

Multi-field inflation: Formulation, effective theory and phenomenology

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Abstract. We have described how to obtain the non-perturbative low energy effective field theory of single field inflation from a generic multi-field model by integrating out heavy fields. The features of heavy physics is described by the effective speed of sound, which leaves distinctive observational signatures in the correlation functions of the curvature perturbation.

1. Introduction

The absence of the relevant inflaton in the standard model of particle physics demands that inflation be described in the context of the speculative high energy theories, where plenty of scalar fields which can contribute to the inflationary dynamics exist. Further, in multi-field system, we can obtain interesting observational signatures, which deviate from the predictions of the single field models of inflation and can be detected in near future. Thus, we have both theoretical and phenomenological motivations to study multi-field inflation.

However, the very model of inflation relevant for the observed universe is still veiled yet, based on the unverified high energy theories. To cope with our ignorance, we may take the effective field theory approach to integrate out heavy fields from the parent theory, which contains multiple fields. Then, we are left with the light “inflaton”, giving rise to effectively single field inflation.

Here, we have presented the systematics of effective single field description of multi-field inflation. The effects of the heavy fields are left as a non-trivial speed of sound c_s . This leaves non-trivial features in the correlation functions of the curvature perturbation \mathcal{R} . We explicitly compute the 2- and 3-point functions in a simple but illustrative case. This article is based on [1, 2, 3, 4].

2. Multi-field inflation with a single light field

For clarity, we have considered a 2-field model. But our approach can immediately be generalized to more complicated models. We begin by considering the action:

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\text{Pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi_a - V(\phi^a) \right], \quad (1)$$

where the field indices is raised and lowered by the field space metric. Now, assume that we have a set of background solutions $\phi_0^a(t)$ and $a(t)$ with given boundary conditions. Then, because of



the invariance of the action under finite time translations $t \rightarrow \hat{t} = t + \xi^0$, we can generate a family of non-trivial solutions of the form:

$$\phi_\Delta^a(t) = \phi_0^a(t + \Delta\mathcal{T}), \quad \text{and} \quad (2)$$

$$a_\Delta(t) = a(t + \Delta\mathcal{T})e^{\Delta\mathcal{R}}, \quad (3)$$

where $\Delta\mathcal{T}$ and $\Delta\mathcal{R}$ are arbitrary constants of our choice.

We have now considered perturbations around the background solution. We have begun by giving some definitions as: The fields $\pi = \pi(t, \mathbf{x})$, and $\mathcal{F} = \mathcal{F}(t, \mathbf{x})$ to represent departures from the homogeneous and isotropic background solution $\phi_0^a(t)$ as:

$$\phi^a(t, \mathbf{x}) = \phi_0^a(t + \pi) + N^a(t + \pi)\mathcal{F}, \quad (4)$$

where $N^a(t + \pi)$ stands for the vector N^a evaluated at $t + \pi$. Notice that π represents deviations from $\phi_0^a(t)$ exactly along the path defined by the background solution, whereas \mathcal{F} parametrizes deviations off the trajectory, but evaluated at $t + \pi$. In fact, the field \mathcal{F} lives in the tangent space spanned by $N^a(t + \pi)$. For gravitational sector, we have written the spatial metric h_{ij} as:

$$h_{ij} = a(t + \pi)e^{2\mathcal{R}}\delta_{ij}. \quad (5)$$

Then, it immediately follows that there must exist a non-trivial solution for π , \mathcal{R} and \mathcal{F} such that,

$$\pi = \text{constant}, \quad \mathcal{R} = \text{constant}, \quad \mathcal{F} = 0. \quad (6)$$

Furthermore, provided that \mathcal{F} is sufficiently massive, integrating it out will necessarily result in a theory where $\pi = \text{constant}$ and $\mathcal{R} = \text{constant}$ are preserved as non-trivial solutions to all orders in perturbation theory. The structure of the perturbation action must be in the form which has these non-trivial solutions.

Given that we have been eventually interested in integrating out \mathcal{F} , it is convenient to work in the comoving gauge, where $\pi = 0$. A special advantage of the comoving gauge is that the gauge invariant comoving curvature perturbation exactly coincides with \mathcal{R} , thus, any property which our original \mathcal{R} possesses is inherent in the comoving gauge. This is not the case, for example, in the flat gauge. Then, we can systematically expand the action in powers of \mathcal{R} and \mathcal{F} . For definiteness, let us explicitly compute up to cubic action. Then, to integrate out \mathcal{F} , we have solved for it to *linear* order and plugged the formal solution back into the action wherever \mathcal{F} appears¹, and expanded in powers of M_{eff}^2 , the effective mass of \mathcal{F} , which to first order means dis-regarding spacetime derivatives acting on \mathcal{F} . This gives:

$$\mathcal{F} = -\frac{2\dot{\phi}_0\dot{\theta}}{HM_{\text{eff}}^2}\dot{\mathcal{R}}, \quad (7)$$

where $\dot{\phi}_0^2 = \dot{\phi}^a\dot{\phi}_a$ and $\dot{\theta} = N^a V_a / \dot{\phi}_0$ is the angular velocity for the trajectory. Plugging \mathcal{F} back into the action gives us the desired effective single field theory for \mathcal{R} , which we have written as: $S_{\text{eff}} = S_2 + S_3$ with:

$$S_2 = \int d^4x \frac{a^3 \epsilon m_{\text{Pl}}^2}{c_s^2} \left[\dot{\mathcal{R}}^2 - c_s^2 \frac{(\nabla \mathcal{R})^2}{a^2} \right], \quad \text{and} \quad (8)$$

$$S_3 = \int d^4x a^3 \left[-\epsilon m_{\text{Pl}}^2 \mathcal{R} \frac{(\nabla \mathcal{R})^2}{a^2} + 3 \frac{\epsilon m_{\text{Pl}}^2}{c_s^2} \dot{\mathcal{R}}^2 \mathcal{R} + \epsilon m_{\text{Pl}}^2 \frac{(1 - c_s^2)^2 - 2}{2c_s^4} \frac{\dot{\mathcal{R}}^3}{H} \right. \\ \left. + \frac{m_{\text{Pl}}^2}{2a^4} \left\{ \left(3\mathcal{R} - \frac{\dot{\mathcal{R}}}{H} \right) [\psi^{,ij}\psi_{,ij} - (\Delta\psi)^2] - 4\mathcal{R}^{,i}\psi_{,i}\Delta\psi \right\} \right], \quad (9)$$

where $\psi = -\mathcal{R}/H + \epsilon \Delta^{-1} \dot{\mathcal{R}}/c_s^2$, and $c_s^{-2} = 1 + \dot{\theta}^2/M_{\text{eff}}^2$.

¹ This prescription exactly reproduces the tree level effective action, while loop contributions are not captured.

3. Features from heavy physics

Given the quadratic action by Eq. (8), we may insert the canonical quadratic action as:

$$S_{2,\text{canonical}} = \int d^4x a^3 \epsilon m_{\text{Pl}}^2 \left[\dot{\mathcal{R}}^2 - \frac{(\nabla \mathcal{R})^2}{a^2} \right], \quad (10)$$

where $c_s = 1$, by writing the whole effective action as:

$$S_{\text{eff}} = S_{2,\text{canonical}} + \underbrace{\int d^4x a^3 \epsilon m_{\text{Pl}}^2 \left(\frac{1}{c_s^2} - 1 \right) \dot{\mathcal{R}}^2}_{\equiv S_{2,\text{int}}} + S_3. \quad (11)$$

That is, instead of absorbing the second term into the “free” part of the action, and hence, being able to recast the quadratic effective action for the curvature mode as one with a modified speed of sound c_s^2 , we can also consider it as a perturbation. A particular advantage of working in this manner is that the Green’s functions for the free part of our theory are simple, corresponding to those of a light scalar field in quasi de Sitter spacetime, with $d\tau = dt/a$ being the conformal time,

$$\mathcal{R}_k(\tau) = \frac{iH}{\sqrt{4\epsilon k^3 m_{\text{Pl}}}} (1 + ik\tau) e^{-ik\tau}. \quad (12)$$

This is in contrast to the Green’s functions for the theory, where we have included the interaction term in the free part, which exhibits a modified speed of sound. The Green’s function for this theory would only be available if c_s^2 were a *constant*, a situation we are not particularly interested in restricting ourselves to. The two approaches would, of course, give us the same answers if the latter were implementable.

Given an interaction Lagrangian, it is convenient to adopt the in-in formalism. The expectation value evaluated at a time t of a time dependent operator $\hat{\mathcal{O}}(t)$ is written as:

$$\langle \hat{\mathcal{O}}(t) \rangle = \sum_{n=1}^{\infty} i^n \int_{t_{\text{in}}}^t dt_n \int_{t_{\text{in}}}^{t_n} dt_{n-1} \cdots \int_{t_{\text{in}}}^{t_2} dt_1 \langle 0 | [H_{\text{int}}(t_1), [H_{\text{int}}(t_2), \cdots [H_{\text{int}}(t_n), \hat{\mathcal{O}}(t)] \cdots]] | 0 \rangle, \quad (13)$$

where t_{in} is some early “in” time when the interaction is turned on. It is described by the interaction Hamiltonian $H_{\text{int}}(t) = -L_{\text{int}}(t)$ for cubic or higher order. We can then straightforwardly compute the correction to the featureless power spectrum, which is easily evaluated via Eq. (13). We have found that the contributions to the featureless power spectrum $\mathcal{P}_{\mathcal{R}}$ arising from the interaction $S_{2,\text{int}}$ is given by:

$$\frac{\Delta \mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} = \kappa \int_0^{\infty} dt \left(1 - \frac{1}{c_s^2} \right) \sin(2\kappa t), \quad (14)$$

where $\kappa \equiv k/k_*$ with k_* being a fiducial reference, and the leading bispectrum by:

$$\begin{aligned} B_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = & 2\Re \left\{ -2i \hat{\mathcal{R}}_{k_1}(0) \hat{\mathcal{R}}_{k_2}(0) \hat{\mathcal{R}}_{k_3}(0) \left[\frac{3}{2} \epsilon \frac{m_{\text{Pl}}^2}{H^2} \int_{-\infty}^0 d\tau \left(1 - \frac{1}{c_s^2} \right)^2 \tau^{-1} \frac{d\hat{\mathcal{R}}_{k_1}^*(\tau)}{d\tau} \frac{d\hat{\mathcal{R}}_{k_2}^*(\tau)}{d\tau} \frac{d\hat{\mathcal{R}}_{k_3}^*(\tau)}{d\tau} \right. \right. \\ & - \epsilon \frac{m_{\text{Pl}}^2}{H^2} \int_{-\infty}^0 d\tau \frac{\epsilon - 3 + 3c_s^2}{c_s^4} \tau^{-2} \frac{d\hat{\mathcal{R}}_{k_1}^*(\tau)}{d\tau} \frac{d\hat{\mathcal{R}}_{k_2}^*(\tau)}{d\tau} \hat{\mathcal{R}}_{k_3}^*(\tau) + 2 \text{ perm} \\ & + \epsilon \frac{m_{\text{Pl}}^2}{H^2} (\mathbf{k}_1 \cdot \mathbf{k}_2 + 2 \text{ perm}) \int_{-\infty}^0 d\tau \frac{\epsilon - 2s + 1 - c_s^2}{c_s^2} \tau^{-2} \hat{\mathcal{R}}_{k_1}^*(\tau) \hat{\mathcal{R}}_{k_2}^*(\tau) \hat{\mathcal{R}}_{k_3}^*(\tau) \\ & \left. + 2\epsilon(\epsilon - \eta_{\parallel}) \frac{m_{\text{Pl}}^2}{H^2} \int_{-\infty}^0 d\tau \frac{s}{c_s^4} \tau^{-3} \frac{d\hat{\mathcal{R}}_{k_1}^*(\tau)}{d\tau} \hat{\mathcal{R}}_{k_2}^*(\tau) \hat{\mathcal{R}}_{k_3}^*(\tau) + 2 \text{ perm} \right\}, \quad (15) \end{aligned}$$

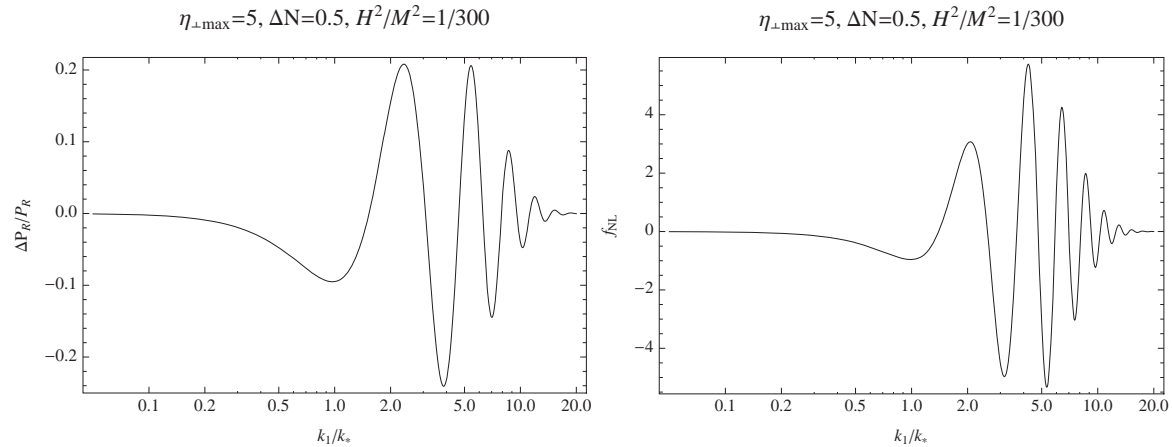


Figure 1. Features of the (left) power spectrum and (right) f_{NL} from the curvilinear trajectory given by Eq. (16). As the turn is experienced ($k = k_*$) oscillatory features arise, and vanish as we probe smaller scales.

with $s = \dot{c}_s/(Hc_s)$ and $\eta_{\parallel} = -\ddot{\phi}_0/(H\dot{\phi}_0)$.

As an illustrative example, we have considered the case that there is a smooth single turn in the otherwise straight trajectory. We have parametrized the turn using the hyperbolic cosine function as:

$$\eta_{\perp} = \frac{\eta_{\perp\text{max}}}{\cosh^2[2\log(\tau/\tau_*)/\Delta N]}, \quad (16)$$

where $\eta_{\perp} = \dot{\theta}/H$ and \star denotes the moment when the bending is at its maximum. Then, the resulting features in the power spectrum and the non-linear parameter f_{NL} are shown in Figure 1 [4]. We have seen that the features are correlated, which could be a distinctive observational signature for future experiments.

4. Conclusions

We have presented the effective single field theory of multi-field inflation. When the mass hierarchy is large, we can integrate out the heavy fields, and have single field description of inflation. The footprints of the heavy fields are given as a non-trivial speed of sound c_s . This leaves features in the correlation functions of the curvature perturbation. Importantly, these features are correlated with the degree of non-Gaussianity enhanced, which may be detected in near future observations.

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References

- [1] Achucarro A, Gong J O, Hardeman S, Palma G A and Patil S P 2011 *Phys. Rev. D* **84** 043502 [1005.3848 [hep-th]]
- [2] Achucarro A, Gong J O, Hardeman S, Palma G A and Patil S P 2011 *JCAP* **01** 030 [1010.3693 [hep-ph]]
- [3] Achucarro A, Gong J O, Hardeman S, Palma G A and Patil S P 2011 [1201.6342 [hep-th]]
- [4] Achucarro A, Gong J O, Palma G A and Patil S P *To appear*