Thermalisation of light sterile neutrinos

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Abstract. We calculate the thermalisation process of light, sterile neutrinos in the early Universe for a range of mixing angles and mass-differences. We show that adding a large initial lepton asymmetry suppress thermalisation significantly. In particular, a sterile neutrino that explains the anomalies seen in ground based neutrino experiments can be made compatible with cosmology. We compute the degree of thermalisation in the (1 active + 1 sterile) scenario and give the effective number of thermalised species just before BBN begins. Resonances are taken properly taken into account, since we solve the full, momentum dependent equations, knowns as the Quantum Kinetic Equations. These proceedings are partly based on the paper [1].

1. Introduction

Recent cosmological data favour additional relativistic degrees of freedom in addition to photons and the three active neutrinos [2, 3, 4]. This extra radiation is often referred to as dark radiation, and one of the prime candidates is a light, sterile neutrino. Sterile neutrinos are spin-$\frac{1}{2}$ fermions that has no standard model interactions; they are not charged under the $SU(3) \times SU(2)_L \times U(1)$ gauge group of the standard model.

Low-mass sterile neutrinos may also explain the excess $\bar{\nu}_e$ events in the LSND experiment [5, 6, 7] as well as the MiniBooNE excess events in both neutrino and antineutrino channels. Interpreted in terms of flavour oscillations, the MiniBooNE data require CP violation and thus no less than two sterile families [8, 9, 10] or additional ingredients such as non-standard interactions [11]. Very recently, the MiniBooNE collaboration published new results [12] which no longer seems to require CP violation and is fully consistent with the LSND anomaly.

A new calculation of the expected flux of $\bar{\nu}_e$ from nuclear reactors reveals a deficit in the measured flux. This deficit can also be explained by a low mass sterile neutrino [13, 14, 15].

The cosmic radiation content is usually expressed in terms of the effective number of thermally excited neutrino species, N_{eff}. Its standard value, $N_{\text{eff}} = 3.046$, slightly exceeds 3 because of e^+e^- annihilation providing residual neutrino heating [16]. The Wilkinson Microwave Anisotropy Probe (WMAP) collaboration found $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$ based on their 7-year data release and additional LSS data [17] at 1σ. Including the Sloan Digital Sky Survey (SDSS) data release 7 (DR7) halo power spectrum, [3] found $N_{\text{eff}} = 4.78^{+1.86}_{-1.79}$ at 2σ. Measurements of the CMB anisotropy on smaller scales by the ACT [18] and SPT [19] collaborations also find tentative evidence for a value of N_{eff} higher than predicted by the standard model (see also [20, 21, 22, 23, 24] for recent discussions of N_{eff}).

2. Quantum Kinetic Equations

If we wanted to calculate the freeze-out process of neutrinos and were not interested in neutrino oscillations, we should just solve the Boltzmann equation for the neutrino distribution function. However, since we are interested in both oscillations and scattering, we must solve a Boltzmann-like equation for the density matrices [25]. This system of equations is called the Quantum Kinetic Equations (QKE).

We restrict ourselves to a system of one active and one sterile neutrino. The density matrices are Hermitian, so they have 4 independent degrees of freedom. They are conveniently expressed in terms of Pauli-matrices $\boldsymbol{\sigma}$:

$$\rho = \frac{1}{2} f_0 (P_0 + \boldsymbol{P} \cdot \boldsymbol{\sigma}), \qquad \overline{\rho} = \frac{1}{2} f_0 (\overline{P}_0 + \overline{\boldsymbol{P}} \cdot \boldsymbol{\sigma}), \tag{1}$$

where $\boldsymbol{P}$ and $\overline{\boldsymbol{P}}$ are real vectors and $f_0 = (e^x + 1)^{-1}$ is the un-normalised Fermi-Dirac distribution with zero chemical potential for comoving momentum $x = p/T$. The evolution equations for P_0 and $\boldsymbol{P}$ are given by

$$\frac{\mathrm{d}P_0}{\mathrm{d}t} = \Gamma \left[\frac{f_{\mathrm{eq}}}{f_0} - \frac{1}{2} (P_0 + P_z) \right], \tag{2a}$$

$$\frac{\mathrm{d}P_x}{\mathrm{d}t} = -V_z P_y, \tag{2b}$$

$$\frac{\mathrm{d}P_y}{\mathrm{d}t} = V_z P_x - V_x P_z, \tag{2c}$$

$$\frac{\mathrm{d}P_z}{\mathrm{d}t} = V_x P_y, \tag{2d}$$

where the potentials V_x and V_z can be found in [26, 1].

3. Lepton asymmetry

The effective lepton number $L^{(a)}$ is in general a combination of the different lepton numbers L_f. The effective lepton number in our system evolves only due to L_{ν_a}, which changes because of active-sterile oscillations. We approximate the scattering kernel by an effective equilibration rate $\Gamma = C_a G_F^2 x T^5$, where $C_e \simeq 1.27$ and $C_{\mu,\tau} = 0.92$. Unfortunately this breaks lepton number conservation, so to circumvent this problem we derive an evolution equation for L which does not explicitly depend on Γ. From the definition of L_f we find:

$$L^{(a)} = \frac{2}{8\zeta(3)} \int_0^\infty \mathrm{d}x x^2 \left(\rho_{aa} - \bar{\rho}_{aa} \right) = \frac{1}{8\zeta(3)} \int_0^\infty \mathrm{d}x x^2 f_0 \left(P_0 - \bar{P}_0 + P_z - \bar{P}_z \right) \Rightarrow \tag{3}$$

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \frac{1}{8\zeta(3)} \int_0^\infty \mathrm{d}x x^2 f_0 \left(\frac{\mathrm{d}P_z}{\mathrm{d}t} - \frac{\mathrm{d}\bar{P}_z}{\mathrm{d}t} \right) = \frac{1}{8\zeta(3)} \int_0^\infty \mathrm{d}x x^2 f_0 V_x \left(P_y - \bar{P}_y \right). \tag{4}$$

The damping term D is given by $D \simeq \Gamma$ to a good approximation. This is the term responsible for destroying coherence and it is easy to see that it appears as a damping factor in the equations for the off-diagonal elements parametrised by P_x and P_y. The equilibrium distributions f_{eq} and $\bar{f}_{\mathrm{eq}}$ are the Fermi-Dirac distributions with a finite chemical potential ξ calculated from the neutrino lepton number. ξ is given by the equation

$$L_{\mathrm{eq}}^{(a)} = \frac{1}{4\zeta(3)} \int_0^\infty \mathrm{d}x \; x^2 \left[\frac{1}{1+e^{x-\xi}} - \frac{1}{1+e^{x+\xi}} \right] = \frac{1}{12\zeta(3)} \left(\pi^2 \xi + \xi^3 \right). \tag{5}$$

The real root of this equation can be expressed entirely using real functions. It is given by the Chebyshev cube root, and we find

$$\xi = \frac{-2\pi}{\sqrt{3}} \sinh\left(\frac{1}{3}\text{arcsinh}\left[-\frac{18\sqrt{3}\zeta(3)}{\pi^3}L^{(a)}\right]\right). \tag{6}$$

4. Momentum grid and resonances

This system exhibits resonances similar to the well-known MSW effect. The resonance condition is $V_z = 0$ for particles and $\bar{V}_z = 0$ for anti-particles. Knowing where the resonances reside in momentum space is important, both for numerical work and for understanding the evolution of the system qualitatively. See appendix A in [1] for a careful analysis of the position of resonances.

In order to numerically solve the QKEs, we define the momentum grid in comoving coordinates ($x = p/T$). Therefore the grid becomes stationary and the partial differential equations become ordinary differential equations coupled through integrated quantities only. Using the temperature T as the evolution parameter, time derivatives, d_t, are replaced by $\rightarrow -HT\partial_T$ in the above equations, provided that the time derivative of the effective number of degrees of freedom can be ignored.

5. Results: zero initial lepton asymmetry

In Fig. 1 we show the fraction of thermalised neutrinos, δN_{eff}, for a range of mixing parameters and an initial asymmetry of $L^{(\mu)} = 0$. The top panel shows the normal hierarchy, $\delta m_s^2 > 0$, and the bottom panel the inverted hierarchy, $\delta m_s^2 < 0$. The dashed rectangle shows the range of parameters plotted in Fig. 2.

We mark with a green hexagon the best fit point of the $3+1$ global analysis presented in [27], obtained from a joint analysis of Solar, reactor, and short-baseline neutrino oscillation data $(\delta m_s^2, \sin^2 2\theta_s) = (0.9\ \text{eV}^2, 0.089)$. For that point $\delta N_{\text{eff}} = 1$ in both hierarchies, i.e. complete thermalization occurs. In addition we show the parameter range preferred by CMB and large scale structure (LSS) data. The $1-2-3\sigma$ contours have been obtained interpolating the likelihood function obtained in [28] for each fixed δm_s^2 and N_{eff}. In both cases the lower left corners of parameter space where little thermalization occurs are disfavoured because of the CMB+LSS preference for extra energy density.

In the inverted hierarchy the resonance conditions are always satisfied. Therefore, we expect full thermalization for a larger region of the mass-mixing parameters than in NH, as confirmed in Fig. 1. In this case, thermalisation may proceed through resonant conversions alone.

6. Results: large initial lepton asymmetry

We now give the results for initial large lepton asymmetry. Figure 2 shows the δN_{eff} contour plot for $L^{(\mu)} = 10^{-2}$ and $\delta m_s^2 > 0$ (top panel) and $\delta m_s^2 < 0$ (bottom panel). The region with full thermalisation is now much smaller than in Fig. 1. In principle, one would expect a lepton asymmetry of the same order of magnitude as the baryon asymmetry ($\eta \simeq 10^{-10}$). However, since neutrinos are neutral particles, $L^{(a)} = 10^{-2} - 10^{-1}$ is not presently excluded [29, 30, 31] by the requirement of charge neutrality. A large lepton number can be generated by e.g. an Affleck-Dine mechanism [32] or other models that are able to produce large lepton asymmetries and small baryonic ones [33, 34]. Another interesting possibility is to grow the lepton asymmetry from some initial $L^{(a)} \sim \mathcal{O}(10^{-10})$ using active-sterile oscillations [35, 36, 37]. Solving the QKE's in IH and with an initially small but non-zero lepton number, our preliminary results point toward a final lepton number varying between 10^{-5} and 10^{-2} depending on the mixing parameters. For illustrative purposes, we choose to adopt $L^{(a)} = 10^{-2}$.

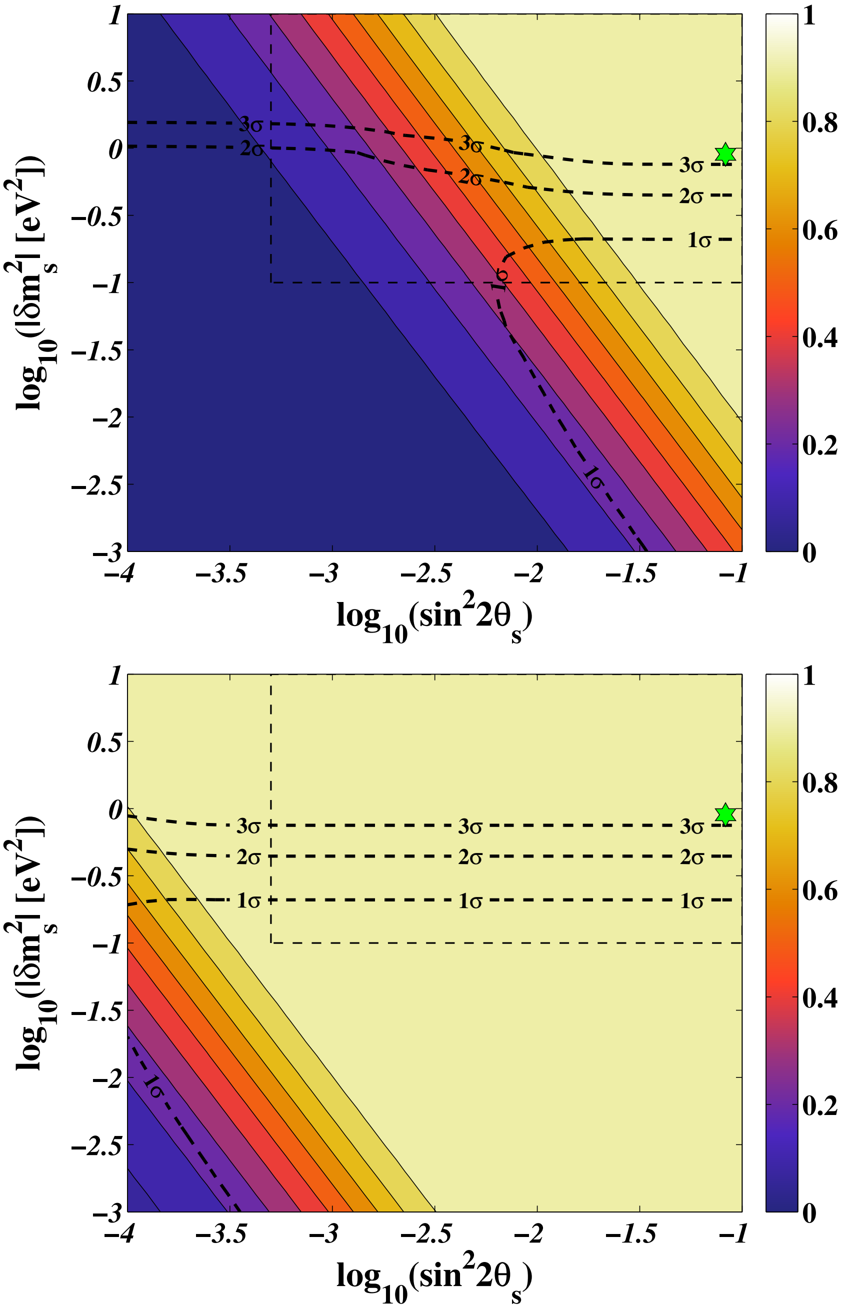

Figure 1. Iso-$\delta N_{\rm eff}$ contours in the $\sin^2 2\theta_s - \delta m_s^2$ plane for $L^{(\mu)} = 0$ and $\delta m_s^2 > 0$ (top panel) and $\delta m_s^2 < 0$ (bottom panel). The green hexagon denotes the ν_s best-fit mixing parameters as in the $3+1$ global fit in [27]: $(\delta m_s^2, \sin^2 2\theta_s) = (0.9\ {\rm eV}^2, 0.089)$. The $1-2-3\sigma$ contours denote the CMB+LSS allowed regions for ν_s with sub-eV mass as in [28].

7. Conclusions

Recent cosmological data seem to favor an excess of radiation beyond three neutrino families and photons, and light sterile neutrinos are possible candidates. The upcoming measurement of $\delta N_{\rm eff}$ by Planck will confirm or rule out the existence of such extra radiation with high precision [38, 39].

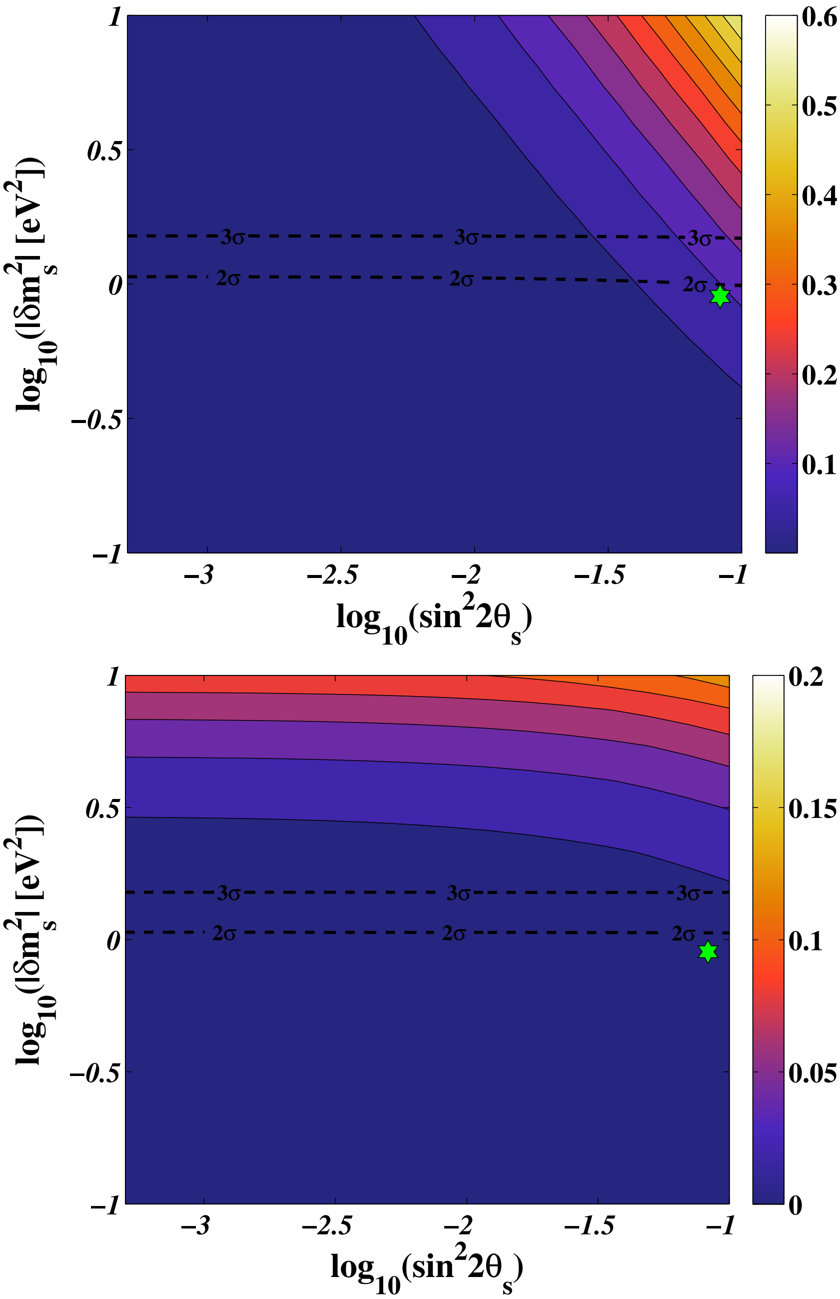

Figure 2. Iso-$\delta N_{\rm eff}$ contours in the $\sin^2 2\theta_s - \delta m_s^2$ plane for $L^{(\mu)} = 10^{-2}$ and $\delta m_s^2 > 0$ (top panel) and $\delta m_s^2 < 0$ (bottom panel), as in Fig. 1.

Light sterile neutrinos could thermalise prior to neutrino decoupling, contributing to the relativistic energy density in the early universe. Present data coming from CMB+LSS, and BBN allow the existence of one sub-eV mass sterile family but do not prefer extra fully thermalised sterile neutrinos in the eV-mass range since they violate the hot dark matter limit on the neutrino mass. However, for large initial lepton asymmetries, light sterile neutrinos are not (or only partially) thermalised for almost all the scanned parameter space. This provides a loophole for eV sterile neutrinos to be compatible with CMB+LSS constraints. For lepton asymmetries around 10^{-2} almost no thermalisation occurs for the parameters preferred by Solar, reactor and short-baseline data, and the sterile neutrinos would contribute very little to the current dark matter density.

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