Dynamical interaction between astrophysical systems and dark substructure

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Abstract. We explore the possibility to constrain the dark matter (DM) sub-structure properties, mass and distribution, at sub-galactic scales, by considering the dynamics of some astrophysical system under the dynamical influence of such sub-structure. We present preliminary results discussing the sensitivity of the Solar System dynamics, and dSphs wide binaries abundance, to the small scale DM distribution.

1. Introduction

The Λ-CDM cosmological scenario is consistent with the large scale Universe observations; however, one of the main ingredients, the Dark Matter, has not yet been detected. One of the principle uncertainties, regarding the direct and indirect detection experiments, is the distribution of DM at the relevant scales [1]. Specifically, models based on WIMP's DM predict a large abundance of substructure at sub-galactic scales. This has been considered as a potential problem for the Λ-CDM scenario because it is not obvious that the predicted dark substructure has an observational counterpart [2], see however [3].

On the other hand, the properties of the dark matter substructure, mass and abundance, are related to the properties of the power spectrum for primordial fluctuations; the nature of the dark matter; and the dynamical evolution of the fluctuations. In the Λ-CDM model the primordial mass power spectrum is fixed by inflation, and the dynamical evolution is followed trough $N-$body simulations, so that to constrain the properties of the substructure can constitute a test for the nature of the DM particle. Once a specific DM particle is assumed, the survival of the smallest structures trough the structure formation history is by itself a topic of discussion, being the extrapolation from N -body simulations at larger scales and the semi-analytic models the most used tools used to estimate the amount of substructure that should be present today [4, 5].

It has been shown that the free streaming, and/or the kinetic decoupling, of WIMP's like DM candidates leads to a mass-power spectrum cut off at approximately $M_{min} \sim 10^{-4} - 10^{-12} M_\odot$ [6, 7], while in the case of Axion like DM the cut off could be as small as $10^{-18} - 10^{-20}\, M_\odot$ [8, 9].[1] This small scale cut-off is unreachable to study with current N-body simulations, nor with analytical models, and it is also undetectable through current observational techniques.

[1] Other DM candidates like the Warm Dark Matter are also consistent with the large scale but they predict a PS cut off at approximately the dwarf galaxies mass [10]).

Here we explore an strategy that could allow us to set upper limits to the the existence of the dark small structure — microhalos and streams— in the Galactic halo. Since the orbital elements of planets in our Solar System are known with outstanding accuracy [11], they might be sensitive to dynamical perturbations triggered by dark matter substructure. In this work we focus on the variation of the astronomical unit, due to the presence of dark substructure and how this compares with the reported anomaly in [12]. We also use our methodology to study the stability of wide binary stars in dSph's of the Milky Way. These kind of binaries has not been detected yet, but previous works suggest that their presence could be in contradiction with the dark matter hypothesis. Here we present some preliminary results on this direction. This proposal was first presented in [13] and will be presented with more detail in [14].

2. Target tidal interactions with mini-halos and streams

The dynamics of some astrophysical systems is known with great accuracy that allows to constrain the existence of incredibly small gravitational perturbations. Here we consider the dynamical perturbations that such systems (the target) could undergo in the presence of dark substructure. Dark substructure could be in the form of mini-halos or streams, —former DM sub-halos that had been already disrupted by tidal forces or encounters—. We will treat the encounters with mini-halos, and streams, as independent of each other and separately. First, we present the equations we will use for independent single encounters, and we then explain how we follow the evolution of the target under multiple encounters.

2.1. One single encounter with the target

Our first approach is to consider spherical micro-halos: the impulsive energy injection to a target orbit during one single encounter, is

$$\frac{\langle \Delta E \rangle_{mh}}{|E_b|} = \frac{14\, G\, M_p^2}{3\, M_c\, v_0^2\, p^4} a^3, \tag{1}$$

where, the impact parameter p is the minimal distance between the target and the pertuber; M_p is the perturber mass; M_c is the mass of the central body in the target; v_0 is the relative encounter velocity; and a is the mean separation of the target system (it coincide with the semi-major axis for some systems) [15, 16]. Equation 1 is valid whenever the characteristic size of the perturber is smaller that the impact parameter (point mass approximation); when the impact parameter is much larger than the extent of target (distant tide approximation); and the encounter velocity is much larger than the internal velocity of the target (impulsive regime). We have verified that the systems under study falls in these approximations.

In our second approach, we consider the perturbers to be a sort of dark streams of cylindrical symmetry characterized by their linear mass density. The simplest model for a stream is 1-dimensional, i.e. a line of linear mass density λ_{ST}. In this case the energy injection, normalized to the binding energy of the target, is:

$$\frac{\langle \Delta E \rangle_{ST}}{|E_b|} = \frac{4\, \pi^2\, G\, \lambda_{ST}^2}{M_c\, v_0^2\, p^2 \sin^2(\theta)} a^3 \left(\cos^2(\alpha) \cos^2(\psi) + \sin^2(\psi) \right). \tag{2}$$

The angles α and ψ defines the orientation of the target orbital plane, and θ the relative direction of movement of target, both with respect to the stream. More realistic models include the finite cross section stream with constant density; or with a power law density, either with a cusp or a core in the center. These models reduces to the one dimensional when the impact parameter is larger than the cross-section of the stream. Here we will consider only the 1-dimensional case and the analysis for the other models, as well as the derivation of equation 2, will be presented in [14].

2.2. Multiple encounters during the lifetime of the target

As a result of multiple encounters, either with mini-halos or streams, the fractional energy of the target system will change. Our general approach to study the effect of this multiple encounters consists of Monte Carlo experiments: first, we draw random values for the encounter parameters (p, v_0, α, etc.), and use them to assess the effect of one single encounter trough the fractional energy, $\langle \Delta E \rangle / E_b$. We then increment the experiment timer with $\Delta t_{enc} \propto 2\,p/v_0$ (the time it takes the perturber to traverse a sphere of radius equal to the impact parameter at the encounter velocity), and then we continue sampling encounters until the time limit is reached, (the lifetime of the target), or until the fractional energy reaches the unity (i.e. the target gets unbound). We repeat this procedure a number of times, so that we can construct an ensemble of histories of the multiple encounters between the target and the perturber population. Finally, we find the median value of the total dynamical perturbations the target experimented due to the presence of the dark substructure.

In the case of encounters with mini-halos we sample the impact parameters from a nearest-neighbor distribution, with numerical density $\nu = \rho_{dm} f_s / m_{mh}$, ($f_s$ is the fraction of the mass density, ρ_{dm}, of dark matter that is in form of mini-halos of mass m_{mh}). On the other hand, if the perturbers are streams, we use the cumulative distribution shown in figure A2, and we scale the physical impact parameter using $p = \left(\sqrt{\lambda_{ST}\, \rho^* / f_s\, \rho_{dm}} \right) x$, according to the different systems under study. See Appendix A. For the velocity distribution we use a Maxwellian distribution; and a uniform distribution function for the geometric parameters, (β, α, etc.). In the next section we present a couple of examples that illustrates the applicability of the method we have described.

3. Examples of applicability

3.1. Anomalies in the Solar System

There is some evidence of a variation in the astronomical unit (AU), the increase is reported to be about $15\, cm\, yr^{-1}$ [12]. This anomaly remains unexplained,(see however [17]).

We have applied our formalism of multiple encounters with dark substructure to the Sun-Earth system, to check if this anomaly could be explained by the dynamic perturbations due to the presence of dark substructure. We assumed the canonical value for the local dark matter density, $\rho_{dm} \sim 10^{-18} M_\odot / AU^3$ ($0.3 GeV/cm^3$), and a mean encounter velocity of $v_0 = 200\, Km/s$.

In figure, 1 we show an example of the resultant probability distribution for an ensemble of Monte-Carlo experiments. Each of the runs in the ensemble corresponds to a history of encounters between the Earth-Sun system and dark streams, (with $\lambda = 10^{-6} M_\odot / AU$), for about 4 Gyr. Since the distribution is non-symmetric, we use the median value instead of the mean. In figure 2 we show the median value of the total fractional energy for different values of the perturber linear density. We see that in general these kind of perturbations are not enough to explain the reported anomaly, indicated by the dashed line. A similar trend was found when studying interactions with mini-halos.

3.2. The case of wide binaries in dSphs Galaxies

The system under study now are the wide binaries under the dynamical perturbation due to dark substructure, inside the dwarf spheroidal galaxies in the Milky Way. The survival of such kind of binary systems has been proposed as a test for the dark matter hypothesis, since it is subjected to the dynamical interaction with the dark matter, regardless it is distributed homogeneously [18], or along dark substructure [19].

Our work focuses on the dynamical interaction between such binaries and dark substructure that could be in form of mini-halos or dark streams. In [19] only considered perturbations caused by structures that produce a significant evolution of the binary as a result of one single encounter, that is the catastrophic regime. With the proposed approach in this work, we are able to test the possibility that the dynamical evolution of the binary is caused by the cumulative effect of multiple encounters with substructure that is not massive enough to be in the catastrophic regime, i.e. we consider also the diffusive regime.

We assume the following parameters for the binaries and the dark halo in which they are embedded: a

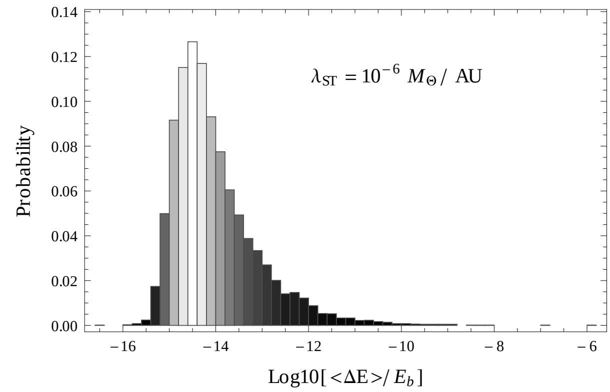

Figure 1. Probability distribution function for the Earth-Moon system binding energy change. Each run in the ensemble correspond to the result after multiple encounters with streams of linear mass density $\lambda_{ST} = 10^{-6} M_\odot/AU$ for about $4\,Gyr$.

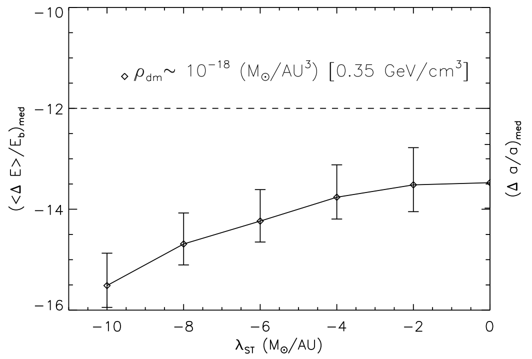

Figure 2. Median value of the Earth-Moon system binding energy change, as a result of multiple encounters with streams with linear mass density λ_{ST}. The dashed line correspond to the reported variation of the AU ($\approx 15\,cm/yr$).

fiducial binary separation of $a = 0.1pc$; a relative velocity distribution centered at $\sqrt{2}\sigma \approx 14\,Km/s$, (given that typical velocity dispersion in dSphs is about $\sigma \approx 10km/s$ [20]); a NFW density profile with virial mass $M_{vir} = 10^9 M_\odot$ and concentration $c = 23.1$ for the dSph (as were considered in [19]),and a radius of $r \approx 0.1\,kpc$ where the binaries could be (this radius is smaller than the typical half-light radius of the dSph's), the dark matter density at this radius is $\rho_{dm} \approx 2.2 * 10^{-17} M_\odot/AU^3$).

In figure 3 we show the results for dynamical interaction between the binaries and mini-halos acting as perturbers. We report the median value of the total injection energy resulted in the ensembles histories of multiple encounters, as a function of the mini-halo mass and for three different times of the total evolution. There is a minimum mass of the mini-halo that causes the total disruption of the binaries, i.e. $\delta E_{tot}/|E_b| = 1$, that depend on the time the binary is subjected to the perturbation field. In the case the evolution time is about $10Gyr$ the minimum mass needed to destroy the binaries is about $10^2 M_\odot$, this mass correspond to the boundary between the catastrophic and the diffusive regime set in [19], in that sense we recover their results. However, we also obtain that it is not enough to consider only the catastrophic encounters because the diffusive ones can also produce significant secular evolution of the binary system. Now, in figure 4 we show the results for the interaction of the binaries with streams like structures. Again we show the cumulative effect of the interactions for three different times of the evolution. Our findings are similar to those for mini-halos, i.e. there is a minimum value of the linear mass density of the streams at which the encounters are energetic enough to unbound the binary system, again this minimum value depends on the time the binary is under perturbation.

4. Discussion

We found that neither the perturbations due to mini-halos nor due to streams, for about the lifetime of the Earth, would be energetic enough to explain the reported anomaly in its variation. We would like to remark that this result is subjected to the mean value of the local dark matter density which is determined at larger scales (about Kilo parsecs). However, it remains open the possibility of having large over-densities at lower scales, still consistent with the mean density. Increasing the value of the dark matter density has the effect of increasing the total fractional energy given, so it would be possible to explain the anomaly in that case.

For the wide binaries, we found that its disruption could be driven by the dynamical contribution of small perturbers, as well as to the presence of dark streams like substructure. Our results strongly depends on the time the binary evolution is under the perturbation field; as the lifetime of this kind of systems is

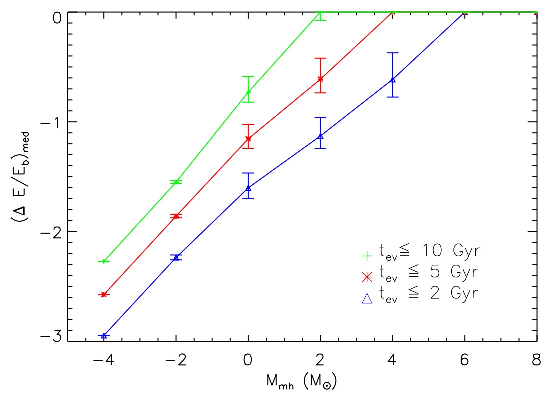

Figure 3. Median value of the binary system binding energy total change as a result of multiple encounters with mini-halos of mass m_{mh}, for three different times of evolution. The minimum mass needed to unbind the binary depends on the total evolution time.

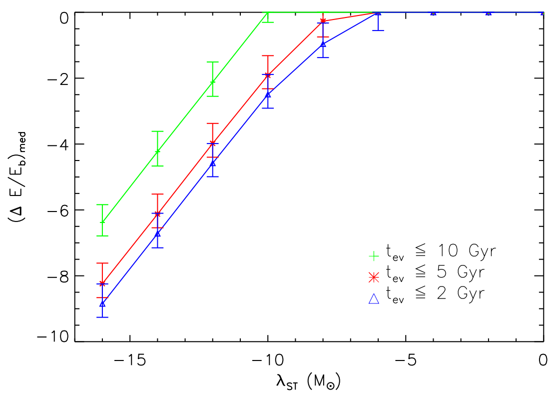

Figure 4. Median value of the binary system binding energy change as a result of multiple encounters with streams of linear mass density λ_{ST}, for three different times of evolution. Again there is a minimum linear mass density needed to unbind the binary.

uncertain thus their existence is not a sharp discriminant of the dark matter hypothesis.

In this contribution we have considered that all the dark matter is in form of mini-halos, or streams, separately. A more realistic situation would be one in which a fraction of the mass is in form of mini-halos and the rest is in form of streams. Such cases, and their implications, are considered in a work that is under revision now as well as the applicability of the proposed procedure to other astrophysical systems.

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Appendix A. Distribution function of impact parameters for stream like perturbers

In the case of having streams as perturbers, the basic problem in using a nearest neighbor like distribution is that we would be linking a sample volume with the determination of the distribution of impact parameters. To break this link we will proceed along the following lines: suppose one is given a random realization of a region of 3-space where lines have been thrown at random and completely uniformly. One can then choose a collection of arbitrary and sufficiently separated points within this region, and determine the minimum impact parameter for each point. This will result in a sampling of the underlying distribution of impact parameters, completely independent of the introduction of a sample volume.
We need to relate the starting realization to a specific universe with a certain spatial mass density and linear mass density for the lines. Then we do have to introduce, now, a sample volume to measure mass density. Exploring this in the same realization, and using different sample volumes we found that the number density of filaments varies with the size of the spherical neighborhood used, but the mass density does not. Thus, the mass density is a well defined physical quantity in this procedure.

For our numerical realization we assumed a linear density of streams $\lambda^* = 1$ and the measured mass density is some value ρ^*, both given some units of mass $[m]$ and longitude $[l]$. By scaling to the physical quantities, λ_{ST} and ρ_{DM}, one finds that the physical impact parameter is given by $p = \left(\sqrt{\lambda_{ST}\,\rho^*/f_s\,\rho}\right) x AU$. For the particular realization used in this work we have $\rho^* = 1205.08$, and the other parameters are given according to the different systems under study. We have separated completely the determination of impact parameters, which is a purely geometric problem, from the mass scaling problem, which is a physical problem.

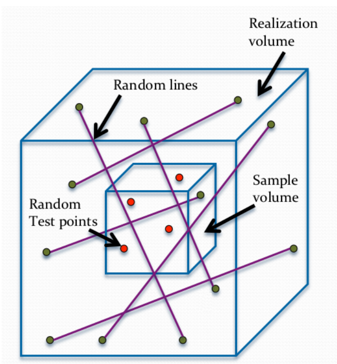

Figure A1. Illustration of the distribution of lines and points used to generate the impact parameter PDF for stream like perturbers.

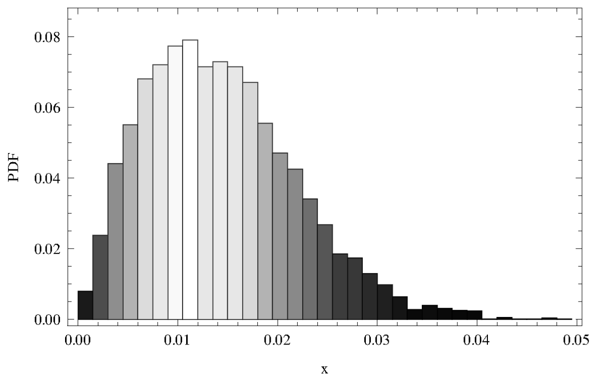

Figure A2. Probability distribution function of impact parameters for stream like perturbers. The impact parameter is scaled according to the particular physical system using: $p = \left(\sqrt{\lambda_{ST}\,\rho^*/f_s\,\rho}\right) x$