

S_3 flavour symmetry and the reactor mixing angle

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Abstract. In the framework of a minimal S_3 symmetric extension of the Standard Model, the mass matrices of all fermions take the same generic form with two texture zeroes. The neutrino mixing matrix, V_{PMNS} , is computed and exact, explicit analytical expressions for the neutrino mixing angles as functions of the masses of neutrinos and charged leptons are obtained in good agreement with the latest experimental data. We also compute the branching ratios of some selected flavour-changing neutral current (FCNC) processes, as well as the contribution of the exchange of neutral flavour-changing scalars to the anomaly of the magnetic moment of the muon, as functions of the masses of charged leptons and the neutral Higgs bosons.

1. Introduction

In the last ten years, flavour mixing research in the leptonic sector progressed from the fascinating exploratory stage of discovery to the more advanced stage of qualitative precision measurements of the observables in neutrino flavour oscillations. A global fit of the observables to the current experimental data in neutrino oscillations of solar, atmospheric, reactor and accelerator neutrinos yields the following results:

a) the differences of the squared masses, to 1σ , are [1]:

$$\begin{aligned}\Delta m_{31}^2 &= 2.47_{-0.067}^{+0.069} \times 10^{-3} \text{ eV}^2 \text{ (Normal hierarchy)}, & \Delta m_{21}^2 &= 7.50 \pm 0.185 \times 10^{-5} \text{ eV}^2, \\ \Delta m_{32}^2 &= -2.43_{-0.065}^{+0.042} \times 10^{-3} \text{ eV}^2 \text{ (Inverted hierarchy)},\end{aligned}\tag{1}$$

b) the magnitudes of the leptonic mixing matrix ($PMNS$) elements, to 3σ , are [1]:

$$|U_{PMNS}| = \begin{pmatrix} 0.795 \leftrightarrow 0.846 & 0.513 \leftrightarrow 0.585 & 0.126 \leftrightarrow 0.178 \\ 0.205 \leftrightarrow 0.543 & 0.416 \leftrightarrow 0.730 & 0.579 \leftrightarrow 0.808 \\ 0.215 \leftrightarrow 0.548 & 0.409 \leftrightarrow 0.725 & 0.567 \leftrightarrow 0.800 \end{pmatrix}.\tag{2}$$

Specially worth of notice is the recent measurement of the reactor mixing angle θ_{13}^{exp} , in the following range of values

$$7.2^\circ \lesssim \theta_{13}^{exp} \lesssim 10.25^\circ,\tag{3}$$

which can discriminate between different possible flavour symmetry groups as realized in neutrino mixing. However, there are some still unsolved problems, such as: i) the absolute value of the



neutrino masses; *ii*) the neutrino mass hierarchy. *iii*) the experimental detection of CP violation in the neutrino oscillation and *iv*) the Dirac or Majorana nature of the neutrinos.

The experimentally determined non vanishing and rather sizeable value of the reactor mixing angle θ_{13}^{exp} [2, 3, 4, 5, 6] opened a gate to the experimental and theoretically important research on CP violation in the leptonic sector as well as the determination of the neutrino masses hierarchy.

2. The S_3 -invariant Extension of the Standard Model

A convenient theoretical framework for a unified interpretation of the experimental data on masses and mixing of quarks and leptons is the minimal S_3 -invariant extension of the Standard Model of J. Kubo, A. Mondragón, M. Mondragón and E. Rodríguez Jauregui [7]. For a recent review see [8, 9]. In this model the concept of flavour and generations is extended to the Higgs sector by introducing in the theory three Higgs fields which are $SU(2)$ doublets, so that, all matter fields, quark, lepton and Higgs fields, have three families. The permutational group S_3 is introduced in the theory as a global flavour symmetry unbroken at the Fermi scale. Then, quark, lepton and Higgs fields are assigned to singlets and doublets of a reducible representation $\mathbf{2} \oplus \mathbf{1}_s$ of the flavour group S_3 . This multi-Higgs model has tree level flavour changing neutral currents whose exchange may give rise to lepton flavour violating processes and may also contribute to the anomalous magnetic moment of the muon [7, 8]. An effective test of the phenomenological success of the model is obtained by verifying that all flavour changing neutral current processes and the magnetic anomaly of the muon, computed in the S_3 -Invariant extended form of the Standard Model, agree with the experimental values [8, 10, 11].

The most general renormalizable Yukawa interactions for leptons are given by $\mathcal{L}_Y = \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}$, where

$$\begin{aligned}\mathcal{L}_{Y_E} = & -Y_1^e \bar{L}_I H_S e_{IR} - Y_3^e \bar{L}_3 H_S e_{3R} - Y_2^e [\bar{L}_I \kappa_{IJ} H_1 e_{JR} + \bar{L}_I \eta_{IJ} H_2 e_{JR}] \\ & - Y_4^e \bar{L}_3 H_I e_{IR} - Y_5^e \bar{L}_I H_I e_{3R} + \text{h. c.} , \\ \mathcal{L}_{Y_\nu} = & -Y_1^\nu \bar{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \bar{L}_3 (i\sigma_2) H_S^* \nu_{3R} - Y_4^\nu \bar{L}_3 (i\sigma_2) H_I^* \nu_{IR} \\ & - Y_2^\nu [\bar{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \bar{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR}] - Y_5^\nu \bar{L}_I (i\sigma_2) H_I^* \nu_{3R} + \text{h. c.} .\end{aligned}\quad (4)$$

In this expressions $I, J = 1, 2$, the singlets are denoted by Q_3 , u_{3R} , d_{3R} , L_3 , e_{3R} , ν_{3R} and H_S , and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \quad (5)$$

Then, the Yukawa interactions, eq. (4), yield the following generic form for the mass matrices of all Dirac fermions in the theory [7],

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}, \quad (6)$$

where the μ 's are, in general, complex since there is no restriction coming from flavour symmetry. For details see [7, 8]. Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos, $\mathcal{L}_M = -\nu_R^T \mathbf{C} \mathbf{M}_{\nu_R} \nu_R$, where $\mathbf{M}_{\nu_R} = \text{diag}\{M_1, M_2, M_3\}$ is the mass matrix for the right-handed neutrinos and \mathbf{C} is the charge conjugation matrix [8, 12]. The small masses of left-handed Majorana neutrinos are obtained through the type I seesaw mechanism: $\mathbf{M}_\nu = \mathbf{M}_{\nu_D} \mathbf{M}_{\nu_R}^{-1} (\mathbf{M}_{\nu_D})^T$, where \mathbf{M}_{ν_D} are the Dirac neutrino mass matrices.

3. The leptonic mass matrices and Z_2 symmetry

The number of parameters in the lepton sector may be further reduced by means of an Abelian Z_2 symmetry, which forbids the following Yukawa couplings [7] $Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$.

Therefore, the mass matrices of the charged leptons and Dirac neutrinos take the form [7, 8]

$$\mathbf{M}_e = m_\tau \begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{M}_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}. \quad (7)$$

respectively. The unitary matrix \mathbf{U}_{eL} that diagonalizes \mathbf{M}_e and enters in the definition of the neutrino mixing matrix \mathbf{U}_{PMNS} may be written in polar form as

$$\mathbf{U}_{eL} = \mathbf{P}_e \mathbf{O}_{eL}, \quad (8)$$

where $\mathbf{P}_e = \text{diag}(1, 1, e^{i\delta_e})$ and the explicit form of orthogonal matrix \mathbf{O}_{eL} (exact to order 10^{-9} in units of the τ mass) is

$$\begin{pmatrix} \frac{1}{\sqrt{2}}x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2-\tilde{m}_\mu^2}\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \quad (9)$$

where $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$ and $x = m_e/m_\mu$.

3.1. The mass matrix of the neutrinos

In the leptonic sector of this extension of Standard Model the mass matrix of the left-handed Majorana neutrinos, \mathbf{M}_ν , is generated by the type I seesaw mechanism. The mass matrix \mathbf{M}_ν takes the form [12, 8]:

$$\mathbf{M}_\nu = \begin{pmatrix} \frac{2(\mu_2^\nu)^2}{\overline{M}} & \lambda \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} \\ \lambda \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2(\mu_2^\nu)^2}{\overline{M}} & \lambda \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} \\ \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} & \lambda \frac{2\mu_2^\nu \mu_4^\nu}{\overline{M}} & \frac{2(\mu_4^\nu)^2}{\overline{M}} + \frac{(\mu_3^\nu)^2}{M_3} \end{pmatrix}, \quad \text{with} \quad \begin{aligned} \lambda &= \left(\frac{M_2 - M_1}{M_1 + M_2} \right), \\ \overline{M} &= 2 \frac{M_1 M_2}{M_2 + M_1}. \end{aligned} \quad (10)$$

M_i ($i = 1, 2, 3$) are the masses of right-handed neutrinos, in this expression the μ_2 , μ_3 and μ_4 are complex parameters that come from the mass matrix of the Dirac neutrinos [12, 8].

The identity of the leptons is encoded in the mass matrices \mathbf{M}_e and \mathbf{M}_ν , for charged leptons and left-handed neutrinos respectively. Nevertheless, the form or texture of these matrices is basis dependent, since given any pair \mathbf{M}_e , \mathbf{M}_ν one can obtain other pairs of equivalent matrices through a unitary rotation, without affecting the physics. On other hand, the phase factors may be factored out of \mathbf{M}_ν if $\phi_4 = \arg\{\mu_4^\nu\}$ and $\phi_3 = \arg\{\mu_3^\nu\}$ satisfy the relation $\phi_4 = \phi_3$ [12]. Hence, the mass matrix of the left-handed Majorana neutrinos can be written as [8, 12]:

$$\mathbf{M}_\nu = \mathbf{Q}^\nu \mathcal{U}_{\frac{\pi}{4}} \left(\mu_0 \mathbf{I}_{3 \times 3} + \widehat{\mathbf{M}} \right) \mathcal{U}_{\frac{\pi}{4}}^\dagger \mathbf{Q}^\nu, \quad (11)$$

where $\mathbf{Q}^\nu = e^{i\phi_2} \text{diag}\{1, 1, e^{i\delta_\nu}\}$ with $\delta_\nu = \phi_4 - \phi_2 = \arg\{\mu_4^\nu\} - \arg\{\mu_2^\nu\}$, $\mu_0 = \xi - d$,

$$\mathcal{U}_{\frac{\pi}{4}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \widehat{\mathbf{M}} = \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 2d \end{pmatrix}. \quad (12)$$

Here, $\xi = 2|\mu_2|^2|\overline{M}|^{-1}$, $a = \sqrt{2}|\mu_2^\nu||\mu_4^\nu||\overline{M}|^{-1}(1-|\lambda|)$, $b \equiv 2|\mu_4^\nu|^2|\overline{M}|^{-1} + |\mu_3^\nu|^2 M_3^{-1} - \xi + d$, $c = \sqrt{2}|\mu_2^\nu||\mu_4^\nu||\overline{M}|^{-1}(1+|\lambda|)$ and $d \equiv 2|\lambda||\mu_2^\nu|^2|\overline{M}|^{-1}$. As mentioned before, the diagonalization of \mathbf{M}_ν is reduced to the diagonalization of the real symmetric matrix $\widehat{\mathbf{M}}$, which is a matrix with two texture zeroes of class I [13]. Hence, the matrix \mathbf{M}_ν is diagonalized by a unitary matrix of the following form

$$\mathbf{U}_\nu = \mathbf{Q}^\nu \mathcal{U}_{\frac{\pi}{4}} \mathbf{O}_\nu^{N[I]}. \quad (13)$$

In the literature, these similarity transformations are also known as weak basis transformations, since they leave invariant the gauge currents [14].

The orthogonal matrix $\mathbf{O}_\nu^{N[I]}$ reparametrized in terms of the neutrino masses is given by [8, 12]

$$\begin{pmatrix} \sqrt{\frac{[-1](m_{\nu_3}-\mu_0)(m_{\nu_2}-\mu_0)f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{(m_{\nu_3[1]}-\mu_0)(\mu_0-m_{\nu_1[3]})f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})(m_{\nu_2}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ \sqrt{\frac{[-1]2d(\mu_0-m_{\nu_1})f_1}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{2d(m_{\nu_2}-\mu_0)f_2^{N[I]}}{\mathcal{D}_2^{N[I]}}} & \sqrt{\frac{[-1]2d(m_{\nu_3}-\mu_0)f_3^{N[I]}}{\mathcal{D}_3^{N[I]}}} \\ -\sqrt{\frac{[-1](\mu_0-m_{\nu_1})f_2^{N[I]}f_3^{N[I]}}{\mathcal{D}_1^{N[I]}}} & \sqrt{\frac{(m_{\nu_2}-\mu_0)f_1f_3^{N[I]}}{\mathcal{D}_2^{N[I]}}} & -\sqrt{\frac{[-1](m_{\nu_3}-\mu_0)f_1f_2^{N[I]}}{\mathcal{D}_3^{N[I]}}} \end{pmatrix}, \quad (14)$$

where: $f_1 = (2d + \mu_0 - m_{\nu_1})$, $f_2^{N[I]} = [-1](2d + \mu_0 - m_{\nu_2})$, $f_3^{N[I]} = [-1](m_{\nu_3} - \mu_0 - 2d)$, $\mathcal{D}_1^{N[I]} = [-1]2d(m_{\nu_2} - m_{\nu_1})(m_{\nu_3} - m_{\nu_1})$, $\mathcal{D}_2^{N[I]} = [-1]2d(m_{\nu_2} - m_{\nu_1})(m_{\nu_3} - m_{\nu_2})$, and $\mathcal{D}_3^{N[I]} = 2d(m_{\nu_3[1]} - m_{\nu_1[3]})(m_{\nu_3[2]} - m_{\nu_2[3]})$. The allowed values for the parameters μ_0 and $2d + \mu_0$ are in the following ranges: $m_{\nu_2[1]} > \mu_0 > m_{\nu_1[3]}$ and $m_{\nu_3[2]} > 2d + \mu_0 > m_{\nu_2[1]}$. The superscripts N and I denote the normal and inverted hierarchies respectively.

4. The leptonic mixing matrix $PMNS$

The leptonic mixing matrix $PMNS$ is the product $\mathbf{U}_{eL}^\dagger \mathbf{U}_\nu \mathbf{K}$, where \mathbf{K} is the diagonal matrix of the Majorana phase factors, defined by $\mathbf{K} = \text{diag}(1, e^{i\alpha}, e^{i\beta})$ [15]. In this case, the theoretical expression for the lepton mixing matrix, \mathbf{U}_{PMNS}^{th} , is:

$$\begin{pmatrix} \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{11}^{N[I]} - O_{21}^{N[I]} e^{i\delta_l} & \left(\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{12}^{N[I]} - O_{22}^{N[I]} e^{i\delta_l} \right) e^{i\alpha} & \left(\frac{\tilde{m}_e}{\tilde{m}_\mu} O_{13}^{N[I]} - O_{23}^{N[I]} e^{i\delta_l} \right) e^{i\beta} \\ -O_{11}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{21}^{N[I]} e^{i\delta_l} & \left(-O_{12}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{22}^{N[I]} e^{i\delta_l} \right) e^{i\alpha} & \left(-O_{13}^{N[I]} - \frac{\tilde{m}_e}{\tilde{m}_\mu} O_{23}^{N[I]} e^{i\delta_l} \right) e^{i\beta} \\ O_{31}^{N[I]} & O_{32}^{N[I]} e^{i\alpha} & O_{33}^{N[I]} e^{i\beta} \end{pmatrix}, \quad (15)$$

where the elements $O_{ij}^{N[I]}$ ($i, j = 1, 2, 3$) are given in eq. (14) and the phase $\delta_l = \delta_\nu - \delta_e$ is associated with the CP violation in the leptonic sector.

Now, in the case of an inverted neutrino mass hierarchy ($m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$), a numerical fit of eq. (15) to the magnitude of the entries of the matrix U_{PMNS} given in [1] yields the numerical values for the solar and reactor mixing angle as:

$$\sin^2 \theta_{12}^l \approx 0.307 \longrightarrow \theta_{12}^l \approx 33.7^\circ, \quad \sin^2 \theta_{13}^l \approx 0.028 \longrightarrow \theta_{13}^l \approx 9.6^\circ, \quad (16)$$

with the following values for the neutrino masses $m_{\nu_2} = 0.088$ eV, $m_{\nu_1} = 0.087$ eV and $m_{\nu_3} = 0.073$ eV, and the parameter values $\delta_l = 75^\circ$, $\mu_0 = 0.0727$ eV and $d = 7.34 \times 10^{-3}$ eV.

5. Flavour Changing Neutral Currents (FCNC)

The Yukawa couplings that enter in the computation of the flavour changing neutral currents are defined in the mass basis, according to $\tilde{Y}_m^{EI} = \mathbf{U}_{eL}^\dagger \mathbf{Y}_w^{EI} \mathbf{U}_{eR}$, where \mathbf{U}_{eL} and \mathbf{U}_{eR} are the

matrices that diagonalize the charged lepton mass matrix. We obtain [10]

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m \quad \text{and} \quad \tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m. \quad (17)$$

All the non-diagonal elements are responsible for tree-level FCNC processes. For example, the amplitude of the flavour violating process $\mu \rightarrow 3e$, is proportional to $\tilde{Y}_{\mu e}^E \tilde{Y}_{ee}^E$ [16]. Then, in the leading order of perturbation theory, the leptonic branching ratio for this process

$$Br(\mu \rightarrow 3e) \approx 2(2 + \tan^2 \beta)^2 \left(\frac{m_e m_\mu}{m_\tau^2} \right)^2 \left(\frac{m_\tau}{M_H} \right)^4, \quad (18)$$

taking for $M_H \approx 120 \text{ GeV}$ and $\tan \beta = 1$ we obtain $Br(\mu \rightarrow 3e) = 2.53 \times 10^{-16}$, well below the experimental upper bound for this process, which is 1×10^{-12} [17]. Similar computations give the numerical estimates of the branching ratios for some other flavour violating processes in the leptonic sector. These results and the corresponding experimental upper bounds are shown in following table

FCNC processes	Theoretical BR	Experimental upper bound BR
$\tau \rightarrow 3\mu$	8.43×10^{-14}	2.0×10^{-7} [18]
$\tau \rightarrow \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7} [18]
$\tau \rightarrow \mu \gamma$	9.24×10^{-15}	6.8×10^{-8} [19]
$\tau \rightarrow e \gamma$	5.22×10^{-16}	1.1×10^{-11} [20]
$\mu \rightarrow 3e$	2.53×10^{-16}	1.0×10^{-12} [17]
$\mu \rightarrow e \gamma$	2.42×10^{-20}	1.2×10^{-11} [21]

In all cases considered, the theoretical estimations made in the framework of the minimal S_3 -invariant extension of the SM are well below the corresponding experimental upper bounds [10].

6. Muon anomalous magnetic moment

In the minimal S_3 -invariant extension of the Standard Model that we are considering here, we have three Higgs $SU(2)$ doublets, one in the singlet and the other two in the doublet representations of the S_3 flavour group. The Z_2 symmetry decouples the charged leptons from the Higgs boson in the S_3 singlet representation. Therefore, in the leading order of perturbation theory there are only two neutral scalars and two neutral pseudoscalars whose exchange will contribute to the anomalous magnetic moment of the muon. Since the heavier generations have larger flavour-changing couplings, the largest contribution comes from the heaviest charged leptons coupled to the lightest of the neutral Higgs bosons. A straightforward computation gives

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau} Y_{\tau\mu}}{16\pi^2} \frac{m_\mu m_\tau}{M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right) = \frac{m_\tau^2}{(246 \text{ GeV})^2} \frac{(2 + \tan^2 \beta)}{32\pi^2} \frac{m_\mu^2}{M_H^2} \left(\log \left(\frac{M_H^2}{m_\tau^2} \right) - \frac{3}{2} \right). \quad (19)$$

Taking again $M_H = 120 \text{ GeV}$ and the upper bound for $\tan \beta = 14$ gives an estimate of the largest possible contribution of the FCNC to the anomaly of the muon's magnetic moment $\delta a_\mu^{(H)} \approx 1.7 \times 10^{-10}$. This number should be compared with the difference between the experimental value and the Standard Model prediction for the anomaly of the muon's magnetic moment [22]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 9.1) \times 10^{-10} \implies \frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx 0.06. \quad (20)$$

Hence, the contribution of the flavour changing neutral currents to the anomaly of the magnetic moment of the muon is smaller than or of the order of 6% of the discrepancy between the experimental value and the Standard Model prediction.

7. Conclusions

The minimal S_3 -invariant extension of the Standard Model permits the calculations of mass and mixing matrices for quark and leptons in a unified way, since the mass matrices of all fermions are brought to the same generic form with two texture zeroes by means of a similarity transformation. In this model we computed the reactor mixing angle, θ_{13} , as function of the masses of the leptons. The numerical values of the neutrino masses extracted from the fit are $m_{\nu_2} = 0.088$ eV, $m_{\nu_1} = 0.087$ eV and $m_{\nu_3} = 0.073$ eV, which in this analysis have an inverted mass hierarchy. The numerical value obtained for θ_{13} is $\theta_{13}^l \approx 9.6^\circ$ in good agreement with the latest analysis of the experimental data on neutrino oscillations [2, 3, 4, 5, 6]. Also, with the help of the Yukawa matrices, we computed the branching ratios of a number of FCNC processes and found that the branching ratios of all FCNC processes considered here are strongly suppressed by powers of the small mass ratios m_e/m_τ and m_μ/m_τ , and by the ratio $(m_\tau/M_{H_{1,2}})^4$, where $M_{H_{1,2}}$ is the mass of the neutral Higgs bosons in the S_3 -doublet. Taking for $M_{H_{1,2}}$ a very conservative value ($M_{H_{1,2}} \approx 120$ GeV), we found that the numerical values of the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds by many orders of magnitude. Finally, the contribution of the flavour changing neutral currents to the anomalous magnetic moment of the muon is small but non-negligible, and it is compatible with the best, state of the art measurements and theoretical computations.

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