Primordial power spectrum features and consequences

G Goswami

IUCAA, Post Bag 4, Pune University Campus, Ganeshkhind, Pune 411007, India

E-mail: `gaurav@iucaa.ernet.in`

Abstract. The present Cosmic Microwave Background (CMB) temperature and polarization anisotropy data is consistent with not only a power law scalar primordial power spectrum (PPS) with a small running but also with the scalar PPS having very sharp features. This has motivated inflationary models with such sharp features. Recently, even the possibility of having nulls in the power spectrum (at certain scales) has been considered. The existence of these nulls has been shown in linear perturbation theory. What shall be the effect of higher order corrections on such nulls? Inspired by this question, we have attempted to calculate quantum radiative corrections to the Fourier transform of the 2-point function in a toy field theory and address the issue of how these corrections to the power spectrum behave in models in which the tree-level power spectrum has a sharp dip (but not a null). In particular, we have considered the possibility of the relative enhancement of radiative corrections in a model in which the tree-level spectrum goes through a dip in power at a certain scale. The mode functions of the field (whose power spectrum is to be evaluated) are chosen such that they undergo the kind of dynamics that leads to a sharp dip in the tree level power spectrum. Next, we have considered the situation in which this field has quartic self interactions, and found one loop correction in a suitably chosen renormalization scheme. Thus, we have attempted to answer the following key question in the context of this toy model (which is as important in the realistic case): In the chosen renormalization scheme, can quantum radiative corrections be enhanced relative to tree-level power spectrum at scales, at which sharp dips appear in the tree-level spectrum?

1. Introduction

It is a well known fact that the correlations of CMB anisotropies depend not only on the values of various (late time) background cosmological parameters (e.g., Ω_b, Ω_c, etc.) but also on the assumed form of the PPS. Cosmological inflation has been a very actively investigated paradigm for explaining the origin of primordial metric perturbations that lead to anisotropies in CMB sky as well as to the large scale structure of the universe. There exist models of inflation that give a smooth, nearly scale invariant scalar PPS (with possibly small running of the spectral index). But, there also exist models [1, 2], in which the PPS is not so smooth, so has very sharp features. Theoretically, at this stage, there is no way to favour models that give smooth (featureless) PPS with those that do not. Interestingly, at this stage, even observationally, it is not possible to favour one of these. In many such theoretical scenarios, the scalar PPS has cuspy dips that sometimes correspond to a null in the PPS, i.e., precisely zero scalar power at some wave number. Also, for a range of modes near such a feature, the tensor power overtakes scalar power. Such cusp-like dips in scalar PPS were reported in the literature but their origin was not satisfactorily understood until recently (see [3] and references therein). An exact null

in scalar PPS can have interesting consequences, such as on processed non-linear matter power spectrum. This leads to an interesting possibility: Since the power spectrum calculation is done perturbatively, and since the leading order answer is too small (or zero) at some scale, could the higher order corrections be important at this scale? If there is no power at some scale, can higher order corrections become so important that they become dominant at this scale? We have attempted to answer such questions in this paper in the context of a toy field theory.

2. Two-point functions

Recall that in cosmological perturbation theory, one is concerned with the theory of fluctuations around a background (inflationary) solution. One quantizes these fluctuations and calculates their correlation functions (on a constant time hyper-surface) using the (well known) in-in formalism. The leading higher order corrections to the 2-point function involve the one loop corrections. In the most extreme case of our interest, the tree-level contribution to the 2-point function is zero and the power spectrum at this scale gets generated purely by radiative corrections. We are interested in the 2-point function given by:

$$\langle \phi(\eta, \vec{x}_1)\phi(\eta, \vec{x}_2) \rangle = \int \frac{d^3\vec{k}_1}{(2\pi)^3}\frac{d^3\vec{k}_2}{(2\pi)^3} e^{i(\vec{k}_1 \cdot \vec{x}_1 + \vec{k}_2 \cdot \vec{x}_2)} \times (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2)\mathcal{G}(k_1, \eta), \qquad (1)$$

where the delta function on the right hand side is due to spatial homogeneity of the background. The connected part of the quantity $\mathcal{G}$, introduced in Eq. (1), can be evaluated perturbatively as:

$$\mathcal{G}(p, \eta) = C_0(p, \eta) + C_1(p, \eta)\lambda + C_2(p, \eta)\lambda^2 + \cdots. \qquad (2)$$

Since the limit $\lambda \to 0$ recovers the free theory, $C_0(p, \eta) = |\phi_p(\eta)|^2$ where $\phi_p(\eta)$ is the mode function which determines the tree level PPS. In this article, we have attempted to calculate $C_1(p, \eta)$ perturbatively.

3. Correlation functions

To illustrate the effect we are interested in, we have tried to calculate correlation functions in a field theory, which mimics the dynamics of comoving curvature perturbation $\mathcal{R}(\eta, \vec{x})$. We have solved for the correlations of a fictitous field σ with the following (classical) action:

$$S = \int d\eta \, d^3\vec{x} \, z^2 \left(\frac{(\sigma')^2}{2} - \frac{(\vec{\nabla}\sigma)^2}{2} - z^2\frac{\lambda\sigma^4}{4!} \right), \qquad (3)$$

where we have replaced all occurances of a in the action of a test scalar field (on an expanding unvierse) by $z = -a\sqrt{2\epsilon}$ (here, a is the scale factor and ϵ is the slow-roll parameter) and the function $z(\eta)$ shall be chosen such that the tree-level power spectrum has sharp features. The realistic case is quite difficult to deal with, so we have made a few simplifying assumptions. To begin with, let us assume quartic self interactions of the fluctuation field (whose power spectrum (PS) has a sharp dip at tree level). This is because the one-loop one-vertex diagram will go as $\mathcal{R}^4$, while the one-loop two-vertex diagram will go as $\mathcal{R}^6$, $\mathcal{R}$ being the comoving curvature perturbation. It is important to realize that the action in Eq. (3) is not as arbitrary as it may look naively. When one writes down the fourth-order action for fluctuations (e.g., comoving curvature perturbation), one gets too many terms (see e.g., [4], where this is done without assuming slow-roll approximation for $\delta\phi$ in spatially flat gauge, in this gauge $\mathcal{R} = \frac{H}{\phi}\delta\phi$, so we can express the action in terms of $\mathcal{R}$). The term with quartic self-interactions is one among many of these terms (corresponding to the first term in Eq. (36) of [4], notice that the coefficient of quartic term is such that we will get the factor of z^4 in the coefficient as is assumed above),

and we shall be finding out what will happen to the loop corrections to the power spectrum due to this term alone. As a second simplifying assumption, let us attempt the case in which though the tree level PS has a sharp dip, this dip does not lead to a null in the PS. This will ensure that the one-loop correction is not dominant over the tree level answer, only gets enhanced (thus, perturbation theory will still be valid). The quadratic part of the above action determines the tree level power spectrum and we have chosen the dynamics of z such that the tree level power spectrum has a sharp dip, but not a null (we have chosen z to take the form it takes in Starobinsky model [2]).

3.1. Correlation functions in canonical formalism

One can canonically quantize such a theory in the usual way. We have calculated the correlation functions in canonical formalism in an interaction picture, in which the time-dependence of the field operators shall be governed by the part of the Hamiltonian quadratic in the field, which satisfies a linear differential equation. Given the mode functions of these free fields, any correlation function can be evaluated.

While evaluating correlation functions at one loop, one encounters (just like in flat spacetime field theory) divergences. Apart from the usual UV divergences and IR divergences (the comoving curvature perturbation behaves very much like a massless test field), one also encounters (extra) late time divergences. This is because when we evaluate the 2-point function in the in-in formalism (for an interaction with an even power of field) we get:

$$\langle\Omega|\phi_H(\eta,\vec{x}_1)\phi_H(\eta,\vec{x}_2)|\Omega\rangle = \langle0|\phi_I(\eta,\vec{x}_1)\phi_I(\eta,\vec{x}_2)|0\rangle$$
$$+i\int_{-\infty}^{\eta}d\eta' a(\eta')\langle0|[H_I(\eta'),\phi_I(\eta,\vec{x}_1)\phi_I(\eta,\vec{x}_2),]|0\rangle + \cdots ,$$

(4)

and the contribution when the external time η is set to zero, diverges.

Due to the presence of the above stated late time divergences, it is natural that the loop corrections to the correlations cannot be kept small. There is a vast literature on this subject, attempting to understand the origin and resolution of these divergences. It is important to understand that this is not the kind of enhancement we are interested in: We are talking about a situation in which the loop correction gets enhanced relative to tree level result, because the tree level contribution itself has a sharp dip with very low power. To get rid of the late time divergences, we shall cut the (one-loop) time integral. The prescription (which we have used) to do this is that, just like the tree level PS, even the loop correction to the power spectrum for the field on de Sitter background is scale invariant.

It is important to note that if one takes into account all the self interactions of the metric fluctuations (e.g., the comoving curvature perturbation) dictated by the symmetries of the problem and evaluates the one loop corrections in such a realistic case carefully, one gets no late time divergences [5] and the loop integral receives most contributions near the times, at which the mode is crossing the Hubble radius. So, by putting a cut-off in time integral (a little after Hubble crossing) in the toy field theory with quartic interactions we are attempting to simulate this.

In renormalized pertrubation theory, the bare Lagrangian (consisting of bare field, bare mass, bare self coupling and bare (non-minimal) coupling to gravity ($\xi R\sigma^2$), all of which depend in the UV regulator) is to be partitioned into a renormalized Lagrangian and a counter-term Lagrangian, the way this partition is done determines the choice of renormalization scheme. For scalar fields, in four dimensions, at one-loop (and polynomial interactions), there is no field strength renormalization. Since, we wish to keep the field massless and minimally coupled, the physical mass and physical value of ξ should both vanish. Unless we are using the so called 'On-Shell' (OS) renormalization scheme, the value of the parameters in the renormalized Lagrangian
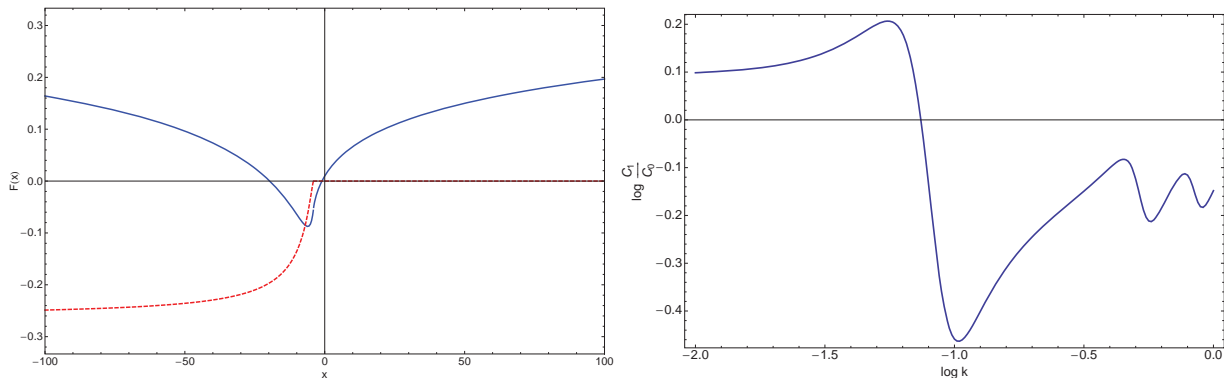
Figure 1. *Left*: For ϕ^3 theory in 6 spacetime (flat) dimensions: The variation of real part of $F(x)$ (blue, continuous) and imaginary part of $F(x)$ (red, dashed) against $x = k^2/m^2$. This function has modest values even when x becomes large. *Right*: For σ theory (Eq. (3)): Logarithm of the ratio C_1/C_0 as a function of $\log k$ for the case in which A_-/A_+ is 0.2 in MS renormalization scheme. Notice that C_1 increases with respect to C_0 and takes up its largest value at a scale just larger than the scale at which the dip in the tree-level power spectrum arises. For this value of A_-/A_+, the enhancement is very small, C_1/C_0 increases from (roughly) 26 to 58 per cent. At other values of A_-/A_+, this enhancement will be larger.

need not be the physical values of those parameters. One can implement a form of minimal subtraction renormalization scheme in a fairly straighforward way. One can easily regularize the UV divergences by putting a cut-off in three dimensional momentum space (and tune the corrections such that the field stays massless).

All this can be easily compared to what happens in the usual flat spacetime field theory. Consider ϕ^3 theory in $d = 6$ spacetime dimensions (the theory does not have a true ground state but perturbation theory does not "know" about it). Suppose we evaluate the one-loop corrections to the (four dimensional Fourier transform of) two-point function in this theory. We know the tree level answer, which is (we are using the signature $(-, +, +, +)$ and $x = k^2/m^2$) $\Delta(k^2) = \frac{1}{k^2+m^2} = \frac{1}{m^2(1+x)}$. Loop corrections are of the order of g^2 and if we call $g^2/(4\pi)^3$ to be α, then the 2-point function (one-loop) is:

$$\Delta(x) = \frac{1}{m^2(1+x)} \left[1 + \alpha F(x)\right]. \tag{5}$$

Since we are working with a weakly coupled theory, we could be working with (say) $g = 0.1$ and then $\alpha = \mathcal{O}(10^{-8})$, and the next order correction is supposed to be of the order of 10^{-16}, even if $F(x)$ becomes fairly large, perturbative calculation shall remain valid. Now, we have used dimensional regularization and OS renormalization scheme to get the function $F(x)$ as shown in the left part of Figure 1. What we are calculating is the equivalent of this for the σ field. There exist a small range of modes, near the dip in the tree-level power spectrum, where the tree-level contribution is at its lowest value while the one-loop contribution is not so small, this causes a relative enhancement in the value of C_1 with respect to C_0 (see Figure 1). The result is shown for the case in which the value of the parameter A_-/A_+ of [2] is 0.2. As one changes this parameter, the depth of the dip in tree level spectrum can be increased, thus, increasing the relative size of loop corrections. We have avoided the cases in which the depth in the dip is too deep because in those cases, the loop correction shall become very large signalling break down of perturbation theory.

4. Conclusion

We have attempted to study how the loop corrections to the correlation functions of cosmological perturbations get affected by the actual dynamics of the mode functions. We have chosen the mode functions of the field such that the tree level power spectrum has a sharp dip, and we have realized that the power spectrum at one loop, renormalized in MS scheme can get enhanced (at scale near the tree level dip) relative to the tree level power spectrum.

References

[1] Jain R K *et al* 2009 *JCAP* **0901** 009 [arXiv:astro-ph/0809.3915], Jain R K *et al* 2010 *Phys. Rev. D* **82** 023509 [arXiv:astro-ph/0904.2518]
[2] Starobinsky A A 1992 *Pis'ma Zh. Éksp. Teor. Fiz.* **55** 477, 1992 *JETP Lett.* **55** 489
[3] Goswami G and Souradeep T 2011 *Phys. Rev. D* **83** 023526
[4] Seery D, Lidsey J E and Sloth M S 2007 *JCAP* **0701** 027 [astro-ph/0610210]
[5] Senatore L and Zaldarriaga M 2010 *JHEP* **1012** 008 [hep-th/0912.2734]