mass terms $\mathcal{M}_- \Phi_- \Phi_- \sim \frac{G}{2} \langle \Phi_+ \Phi_+ \rangle \Phi_- \Phi_-$ and $\mathcal{M}_+ \Phi_+ \Phi_+ \sim \frac{G}{2} \langle \Phi_- \Phi_- \rangle \Phi_+ \Phi_+$, respectively.

We have extended the analysis to consider the option [12]. The gap equations are given by

$$
\begin{aligned}
m_\pm &= \frac{\bar{\eta}_\mp G}{2} I_2(|m_\mp|^2, \tilde{m}_\mp^2, |\eta_\mp|, \Lambda^2) \,, \\
\eta_\pm &= \bar{m}_\mp G \, I_1(|m_\mp|^2, \tilde{m}_\mp^2, |\eta_\mp|, \Lambda^2) - \frac{\bar{\eta}_\mp GB}{2} I_2(|m_\mp|^2, \tilde{m}_\mp^2, |\eta_\mp|, \Lambda^2) \,.
\end{aligned}
\tag{12}
$$

Here we have allowed for unequal soft supersymmetry breaking masses of the two superfields. The first thing to note is that when the latter are equal, $\tilde{m}_+^2 = \tilde{m}_-^2$, the natural outcome will be $m_+ = m_-$ and $\eta_+ = \eta_-$, in which case the gap equations collapsed to one for the common parameters, which is exactly the same as the one for the Dirac case. Naively, it says the Majorana mass generation is feasible. A more careful thinking leads one to worry what will really be the outcome for the HSNJL model taken without assumption. The completing mass generation scenarios will imply plausibly different symmetry breaking directions.

The above has important implication for the extended model of applying to electroweak symmetry breaking Eq.(6), bringing into question if it does give us what we want. To address the question takes a highly nontrivial effort [13]. However, some careful thinking about the results we have so far gives interesting insight into what should be the basic features. Take the above gap equations for the limiting case that one of the soft mass vanishes, say $\tilde{m}_-^2 = 0$ (and $B = 0$), one can easily see that the Majorana mass generation option is killed. Nontrivial solution is no longer possible. If we consider the Dirac mass gap equation under the same condition, nontrivial solution survives well. In fact, the Dirac mass case is sensitive mostly only the the average of the two soft mass parameters [12]. Explicitly, the gap equations with unequal soft masses are given by

$$
\begin{aligned}
m &= \frac{\bar{\eta} G}{2} I_2(|m|^2, \tilde{m}_{av}^2, |\eta'|, \Lambda^2) \,, \\
\eta &= \bar{m} G \, I_1(|m|^2, \tilde{m}_{av}^2, |\eta'|, \Lambda^2) + \frac{\bar{\eta} GB}{2} I_2(|m|^2, \tilde{m}_{av}^2, |\eta'|, \Lambda^2) \,,
\end{aligned}
\tag{13}
$$

with

$$
\begin{aligned}
\tilde{m}_{av}^2 &= \frac{\tilde{m}_+^2 + \tilde{m}_-^2}{2} \,, \\
|\eta'| &= \sqrt{|\eta|^2 + \left(\frac{\tilde{m}_+^2 - \tilde{m}_-^2}{2}\right)^2} \,.
\end{aligned}
\tag{14}
$$

The line of thinking indicates strongly that a split in the soft supersymmetry breaking masses favors Dirac over Majorana masses. Under that situation, we expect for a certain range of the value of coupling, only Dirac type mass term will arise. This is a positive picture for the application to electroweak symmetry breaking as the phenomenologically required large $\tan\beta$ scenario fits well into it. [2] The implications for the phenomenological squark masses will be very interesting.

4. Concluding Remarks
The first message is the Holomorphic Supersymmetric Nambu–Jona-Lasinio model works well as a model of dynamical symmetry breaking and mass generation. The structure can be used

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[2] The presence of the strong QCD coupling also favors the formation of the colorless condensate for Dirac type masses instead of the color and charge breaking Majorana masses among the quark superfields.

to formulate a version of the MSSM as its low energy effective field theory, in which the Higgs superfields arise as composites of the top and bottom superfields. The version is more interesting compared to the one from the old SNJL model, both theoretically and phenomenologically.

From a superfield theoretical perspective, it is interesting to note the formulation of the analysis based on the consideration of the theoretical notion such as generating functions, correlation functions, and mass parameters as superspace entities with supersymmetric as well as supersymmetry breaking parts. The approach may be interesting, for example, for the application to spontaneous supersymmetry breaking.

The HSNJL model in general has more interesting mass generation, and hence symmetry breaking, options. It may generates both Dirac type and Majorana type mass terms. While we have not finish the most general analysis without prior assumptions, our studies so far has given a clear indication that the splitting in value of the input soft supersymmetry breaking masses favor the Dirac type mass term over the Majorana type ones. The latter goes in line with the application to MSSM where the splitting would be responsible for the hierarchy in the Dirac top and bottom masses generated.

Acknowledgments

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