Non-Abelian tensor multiplet in four dimensions

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Abstract. The long-standing problem with a non-Abelian tensor with non-trivial consistent couplings in four dimensions has been solved. The key technique is double-fold: (1) Adding extra Chern-Simons terms for the field strength of non-Abelian tensor, and (2) employing a compensator mechanism. We generalize this mechanism to supersymmetric system. Our system has three multiplets: (i) The usual non-Abelian vector multiplet (VM) $(A_\mu{}^I, \lambda^I)$, (ii) A non-Abelian tensor multiplet (TM) $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$, and (iii) A compensator vector multiplet (CVM) $(C_\mu{}^I, \rho^I)$. The indices ${\scriptstyle I,\, J,\, \cdots}$ are for the adjoint representation of a non-Abelian group G. All of our fields are propagating with kinetic terms. The $C_\mu{}^I$-field plays the role of a compensator absorbed into the longitudinal component of $B_{\mu\nu}{}^I$. We give both the component lagrangian and a corresponding superspace reformulation, reconfirming the total consistency of the system.

1. The Conventional Problem with Non-Abelian Tensors

First, the conventional problem with non-Abelian tensors will be discussed. A solution will be presented. Also presented will be a supersymmetrized physical non-Abelian field with consistent interactions [1], both in component language and superfield language.

The long-standing problem with a non-Abelian tensor is described as follows. Let ${\scriptstyle I}$ be the adjoint index of a non-Abelian group G gauged by the a non-Abelian vector field $A_\mu{}^I$, minimally coupled to the antisymmetric tensor $B_{\mu\nu}{}^I$ with the coupling constant g. Consider the conventional field strength[3]

$$G^{(0)}_{\mu\nu\rho}{}^I \equiv +3D_{[\mu}B_{\nu\rho]}{}^I \equiv +3(\partial_{[\mu}B_{\nu\rho]}{}^I + gf^{IJK}A_{[\mu}{}^J B_{\nu\rho]}{}^K) \quad , \tag{1.1}$$

where D_μ is the usual gauge-covariant derivative with the structure constant f^{IJK} of the group G. Consider an action $I_0 \equiv \int d^4x\, \mathcal{L}_0$ with the lagrangian[4]

$$\mathcal{L}_0 \equiv -\frac{1}{12}(G^{(0)}_{\mu\nu\rho}{}^I)^2 - \frac{1}{4}(F_{\mu\nu}{}^I)^2 \quad , \tag{1.2}$$

with $F_{\mu\nu}{}^I \equiv 2\partial_{[\mu}A_{\nu]}{}^I + gf^{IJK}A_\mu{}^J A_\nu{}^K$. Obviously, the B-field equation is[5]

$$\frac{\delta\mathcal{L}_0}{\delta B_{\mu\nu}{}^I} = +\frac{1}{2}D_\rho G^{(0)\mu\nu\rho I} \doteq 0 \quad . \tag{1.3}$$

[3] The formulation in this section is the so-called 'tensor hierarchy' [2, 3, 4, 5], but we re-formulate their general expressions in terms of our objectives here.
[4] We use the signature $(-,+,+,+)$ for four dimensions (4D) as in [1].
[5] The symbol $\doteq$ stands for a field equation, to be distinguished from an algebraic identity. We also use the symbol $\overset{?}{=}$ for an equality under question.

The problem is that the divergence of this B-field equation does *not* vanish:

$$0 \overset{?}{=} D_\nu \left(\frac{\delta \mathcal{L}_0}{\delta B_{\mu\nu}{}^I} \right) = +\frac{1}{4} g f^{IJK} F_{\nu\rho}{}^J G^{(0)\mu\nu\rho K} \neq 0 \quad , \tag{1.4}$$

unless $F_{\mu\nu}{}^I$ or $G^{(0)}_{\mu\nu\rho}{}^I$ vanishes trivially. This inconsistency arises already at the *classical* level. This is also one of the reasons, why topological formulations with vanishing field strengths $F_{\mu\nu}{}^I \doteq 0$, $G^{(0)}_{\mu\nu\rho}{}^I \doteq 0$ such as in [6] are easier to formulate for non-Abelian tensors.

An additional problem is related to the so-called local tensorial gauge transformation of the B-field:

$$\delta_\beta B_{\mu\nu}{}^I = +D_{[\mu} \beta_{\nu]}{}^I - D_{[\nu} \beta_{\mu]}{}^I \quad , \tag{1.5}$$

because the field strength $G_{\mu\nu}{}^I$ is *not* invariant under δ_β:

$$\delta_\beta G^{(0)}_{\mu\nu\rho}{}^I = +3 g f^{IJK} F_{[\mu\nu}{}^J \beta_{\rho]}{}^K \neq 0 \quad . \tag{1.6}$$

This further implies the *non*-invariance of the action: $\delta_\beta I_0 \neq 0$. These two problems are mutually related, because the non-vanishing of (1.4) is also re-casted into $\delta_\beta I_0 \neq 0$.

2. Solution to the Problem

The solution to the problem above is to introduce a non-trivial Chern-Simons (CS) term into the G-field strength:

$$\begin{aligned} G_{\mu\nu\rho}{}^I &\equiv +3(\partial_{[\mu} B_{\nu\rho]}{}^I + g f^{IJK} A_{[\mu}{}^J B_{\nu\rho]}{}^K) - 3 f^{IJK} C_{[\mu}{}^J F_{\nu\rho]}{}^K \\ &= +3 D_{[\mu} B_{\nu\rho]}{}^I - 3 f^{IJK} C_{[\mu}{}^J F_{\nu\rho]}{}^K \equiv +G^{(0)}_{\mu\nu\rho}{}^I - 3 f^{IJK} C_{[\mu}{}^J F_{\nu\rho]}{}^K \quad , \end{aligned} \tag{2.1}$$

where $C_\mu{}^I$ is a ‘compensator’ vector field, also carrying the adjoint index. The field strength for C is defined by

$$H_{\mu\nu}{}^I \equiv +D_{[\mu} C_{\nu]}{}^I - D_{[\nu} C_{\mu]}{}^I + g B_{\mu\nu}{}^I \quad . \tag{2.2}$$

Now these field strengths G and H are *invariant* under the δ_β-transformation

$$\delta_\beta B_{\mu\nu}{}^I = + D_{[\mu} \beta_{\nu]}{}^I - D_{[\nu} \beta_{\mu]}{}^I \tag{2.3a}$$

$$\delta_\beta C_\mu{}^I = - g \beta_\mu{}^I \quad , \tag{2.3b}$$

which is the ‘proper’ gauge transformation for $B_{\mu\nu}{}^I$, and δ_γ-transformation

$$\delta_\gamma B_{\mu\nu}{}^I = - f^{IJK} F_{\mu\nu}{}^J \gamma^K \quad , \tag{2.4a}$$

$$\delta_\gamma C_\mu{}^I = D_\mu \gamma^I \quad . \tag{2.4b}$$

which is the ‘proper’ gauge transformation for $C_\mu{}^I$. As (2.3b) shows, $C_\mu{}^I$ is a compensator field for the δ_β-transformation.

The role played by the $C \wedge F$-term in (2.1) is to cancel the unwanted term in (1.6). The C-field itself should have its own ‘gauge’ transformation as the covariant gradient (2.4b). The contribution of $\delta_\gamma (2 D_{[\mu} C_{\nu]}{}^I)$ in (2.2) is cancelled by the contribution of $\delta_\gamma (g B_{\mu\nu}{}^I)$. In other words, we have the total invariances

$$\delta_\beta (G_{\mu\nu\rho}{}^I, H_{\mu\nu}{}^I) = (0, 0) \quad , \qquad \delta_\gamma (G_{\mu\nu\rho}{}^I, H_{\mu\nu}{}^I) = (0, 0) \quad . \tag{2.5}$$

Accordingly, we also have the consistency problem (1.4) solved, by considering the lagrangian

$$\mathcal{L}_1 \equiv -\tfrac{1}{12}\,(G_{\mu\nu\rho}{}^I)^2 - \tfrac{1}{4}\,(H_{\mu\nu}{}^I)^2 - \tfrac{1}{4}\,(F_{\mu\nu}{}^I)^2 \quad . \tag{2.6}$$

The total action is also invariant $\delta_\beta I_1 = \delta_\gamma I_1 = 0$. The field equations for B and C-fields are

$$\frac{\delta \mathcal{L}_1}{\delta B_{\mu\nu}{}^I} = +\tfrac{1}{2}\, D_\rho G^{\mu\nu\rho\, I} - \tfrac{1}{2}\, g H^{\mu\nu\, I} \doteq 0 \quad , \tag{2.7a}$$

$$\frac{\delta \mathcal{L}_1}{\delta C_\mu{}^I} = -D_\nu H^{\mu\nu\, I} + \frac{1}{2} f^{IJK} F_{\rho\sigma}{}^J G^{\mu\rho\sigma\, K} \doteq 0 \quad , \tag{2.7b}$$

The divergence of the B-field equation vanishes now:

$$0 \stackrel{?}{=} D_\nu \left(\frac{\delta \mathcal{L}_1}{\delta B_{\mu\nu}{}^I} \right) = +\frac{1}{2} g \left(\frac{\delta \mathcal{L}_1}{\delta C_\mu{}^I} \right) \doteq 0 \quad , \tag{2.8}$$

where the last equality holds because of the C-field equation. In other words, the unwanted FG-term in (1.4) is now cancelled by the contribution of the C-field equation.

Relevantly, the divergence of (2.7b) also vanishes, as it should without any inconsistency:

$$0 \stackrel{?}{=} D_\mu \left(\frac{\delta \mathcal{L}_1}{\delta C_\mu{}^I} \right) = + f^{IJK} F_{\mu\nu}{}^J \left(\frac{\delta \mathcal{L}_1}{\delta B_{\mu\nu}{}^K} \right) \doteq 0 \quad . \tag{2.9}$$

We emphasize repeatedly that these invariances have never been accomplished without the peculiar CS terms both in (2.1) and (2.2) [2, 3, 4, 5].

3. Component Formulation of N=1 TM

The supersymmetrization of the purely bosonic system (2.6) has been accomplished in our recent paper [1]. We need three multiplets: (i) A tensor multiplet (TM) $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$, (ii) A Yang-Mills vector multiplet (YMVM) $(A_\mu{}^I, \lambda^I)$, and (iii) A compensating vector multiplet (CVM) $(C_\mu{}^I, \rho^I)$. Our total action $I \equiv \int d^4x\, g^2 \mathcal{L}$ has the lagrangian [1]

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{12}(G_{\mu\nu\rho}{}^I)^2 + \frac{1}{2}(\overline{\chi}^I \not{D} \chi^I) - \frac{1}{2}(D_\mu \varphi^I)^2 - \frac{1}{2} g^2 (\varphi^I)^2 - g(\overline{\chi}^I \rho^I) \\
& - \frac{1}{4}(H_{\mu\nu}{}^I)^2 + \frac{1}{2}(\overline{\rho}^I \not{D} \rho^I) - \frac{1}{4}(F_{\mu\nu}{}^I)^2 + \frac{1}{2}(\overline{\lambda}^I \not{D} \lambda^I) \\
& - \frac{1}{2} g f^{IJK} (\overline{\lambda}^I \chi^J) \varphi^K + \frac{1}{2} f^{IJK} (\overline{\lambda}^I \gamma^\mu \rho^J) D_\mu \varphi^K + \frac{1}{12} f^{IJK} (\overline{\lambda}^I \gamma^{\mu\nu\rho} \rho^J) G_{\mu\nu\rho}{}^K \\
& + \frac{1}{4} f^{IJK} (\overline{\rho}^I \gamma^{\mu\nu} \chi^J) F_{\mu\nu}{}^K - \frac{1}{4} f^{IJK} (\overline{\lambda}^I \gamma^{\mu\nu} \chi^J) H_{\mu\nu}{}^K - \frac{1}{2} f^{IJK} F_{\mu\nu}{}^I H^{\mu\nu\, J} \varphi^K \quad ,
\end{aligned} \tag{3.1}$$

up to quartic-order terms $\mathcal{O}(\phi^4)$, and the coupling constant g has the dimension of mass.

The scalar φ^I has its mass g, while there is a mixture between χ^I and ρ^I with the same mass g. As has been mentioned after (2.4), $C_\mu{}^I$ is a compensator field [7], absorbed into the longitudinal component of $B_{\mu\nu}{}^I$. The kinetic term of the C-field becomes the mass term of $B_{\mu\nu}{}^I$. Accordingly, the degrees of freedom (DOF) for the massive TM fields are $B_{\mu\nu}{}^I$ (3), χ with ρ^I (4) and $\varphi^I(1)$.

Our action I is invariant under global $N = 1$ supersymmetry [1]

$$\delta_Q B_{\mu\nu}{}^I = + (\bar\epsilon\gamma_{\mu\nu}\chi^I) - 2f^{IJK} C_{[\mu|}{}^J (\delta_Q A_{|\nu]}{}^K) \quad , \tag{3.2a}$$

$$\begin{aligned}\delta_Q \chi^I = &+ \frac{1}{6}(\gamma^{\mu\nu\rho}\epsilon) G_{\mu\nu\rho}{}^I - (\gamma^\mu\epsilon) D_\mu \varphi^I \\ &+ \frac{1}{2} f^{IJK} \Big[+ \epsilon(\overline{\lambda}{}^J \rho^K) - (\gamma_5\gamma^\mu\epsilon)(\overline{\lambda}{}^J\gamma_5\gamma_\mu\rho^K) - (\gamma_5\epsilon)(\overline{\lambda}{}^J\gamma_5\rho^K)\Big] \quad ,\end{aligned} \tag{3.2b}$$

$$\delta_Q \varphi^I = + (\bar\epsilon\chi^I) \quad , \tag{3.2c}$$

$$\delta_Q C_\mu{}^I = + (\bar\epsilon\gamma_\mu\rho^I) + f^{IJK} (\bar\epsilon\gamma_\mu\lambda^J)\varphi^K \quad , \tag{3.2d}$$

$$\begin{aligned}\delta_Q \rho^I = &+ \frac{1}{2}(\gamma^{\mu\nu}\epsilon) H_{\mu\nu}{}^I - g\epsilon\varphi^I - \frac{1}{2} f^{IJK} (\gamma^{\mu\nu}\epsilon) F_{\mu\nu}{}^J \varphi^K \\ &+ \frac{1}{4} f^{IJK} \Big[+ \epsilon(\overline{\lambda}{}^J\chi^K) - (\gamma^\mu\epsilon)(\overline{\lambda}{}^J\gamma_\mu\chi^K) + \frac{1}{2}(\gamma^{\mu\nu}\epsilon)(\overline{\lambda}{}^J\gamma_{\mu\nu}\chi^K) \\ &\qquad - (\gamma_5\gamma^\mu\epsilon)(\overline{\lambda}{}^J\gamma_5\gamma_\mu\chi^K) - (\gamma_5\epsilon)(\overline{\lambda}{}^J\gamma_5\chi^K)\Big] \quad ,\end{aligned} \tag{3.2e}$$

$$\delta_Q A_\mu{}^I = + (\bar\epsilon\gamma_\mu\lambda^I) \quad , \tag{3.2f}$$

$$\delta_Q \lambda^I = + \frac{1}{2}(\gamma^{\mu\nu}\epsilon) F_{\mu\nu}{}^I + \frac{1}{2} f^{IJK} (\gamma_5\epsilon)(\overline{\rho}{}^J\gamma_5\chi^K) \quad , \tag{3.2g}$$

up to $\mathcal{O}(\phi^3)$-terms.

Our tensorial gauge transformation δ_β, and δ_γ-transformation are exactly the same as (2.3) and (2.4), while all the fermionic fields transform only under the usual non-Abelian gauge transformation δ_α, as the B and C-fields do, so that there is no problem with the $\delta_\beta I = 0$ and $\delta_\gamma I = 0$ of the field strengths as in (2.1) and (2.2), *via* (2.5).

The δ_Q-transformations of the field strengths reflect their CS terms:

$$\delta_Q G_{\mu\nu\rho}{}^I = + 3(\bar\epsilon\gamma_{[\mu\nu} D_{\rho]}\chi^I) + 3f^{IJK} (\delta_Q A_{[\mu}{}^J) H_{\nu\rho]}{}^K - 3f^{IJK} (\delta_Q C_{[\mu}{}^J) F_{\nu\rho]}{}^K \quad , \tag{3.3a}$$

$$\delta_Q H_{\mu\nu}{}^I = - 2(\bar\epsilon\gamma_{[\mu} D_{\nu]}\rho^I) + g(\bar\epsilon\gamma_{\mu\nu}\chi^I) + 2f^{IJK} D_{[\mu|} \Big[(\delta_Q A_{|\nu]}{}^J)\varphi^K \Big] \quad , \tag{3.3b}$$

$$\delta_Q F_{\mu\nu}{}^I = - 2(\bar\epsilon\gamma_{[\mu} D_{\nu]}\lambda^I) \quad . \tag{3.3c}$$

Since we have *not* added the D-auxiliary field, our YMVM and CVM have *on-shell* DOF 2+2, while *off-shell* DOF 3+4. However, our TM is in the *off-shell* formulation with the total off-shell DOF $4 + 4$, because the off-shell DOF of each field are $[(4-1)\cdot(4-2)]/2 = 3$ for $B_{\mu\nu}$, 4 for χ and 1 for φ.

The field equations for all of our fields are[6] [1]

$$\begin{aligned}&+ \not{D}\lambda^I - \frac{1}{2} g f^{IJK} \chi^J \varphi^K + \frac{1}{2} f^{IJK} (\gamma^\mu\rho^J) D_\mu\varphi^K \\ &\qquad - \frac{1}{4} f^{IJK} (\gamma^{\mu\nu}\chi^J) H_{\mu\nu}{}^K + \frac{1}{12} f^{IJK} (\gamma^{\mu\nu\rho}\rho^J) G_{\mu\nu\rho}{}^K \doteq 0 \quad ,\end{aligned} \tag{3.4a}$$

$$+ \not{D}\chi^I - g\rho^I + \frac{1}{2} g f^{IJK} \lambda^H \varphi^K - \frac{1}{4} f^{IJK} (\gamma^{\mu\nu}\lambda^J) H_{\mu\nu}{}^K + \frac{1}{4} f^{IJK} (\gamma^{\mu\nu}\rho^J) F_{\mu\nu}{}^K \doteq 0 \quad , \tag{3.4b}$$

$$+ \not{D}\rho^I - g\chi^I + \frac{1}{2} f^{IJK} (\gamma^\mu\lambda^J) D_\mu\varphi^K$$

[6] These equations are fixed up to $\mathcal{O}(\phi^3)$-terms, due to the quartic fermion terms in the lagrangian.

$$- \frac{1}{12} f^{IJK} (\gamma^{\mu\nu\rho} \lambda^J) G_{\mu\nu\rho}{}^K + \frac{1}{4} f^{IJK} (\gamma^{\mu\nu} \chi^J) F_{\mu\nu}{}^K \doteq 0 \ \ , \tag{3.4c}$$

$$\begin{aligned} &+ D_\nu F_\mu{}^{\nu\, I} + g f^{IJK} \varphi^J D_\mu \varphi^K + \frac{1}{2} g f^{IJK} (\overline{\lambda}^J \gamma_\mu \lambda^K) + f^{IJK} H_{\mu\nu}{}^J D^\nu \varphi^K \\ &\qquad - \frac{1}{2} f^{IJK} G_{\mu\rho\sigma}{}^J H^{\rho\sigma\, K} + \frac{1}{2} f^{IJK} (\overline{\chi}^J D_\mu \rho^K) + \frac{1}{2} f^{IJK} (\overline{\rho}^J D_\mu \chi^K) \doteq 0 \ \ , \end{aligned} \tag{3.4d}$$

$$\begin{aligned} &+ D_\rho G^{\mu\nu\rho\, I} - g H^{\mu\nu\, I} - \frac{1}{2} f^{IJK} D_\rho (\overline{\lambda}^J \gamma^{\mu\nu\rho} \rho^K) \\ &\qquad + g f^{IJK} F^{\mu\nu\, J} \varphi^K - \frac{1}{2} g f^{IJK} (\overline{\lambda}^J \gamma^{\mu\nu} \chi^K) \doteq 0 \ \ , \end{aligned} \tag{3.4e}$$

$$+ D_\mu^2 \varphi^I - g f^{IJK} (\overline{\lambda}^J \chi^K) - g^2 \varphi^I - \frac{1}{2} f^{IJK} F_{\mu\nu}{}^J H^{\mu\nu\, K} \doteq 0 \ \ , \tag{3.4f}$$

$$\begin{aligned} &+ D_\nu H^{\mu\nu\, I} - \frac{1}{2} f^{IJK} F_{\rho\sigma}{}^J G^{\mu\rho\sigma\, K} - \frac{1}{2} f^{IJK} (\overline{\chi}^J D^\mu \lambda^K) - \frac{1}{2} f^{IJK} (\overline{\lambda}^J D^\mu \chi^K) \\ &\qquad + \frac{1}{2} g f^{IJK} (\overline{\lambda}^J \gamma^\mu \rho^K) - f^{IJK} F^{\mu\nu\, J} D_\nu \varphi^K \doteq 0 \ \ . \end{aligned} \tag{3.4g}$$

In deriving each of these equations, we have also used other field equations.

4. Superspace Reformulation of N=1 TM

We can re-formulate our theory in superspace, as an independent consistency-reconfirmation.

Our superspace BIds for the superfield strengths $F_{AB}{}^I$, $G_{ABC}{}^I$ and $H_{AB}{}^I$ are[7]

$$+ \frac{1}{6} \nabla_{[A} G_{BCD)}{}^I - \frac{1}{4} T_{[AB|}{}^E G_{E|CD)} - \frac{1}{4} f^{IJK} F_{[AB}{}^J H_{CD)}{}^K \equiv 0 \ \ , \tag{4.1a}$$

$$+ \frac{1}{2} \nabla_{[A} H_{BC)}{}^I - \frac{1}{2} T_{[AB|}{}^D H_{D|C)}{}^I - g\, G_{ABC}{}^I \equiv 0 \ \ , \tag{4.1b}$$

$$+ \frac{1}{2} \nabla_{[A} F_{BC)}{}^I - \frac{1}{2} T_{[AB|}{}^D F_{D|C)}{}^I \equiv 0 \ \ . \tag{4.1b}$$

Our relevant superspace constraints at the mass dimensions $0 \le d \le 1$ are

$$T_{\alpha\beta}{}^c = +2(\gamma^c)_{\alpha\beta} \ \ , \quad G_{\alpha\beta c}{}^I = +2(\gamma_c)_{\alpha\beta}\, \varphi^I \ \ , \tag{4.2a}$$

$$G_{\alpha bc}{}^I = -(\gamma_{bc} \chi^I)_\alpha \ \ , \quad H_{\alpha b}{}^I = -(\gamma_b \rho^I)_\alpha - f^{IJK} (\gamma_b \lambda^J)_\alpha\, \varphi^K \ \ , \tag{4.2b}$$

$$F_{\alpha b}{}^I = -(\gamma_b \lambda^I)_\alpha \ \ , \quad \nabla_\alpha \varphi^I = -\chi_\alpha{}^I \ \ , \tag{4.2c}$$

$$\begin{aligned} \nabla_\alpha \chi_\beta{}^I = &- \frac{1}{6} (\gamma^{cde})_{\alpha\beta} G_{cde}{}^I - (\gamma^c)_{\alpha\beta} \nabla_c \varphi^I \\ &- \frac{1}{2} f^{IJK} \Big[+ C_{\alpha\beta} (\overline{\lambda}^J \rho^K) - (\gamma_5 \gamma^c)_{\alpha\beta} (\overline{\lambda}^J \gamma_5 \gamma_c \rho^K) - (\gamma_5)_{\alpha\beta} (\overline{\lambda}^J \gamma_5 \rho^K) \Big] \ \ , \end{aligned} \tag{4.2d}$$

$$\begin{aligned} \nabla_\alpha \rho_\beta{}^I = &+ \frac{1}{2} (\gamma^{cd})_{\alpha\beta} H_{cd}{}^I + g\, C_{\alpha\beta}\, \varphi^I - \frac{1}{2} f^{IJK} (\gamma^{cd})_{\alpha\beta} F_{cd}{}^J \varphi^K \\ &- \frac{1}{4} f^{IJK} \Big[+ C_{\alpha\beta} (\overline{\lambda}^J \chi^K) + (\gamma^c)_{\alpha\beta} (\overline{\lambda}^J \gamma_c \chi^K) - \frac{1}{2} (\gamma^{cd})_{\alpha\beta} (\overline{\lambda}^J \gamma_{cd} \chi^K) \\ &\qquad - (\gamma_5 \gamma^c)_{\alpha\beta} (\overline{\lambda}^J \gamma_5 \gamma_c \chi^K) - (\gamma_5)_{\alpha\beta} (\overline{\lambda}^J \gamma_5 \chi^K) \ \ , \end{aligned} \tag{4.2e}$$

$$\nabla_\alpha \lambda_\beta{}^I = + \frac{1}{2} (\gamma^{cd})_{\alpha\beta} F_{cd}{}^I - \frac{1}{2} (\gamma_5)_{\alpha\beta}\, f^{IJK} (\overline{\rho}^J \gamma_5 \chi^K) \ \ . \tag{4.2f}$$

[7] In this superspace section, we use the indices $A = (a,\alpha)$, $B = (b,\beta)$, $\cdots$ for superspace coordinates, where $a,\ b,\ \cdots = 0, 1, 2, 3$ (or $\alpha,\ \beta,\ \cdots = 1, 2, 3, 4$) are for bosonic (or fermionic) coordinates. In superspace, we use the (anti)symmetrization convention, *e.g.*, $X_{[AB)} \equiv X_{AB} - (-1)^{AB} X_{BA}$, different from our component formulation.

All other components, such as $G_{\alpha\beta\gamma}{}^I$ or $T_{\alpha\beta}{}^\gamma$ *etc.* at $d \le 1$ are zero.

Although most of technical details associated with superspace formulation are skipped here, we presented a rather independent confirmation for the total consistency of our $N = 1$ non-Abelian tensor multiplet in superspace.

5. Concluding Remarks

In this talk, we have explained how to formulate the $N = 1$ supersymmetrization in 4D of a physical non-Abelian tensor with consistent couplings [1]. This is the supersymmetrization of the special case [4] of the minimal tensor hierarchy [5], which, in turn, is a special case of more general hierarchy in [2][3]. Both the component and superspace formulations of our system are given, as the cross-verification of our system. Our CVM $(C_\mu{}^I, \rho^I)$ plays the role of a compensator multiplet, absorbed into the TM $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$, making the latter massive.

There exists certain problem for the quantization of Stueckelberg theory [7] for non-Abelian gauge groups [9]. This is because the longitudinal components of the gauge field do not decouple from the physical Hilbert space, so that the renormalizability and unitarity of the system are spoiled [9]. We take rather an optimistic standpoint about this potential problem for the following reasons. First, we mention that our theory is *not* renormalizable due to Pauli couplings. This feature is *not* necessarily a fatal drawback for our theory, because certain theories exist in 4D, such as non-linear sigma models that are *not* renormalizable, but are *not* rejected from the outset. Second, $N = 1$ supersymmetry may well improve quantum behavior of our theory, compared with non-supersymmetric systems. There is good chance that supersymmetries solve the quantum problem of non-Abelian Stueckelberg theories.

The importance of our result [1] is double-fold: (i) A new *supersymmetric* physical system with Stueckelberg mechanism that solves the problem with non-Abelian tensor is presented. (ii) The problem with extra vector fields in the non-singlet representation of a non-Abelian gauge group is now solved. We should also consider the possibility that $N = 1$ supersymmetry may well provide better quantum behavior compared with non-supersymmetric cases.

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