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Density perturbations from curvatons revisited

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Abstract. We investigate density perturbations sourced by a curvaton with a generic energy potential. The key feature of a curvaton potential which deviates from a quadratic is that the curvaton experiences a non-uniform onset of its oscillation. This sources additional contributions to the resulting density perturbations, and we especially find that curvaton potentials that are flatter compared to a quadratic lead to enhancement of the linear and second-order density perturbations, while steepened potentials can generate strongly scale-dependent non-Gaussianity. As such examples, we study pseudo-Nambu-Goldstone curvatons and self-interacting curvatons. Our analyses are analytic, and thus provide a systematic framework for studying curvatons in general. The discussion in this paper are based on [1, 2].

1. Introduction

The curvaton mechanism [3, 4, 5] is an attractive mechanism for creating the primordial density perturbations of our universe. In this scenario, the curvaton field possessing large-scale field fluctuations generates the density perturbations while it oscillates about its potential minimum and comes close to dominating the universe. The mechanism also has the merit that when embedded in inflationary cosmology, it frees the inflaton from being responsible for generating the perturbations and therefore drastically relaxes constraints on inflationary model building.

In this paper, we explore density perturbations sourced by a curvaton rolling along an arbitrary energy potential. When the potential deviates from a quadratic one, the curvaton field with large-scale field fluctuations starts its oscillation at different times at different patches of the universe. Such behaviour contributes to the curvaton energy density perturbations in addition to that sourced directly from the original field fluctuations. We find that such non-trivial conversion processes of the field fluctuations into the density perturbations can lead to strong enhancement/suppression of the density perturbations of the universe, as well as for their non-Gaussianities.

It is worthwhile to investigate curvaton potentials which deviate from simple quadratic ones, both from observational and theoretical reasons: Considering a curvaton whose field fluctuations were generated during the inflationary era, in order for the curvaton to produce the red-tilted density perturbation spectrum as suggested by latest CMB observations [6] without relying on specific inflation mechanisms (such as large-field models), the curvaton needs to be located along a potential with negative curvature during inflation. This obviously suggests the curvaton potential to take non-trivial forms. Furthermore, explicit curvaton models constructed in the framework of microscopic physics can naturally possess intricate energy potentials.

We first derive analytic expressions for the density perturbations sourced by curvatons with generic energy potentials. Then we look into pseudo-Nambu-Goldstone curvatons and self-interacting curvatons, which provide, respectively, typical examples of curvaton potentials that flatten/steepen compared to a quadratic one. We find that flat potentials can lead to enhancement of the linear and second-order density perturbations, while steepened potentials can generate strongly scale-dependent non-Gaussianity.

The results presented in the following are based on the works [1, 2], where one can find detailed discussions. (For related discussions, see also [7, 8, 9, 10, 11, 12] and references therein.)

2. Density Perturbations from Curvatons

A light curvaton acquires nearly scale-invariant field fluctuations during inflation, that are converted into the cosmological density perturbations as the curvaton oscillates and decays in the post-inflationary era. Let us first lay out analytic expressions for the density perturbations generated by a curvaton σ with a generic energy potential $V(\sigma)$. The potential is assumed to have no explicit dependence on time, and also that it is well approximated by a quadratic one around its minimum (which we set as $\sigma = 0$) so that the curvaton oscillations are sinusoidal.[1] Hence the curvaton energy density is considered to redshift similarly to nonrelativistic matter after the onset of the oscillations until when the curvaton decays into radiation, whereas we suppose the inflaton to behave as matter from the end of inflation until reheating when the inflaton decays into radiation. The energy density of the curvaton before the beginning of its oscillation is assumed to be negligibly tiny compared to the total energy of the universe, having little effect on the expansion history. Furthermore, we neglect density perturbations sourced from the inflaton.

2.1. Linear and Second Order Perturbations

Supposing the curvaton field fluctuations to be nearly Gaussian and to satisfy $\mathcal{P}_{\delta\sigma}(k) = (H|_{k=aH}/2\pi)^2$ at the time when the scale k of interest exits the horizon, then using the $\delta\mathcal{N}$-formalism, the power spectrum of the linear order density perturbations can be expressed in terms of the curvaton potential as

$$\mathcal{P}_\zeta(k_*) = \left(\frac{\partial\mathcal{N}}{\partial\sigma_*}\frac{H_*}{2\pi}\right)^2, \tag{1}$$

with

$$\frac{\partial\mathcal{N}}{\partial\sigma_*} = \frac{\hat{r}}{4+3\hat{r}}\left(1 - X(\sigma_{\mathrm{osc}})\right)^{-1}\left\{\frac{V'(\sigma_{\mathrm{osc}})}{V(\sigma_{\mathrm{osc}})} - \frac{3X(\sigma_{\mathrm{osc}})}{\sigma_{\mathrm{osc}}}\right\}\frac{V'(\sigma_{\mathrm{osc}})}{V'(\sigma_*)}. \tag{2}$$

Here, the subscript $*$ denotes values when the CMB scale k_* exits the horizon, "osc" represents values at the onset of the curvaton oscillation, and a prime denotes a derivative with respect to σ. An overdot will be used below to denote a time derivative. $\hat{r}$ is the energy density ratio between the curvaton and radiation (which originates from the inflaton) upon curvaton decay

$$\begin{aligned}\hat{r} &\equiv \left.\frac{\rho_\sigma}{\rho_r}\right|_{\mathrm{dec}} \\ &= \mathrm{Max.}\left[\frac{V(\sigma_{\mathrm{osc}})}{3M_p^2 H_{\mathrm{osc}}^{3/2}\Gamma_\sigma^{1/2}} \times \mathrm{Min.}\left(1,\ \frac{\Gamma_\phi^{1/2}}{H_{\mathrm{osc}}^{1/2}}\right), \left\{\frac{V(\sigma_{\mathrm{osc}})}{3M_p^2 H_{\mathrm{osc}}^{3/2}\Gamma_\sigma^{1/2}} \times \mathrm{Min.}\left(1,\ \frac{\Gamma_\phi^{1/2}}{H_{\mathrm{osc}}^{1/2}}\right)\right\}^{4/3}\right],\end{aligned} \tag{3}$$

where the first and second terms in the Max. parentheses correspond to the curvaton being subdominant and dominant at its decay, respectively, while the Min. parentheses are due to

[1] The formulae can be generalized to cases with non-sinusoidal oscillations as well, see Appendix B of [1].

whether the onset of oscillation is after or before reheating. Γ_ϕ and Γ_σ are constants denoting respectively the decay rates of the inflaton and the curvaton. We adopt the sudden decay approximation where the scalar fields suddenly decay into radiation when $H = \Gamma$.

The function X in (2) denotes effects due to the non-uniform onset of the curvaton oscillations (which are absent for a purely quadratic curvaton potential), defined as follows:

$$X(\sigma_{\rm osc}) \equiv \frac{1}{2(c-3)} \left(\frac{\sigma_{\rm osc} V''(\sigma_{\rm osc})}{V'(\sigma_{\rm osc})} - 1 \right). \tag{4}$$

Here, c is a constant whose value is set by whether reheating ($=$ inflaton decay, at $t_{\rm reh}$) is earlier/later than the onset of the curvaton oscillation:

$$c = \begin{cases} 5 & (t_{\rm reh} < t_{\rm osc}) \\ 9/2 & (t_{\rm reh} > t_{\rm osc}). \end{cases} \tag{5}$$

We can define the onset of the oscillation as when the time scale of the curvaton rolling becomes comparable to the Hubble time, i.e. $|\dot{\sigma}/H\sigma| = 1$. This gives the Hubble parameter at the time as

$$H_{\rm osc}^2 = \frac{V'(\sigma_{\rm osc})}{c\sigma_{\rm osc}}, \tag{6}$$

where c is given in (5). Furthermore, the curvaton field value at the onset of the oscillations $\sigma_{\rm osc}$ is obtained as a function of σ_* by solving

$$\int_{\sigma_*}^{\sigma_{\rm osc}} \frac{d\sigma}{V'} = -\frac{\mathcal{N}_*}{3H_{\rm inf}^2} - \frac{1}{2c(c-3)H_{\rm osc}^2}, \tag{7}$$

where $\mathcal{N}_*$ is the number of e-folds during inflation between the horizon exit of the CMB scale and the end of inflation, and $H_{\rm inf}$ is the inflationary Hubble scale (we are assuming a nearly constant Hubble parameter during inflation, thus $H_{\rm inf} \simeq H_*$).

Thus by combining the above expressions, one can compute the power spectrum produced by a curvaton with a generic potential $V(\sigma)$, given the parameters σ_*, Γ_ϕ, Γ_σ, $H_{\rm inf}$, and $\mathcal{N}_*$.

As for the local-type bispectrum produced by the curvaton, we parametrize its overall amplitude by the non-linearity parameter $f_{\rm NL}$ on the equilateral configuration, which can be shown to take the form

$$\begin{aligned} f_{\rm NL}(k_*) &= \frac{5}{6}\frac{\partial^2 \mathcal{N}}{\partial \sigma_*^2} \left(\frac{\partial \mathcal{N}}{\partial \sigma_*} \right)^{-2} \\ &= \frac{40(1+\hat{r})}{3\hat{r}(4+3\hat{r})} + \frac{5(4+3\hat{r})}{6\hat{r}} \left\{ \frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}} \right\}^{-1} \Bigg[(1 - X(\sigma_{\rm osc}))^{-1} X'(\sigma_{\rm osc}) \\ &\quad + \left\{ \frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}} \right\}^{-1} \left\{ \frac{V''(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{V'(\sigma_{\rm osc})^2}{V(\sigma_{\rm osc})^2} - \frac{3X'(\sigma_{\rm osc})}{\sigma_{\rm osc}} + \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}^2} \right\} \\ &\quad + \frac{V''(\sigma_{\rm osc})}{V'(\sigma_{\rm osc})} - (1 - X(\sigma_{\rm osc})) \frac{V''(\sigma_*)}{V'(\sigma_{\rm osc})} \Bigg]. \end{aligned} \tag{8}$$

2.2. Scale-Dependence

The scale-dependence of the linear and second order perturbations are calculated by using the slow-roll approximation for the curvaton $3H_*\dot{\sigma}_* \simeq -V'(\sigma_*)$. The spectral index of the linear order perturbations is

$$n_s - 1 \equiv \frac{d}{d\ln k} \ln \mathcal{P}_\zeta \simeq 2\frac{\dot{H}_*}{H_*^2} + \frac{2}{3}\frac{V''(\sigma_*)}{H_*^2}, \tag{9}$$

and its running is

$$\alpha \equiv \frac{dn_s}{d\ln k} \simeq 2\frac{\ddot{H}_*}{H_*^3} - 4\frac{\dot{H}_*^2}{H_*^4} - \frac{4}{3}\frac{\dot{H}_*}{H_*^2}\frac{V''(\sigma_*)}{H_*^2} - \frac{2}{9}\frac{V'(\sigma_*)V'''(\sigma_*)}{H_*^4}. \tag{10}$$

In order to parametrize the scale-dependence of the non-Gaussianity, we define the spectral index of $f_{\rm NL}$ as follows:

$$\begin{aligned} n_{f_{\rm NL}} &\equiv \frac{d\ln|f_{\rm NL}|}{d\ln k} \\ &\simeq \frac{1}{f_{\rm NL}}\frac{5(4+3\hat{r})}{18\hat{r}}\left\{\frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}}\right\}^{-1}(1-X(\sigma_{\rm osc}))\frac{V'(\sigma_*)}{V'(\sigma_{\rm osc})}\frac{V'''(\sigma_*)}{H_*^2}. \end{aligned} \tag{11}$$

We remark that a totally scale-invariant $f_{\rm NL}$ corresponds to $n_{f_{\rm NL}} = 0$ (instead of 1).

3. Pseudo-Nambu-Goldstone Curvatons

As a simple example, let us examine the case where the curvaton is a pseudo-Nambu-Goldstone (NG) boson of a broken U(1) symmetry, possessing a potential of the form

$$V(\sigma) = \Lambda^4\left[1 - \cos\left(\frac{\sigma}{f}\right)\right]. \tag{12}$$

Here, f and Λ are mass scales. Without loss of generality, we set the field value of the curvaton at horizon exit during inflation to lie within the range $0 < \sigma_* < \pi f$, and consider its oscillation about the origin $\sigma = 0$. (For an investigation of NG curvatons located close to their potential minimum, see also [13].) Supposing that the coupling of the NG curvaton with its decay product is suppressed by the scale of symmetry breaking f, we set the curvaton decay rate as

$$\Gamma_\sigma = \frac{1}{16\pi}\frac{m^3}{f^2} = \frac{1}{16\pi}\frac{\Lambda^6}{f^5}, \tag{13}$$

where m is the mass at the minimum, i.e. $m^2 = V''(0)$.

We show the resulting density perturbations from NG curvatons in Figures 1 and 2, where the analytic estimations given by the expressions in the previous section are plotted as solid lines. We also plotted numerically computed results shown as dots. The potential parameters and the inflation/reheating scales are chosen such that the curvaton with $\sigma_* = \frac{3}{4}\pi f$ generates a density perturbation spectrum whose amplitude takes the COBE normalization value $\mathcal{P}_\zeta \approx 2.4 \times 10^{-9}$, with the spectral index lying at the central value of the WMAP7 [6] bound (for a power-law spectrum with no tensor modes) $n_s \approx 0.96$, assuming constant H during inflation. In the figures, we have fixed all the parameters except for the initial field value σ_*, and plotted the perturbations as functions of σ_*.

In the region $\sigma_* \ll \pi f$, the curvaton potential is well approximated by a quadratic, thus the density perturbations behave similarly as from quadratic curvatons. For the chosen parameters, the curvaton energy fraction at decay $\hat{r}$ is smaller than unity for $\sigma_*/\pi f \lesssim 0.5$. Therefore, as $\hat{r}$ increases with σ_*, the power spectrum also increases while $f_{\rm NL}$ decreases, which are familiar behaviors of quadratic curvatons. On the other hand, as one approaches $\sigma_*/\pi f \to 1$, it can be seen that the linear order perturbation as well as the non-Gaussianity increase. This is due to the non-uniform onset of the curvaton oscillation, and it can further be shown that the linear order perturbations blow up in the hilltop limit, accompanied by a mild increase of the non-linearity parameter $f_{\rm NL}$. This is actually a generic feature of hilltop curvatons.

NG curvatons at the hilltop can work with a wide range of inflation/reheating scales, and a detailed study of the parameter space reveals that such hilltop NG curvatons, even though it well dominates the universe before it decays, predict the non-Gaussianity to lie in the range $10 \lesssim f_{\rm NL} \lesssim 30$ [1]. (This is in strong contrast to quadratic curvatons, which produce large $f_{\rm NL}$ only when $\hat{r} < 1$.)

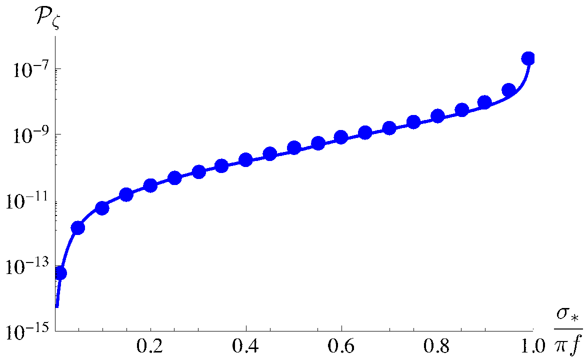

Figure 1. Power spectrum.

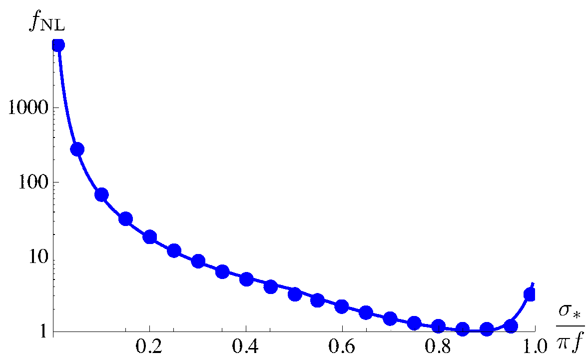

Figure 2. Non-Gaussianity.

4. Self-Interacting Curvatons

A simple example of curvaton potentials that steepen more rapidly than a quadratic is given by curvatons with self-interactions of the form

$$V(\sigma) = \Lambda^4 \left[\left(\frac{\sigma}{f}\right)^2 + \left(\frac{\sigma}{f}\right)^m \right], \tag{14}$$

where Λ and f are positive constants with mass dimension, and m is an even integer with $m > 2$. f denotes where the higher-order term becomes important, while Λ sets the overall scale of the curvaton potential. Such self-interacting curvatons do not produce a red-tilted perturbation spectrum unless relying on the time-variation of H during inflation, however, they have an interesting feature such that the resulting $f_{\rm NL}$ can be strongly scale-dependent, even when the linear order perturbations are nearly scale-invariant [2].

We plot the density perturbations produced from self-interacting curvatons with $m = 8$ in Figures 3 and 4. Here the parameters are chosen such that the power spectrum obtains the COBE normalization value at around $\sigma_*/f \approx 0.6$ (where the power spectrum obtains a peak). As in the previous section, we have fixed all parameters except for σ_*, and shown how the perturbations depend on σ_*. The blue solid lines denote the analytic calculations and the blue dots the numerically computed results. In the $n_{f_{\rm NL}}$ - $f_{\rm NL}$ plane of Figure 4, we have also shown as black dashed lines the detection limit of the Planck satellite [14], which can probe the running of the local-type $f_{\rm NL}$ if its running is large enough to satisfy $|n_{f_{\rm NL}} f_{\rm NL}| \gtrsim 5$. (See [15] for detailed analyses on the expected constraints on running non-Gaussianity.)

The $\sigma_* \ll f$ region is well-approximated by quadratic curvatons, and since $\hat{r}$ is smaller than unity throughout the plotted region, the power spectrum increases with σ_* in the small field regime due to the increase of $\hat{r}$. However for larger field values, the power spectrum starts to decrease. This is attributed to the steep self-interacting potential forcing the curvaton to roll down to small field values by the onset of the curvaton oscillation. This greatly diminishes the initial differences in the curvaton field values during inflation σ_*, thus suppresses the resulting linear order density perturbations. As a consequence, the power spectrum obtains a peak at $\sigma_*/f \approx 0.6$, and there the non-linearity parameter $f_{\rm NL}$ crosses zero and its running $n_{f_{\rm NL}}$ blows up. Therefore in the vicinity of $\sigma_*/f \approx 0.6$, a large and strongly scale-dependent $f_{\rm NL}$ can be produced, whose running is in the detectable range by upcoming CMB observations.

5. Conclusions

We have analytically explored density perturbations from a curvaton with a generic potential, and shown that non-quadratic potentials have rich phenomenology. Through studying pseudo-

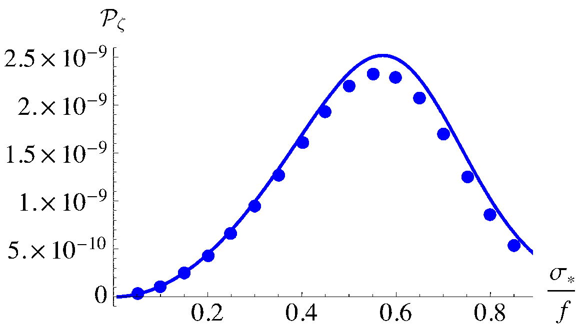

Figure 3. Power spectrum.

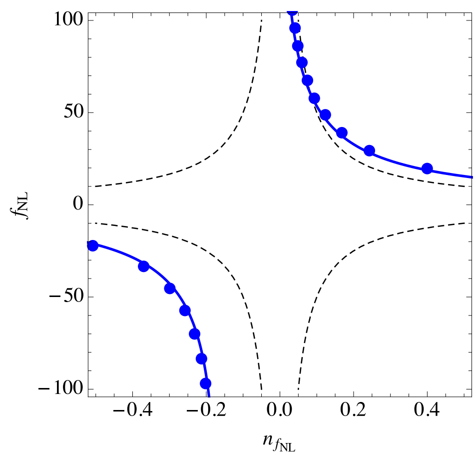

Figure 4. $n_{f_{\rm NL}}$ - $f_{\rm NL}$ plane, showing the region $0.45 \lesssim \sigma_*/f \lesssim 0.65$.

Nambu-Goldstone curvatons and self-interacting curvatons, we have seen that potentials that flatten compared to a quadratic can enhance the linear and second order density perturbations, while steepened potentials are capable of producing running non-Gaussianity. The analytic formulae presented in this paper enable us to go beyond individual case studies and give a systematic treatment of the curvaton scenario in general. This will be helpful for probing the physics of curvatons when combined with upcoming data.

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