Journal of Physics: Conference Series **484** (2014) 012056 doi:10.1088/1742-6596/484/1/012056

Singularity resolution in cosmologies using AdS/CFT

A Ghosh

Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

E-mail: `archisman.ghosh@uky.edu`

Abstract. Singularities in general relativity are expected to be cured by a quantum theory of gravity. String theory is a prospective candidate for quantum gravity and holographic correspondences like AdS/CFT, that arise from string theory, might shed some light into the nature of singularities. While the theory of gravity might not be tractable near the singularity, the corresponding field theory might be well-behaved. We have come up with a model of cosmology, where curvatures become large at some time, thus, resembling a big crunch, but whose field theory dual remains well-behaved. Our analysis in the field theory indicates that we have a bouncing cosmology with a thermal distribution of radiation, and a possible formation of a small black hole after the bounce.

1. Motivation and overview

The theory of general relativity has a unique property of predicting its own failure: It breaks down at places called *singularities*. Our usual notions of spacetime, break down near these singularities. At spacelike or null singularities, like those in the interiors of neutral black holes, “time” begins or ends, the meaning of which is not clear. Particularly, disturbing are the cosmological singularities like the big bang or the big crunch, which no observer can bypass.

Near these singularities, where the curvatures have become large compared to some scale, a quantum theory of gravity is expected to take over. Holographic correspondences like AdS/CFT [1] arising from string theory provide a non-perturbative formulation of quantum gravity. Here, gravity in the bulk is an effective description of a non-gravitational field theory in a lower number of dimensions. We ask the question: Whether the dual description following these correspondences can give us some insight as to what happens near or past these singularities? In these models, the spectrum at late times can be interpreted as the primordial fluctuations that lead to structure formation in the early universe. An incomplete, but long list of attempts made in this direction has been given in [2]. (For reviews, refer [3].) The bulk of this article is based on the work done in [4].

2. The AdS/CFT correspondence

The AdS/CFT correspondence relates a theory of gravity to a gauge theory with no dynamical gravity. In the simplest example of the correspondence relevant to our discussion, the theory of gravity is a type IIB String Theory on the $AdS_5 \times S^5$ background given by the metric:

$$ds^2 = -(1 + \frac{r^2}{R_{AdS}^2})dt^2 + \frac{dr^2}{1 + \frac{r^2}{R_{AdS}^2}} + r^2\, d\Omega_3^2 + R_{AdS}^2\, d\Omega_5^2 \ , \tag{1}$$

and the dual gauge theory is the $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang-Mills theory on $S^3\times\mathbb{R}$. Here, R_{AdS} is the length scale associated with the geometry, which we have eventually set to unity. The Yang-Mills coupling g_{YM} and the N of the gauge group in the gauge theory are related to the parameters of the string theory, the string length and coupling $l_s\equiv\sqrt{\alpha'}$ and $g_s=e^{\Phi}$ respectively[1], via the relations:

$$g_{YM}^2=4\pi g_s \quad and \quad \lambda\equiv g_{YM}^2 N=\frac{R_{AdS}^4}{l_s^4}\,. \tag{2}$$

Here, λ is the 't Hooft coupling, the relevant coupling of gauge theories in the large N limit. In this limit ($N\to\infty$, $g_{YM}\to 0$, $\lambda\equiv g_{YM}^N=$ finite), the dual description is a *classical* string theory with the quantum corrections suppressed by $1/N$. Moreover, the above relations tell us that the large 't Hooft coupling limit ($\lambda\to\infty$), corresponds to the low-energy super-gravity truncation of the string theory, where the stringy modes are suppressed by α'. This is because, the massive string modes, which have masses of $O(1/\sqrt{\alpha'})$, are heavy compared to the energy scale of the theory $1/R_{AdS}$, and are not excited. So we are left only with the massless super-gravity modes.

3. Our setup in super-gravity

We have considered a Yang-Mills theory with a time-dependent 't Hooft coupling $\lambda(t)$. In the dual bulk (via AdS/CFT), this corresponds to a non-normalizable dilaton mode, whose boundary value $\Phi_0(t)$ is being dialled. The dilaton evolves following the super-gravity equations of motion and produces a non-trivial metric by back-reaction. The equations to solve are, thus, the Einstein's dilaton equations:

$$R_{ab}=\Lambda g_{ab}+\frac{1}{2}\partial_a\Phi\partial_b\Phi,\ \ (\text{with } \Lambda=-4), \quad and \quad \nabla^2\Phi=0\,. \tag{3}$$

We have chosen a profile such that the 't Hooft coupling λ is large at early and late times. The dual theory, thus, has a good gravity description at these times. We have chosen the initial state to be the vacuum of the gauge theory, which in the bulk corresponds to pure $AdS_5\times S^5$. At intermediate times, λ is allowed to become $O(1)$, as the curvatures in the bulk reach the string scale at this stage, and super-gravity description breaks down. We have fixed N to remain large, so that the curvatures do not reach the Planck scale, and the system remains classical. Thus, the singularities that we have described are rather restrictive.

Furthermore, we have varied the 't Hooft coupling, or equivalently the boundary value of the dilaton *slowly*. The curvature scale of the AdS or equivalently the radius of S^3 on which the field theory is defined sets a scale in the theory. We can quantify this as:

$$\Phi_0=f(\epsilon t),\ \epsilon\ll 1, \quad and \quad \dot{\Phi}_0=\epsilon f'(\epsilon t)\,, \tag{4}$$

where each time derivative comes with an additional power of ϵ.

Although, the change is slow, the total change in the value of the dilaton accrues over time, and we can eventually have a large change in the value of the dilaton. Our solution, therefore, includes the case where the value of the dilaton becomes small, the bulk curvatures become large compared to the string scale, and super-gravity approximation in the bulk breaks down.

3.1. Our solution

We have expanded the dilaton and the bulk metric order by order in ϵ as:

$$\Phi(r,t)=\Phi_0(t)+\epsilon\Phi_1(r,t)+\epsilon^2\Phi_2(r,t)+\dots\,, \quad and \quad g_{ab}=g_{ab}^{(0)}+\epsilon g_{ab}^{(1)}+\epsilon^2 g_{ab}^{(2)}+\dots\,, \tag{5}$$

[1] In String Theory, the coupling g_s is not an independent parameter, but is dynamically determined by the value of the scalar field Φ, the *dilaton*.

and plug them in the Einstein's equations. Solving to the lowest non-trivial order in ϵ, we get:

$$\Phi_2(r,t) = \Phi_0(t) + \frac{1}{4}\ddot{\Phi}_0(t)\left[\frac{1}{r^2}\log(1+r^2) - \frac{1}{2}\Big(\log(1+r^2)\Big)^2 - \mathrm{dilog}(1+r^2) - \frac{\pi^2}{6}\right],$$

$$g_{tt} = 1 + r^2 - \frac{1}{12}\dot{\Phi}_0^2\Big[3 - \frac{1}{r^2}\ln(1+r^2)\Big] \quad and \quad g^{rr} = 1 + r^2 - \frac{1}{12}\dot{\Phi}_0^2\Big[1 - \frac{1}{r^2}\ln(1+r^2)\Big]. \qquad (6)$$

The solution is smooth everywhere: There are no horizons, and no black holes are formed. This is a consequence of the slow variation. For a variation that is fast enough, we can form black holes even within the super-gravity approximation [5].

We have now considered the situation where the value of the dilaton has become small at intermediate times and super-gravity approximation has broken down. We have turned to the gauge theory to see if it can give us any clue as to what happens in the stringy regime and what happens afterwards when super-gravity is restored.

4. Field theory analysis

The boundary gauge theory is a quantum mechanical system with a time dependent parameter - the coupling - that is *slowly varying*. For the field theory on S^3, the spectrum has a gap of order $1/R_{AdS}$ for all values of 't Hooft coupling. So we can use the *adiabatic approximation* in quantum mechanics to study the system. The standard adiabatic approximation of quantum mechanics will turn out to be valid only if $N\epsilon \ll 1$, which is rather stringent. We have formulated an alternate *adiabatic approximation in terms of coherent states.*

4.1. Standard adiabatic approximation

Consider a quantum mechanical system with Hamiltonian $H(\zeta(t))$ that depends on a slowly-varying parameter $\zeta(t)$. In our case, $\zeta(t)$ is the time dependent 't Hooft coupling $\lambda(t)$ or equivalently, the boundary value of the dilation field $\Phi_0(t)$. The instantaneous eigenstates of the Hamiltonian are $H(\zeta)|\phi_m(\zeta)\rangle = E_m(\zeta)|\phi_m(\zeta)\rangle$. The evolution of the ground state $|\phi_0\rangle$ of $H(\zeta_0)$ in the adiabatic approximation is simply given by: $|\psi_0(t)\rangle \simeq |\phi_0(\zeta)\rangle e^{-i\int_{-\infty}^{t} E_0(\zeta)dt}$.

The approximation is good if the first corrections are small: $\left|\left\langle \phi_n|\frac{\partial H}{\partial \zeta}|\phi_0\right\rangle\dot{\zeta}\right| \ll (\Delta E)^2$, where ΔE is the energy difference between the ground and the first excited states. In our case, $\dot{\zeta}(t) = \dot{\Phi}_0(t) \sim O(\epsilon)$ and using AdS/CFT [2], $\left\langle \phi_n|\frac{\partial H}{\partial \Phi_0}|\phi_0\right\rangle = -\left\langle \phi_n|\hat{\mathcal{O}}_{l=0}|\phi_0\right\rangle \sim N$.

Thus, the adiabatic approximation is valid if $N\epsilon \ll 1$. This condition is, however, more stringent than the condition $\epsilon \ll 1$, which we had in the super-gravity analysis.

4.2. Adiabatic approximation with coherent states

A general coherent state has the form $|\Psi(t)\rangle = \exp\left[\sum_n \lambda_n(t)a_n^\dagger\right]|0\rangle$, where $a_n^\dagger$ are the creation operators, and $\lambda_n(t)$ are the coherent state parameters. For large values of λ_n, the system goes over to a classical configuration. In our problem, the creation and annihilation operators need to be replaced by the creation and annihilation parts $\hat{\mathcal{O}}_{(\pm)}$ of $\hat{\mathcal{O}}$. In general, the operators $\hat{\mathcal{O}}_{(\pm)}$ have a non-trivial operator algebra, which mixes all of them and makes the evolution of $\chi(t)$ and $\lambda_n(t)$, and quite complicated. However, the situation drastically simplifies for a slow variation. The expectation value of the 3-point function taken in a coherent state can be shown to be $\left\langle \psi|\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\mathcal{O}}_3|\psi\right\rangle \sim \epsilon$. For $\epsilon \ll 1$, the three and higher point functions are, thus, parametrically suppressed, and all we are left with the 2-point functions. The operators $\hat{\mathcal{O}}_{(\pm)}$ *decouple*, and their algebra reduces to that of a free harmonic oscillator.

[2] Turning on a field Φ with a boundary value Φ_0 corresponds to the deformation of the gauge theory action by a gauge invariant operator $\hat{\mathcal{O}} = \frac{\delta\mathcal{S}_{\mathrm{Cl}}}{\delta\Phi_0}$, where $\mathcal{S}_{\mathrm{Cl}}$ is the classical super-gravity action.

After plugging in the mode expansion in the Schrödinger equation, subsequent integration by parts leads to a perturbative expansion for λ_n, we get: $\lambda_n(t) = -\frac{iN\dot{\Phi}_0(t)}{(2n)^2} + \frac{N\ddot{\Phi}_0(t)}{(2n)^3} + \cdots$. The above expansion is valid if the second and subsequent terms are small compared to the first term, i.e., $|\frac{\ddot{\Phi}_0}{2n\dot{\Phi}_0}| \ll 1 \; \forall \; n$. This is identical to the condition $\epsilon \ll 1$. Moreover, the state represents a "classical" deformation only when the coherent state parameter λ is large. That holds when $N\epsilon \gg 1$.

4.3. Results

For small 't Hooft coupling, the coupling between the oscillators is still individually suppressed by ϵ and most of the above analysis goes through. However, now we have $O(N^2)$ *stringy modes*, whose frequencies are comparable to those of the super-gravity modes. *So there is a possibility of thermalization.* The energy that has been pumped in might get distributed between the various modes. Then at late times, one could be left with $O(N^2\epsilon^2)$ thermalized energy in the system. It is difficult to say whether thermalization would indeed occur, since the time scale of variation is same as the thermalization time scale.

However, we can assume that all the energy left behind in the system has been thermalized ($E_{\rm Thermal} \sim N^2\epsilon^2$), and from entropic considerations in super-gravity figure out the worst case scenarios for various values of ϵ. For $\epsilon \ll \lambda^{5/4}N^{-1}$, a gas of *super-gravity modes* is favoured. For $\epsilon \gtrsim \lambda^{5/4}N^{-1}$, we can have a gas of *massive string modes*. For $\epsilon \gtrsim \lambda^{-7/8}$, a *small black hole* can form. In order to form a *large black hole*, one would require $\epsilon \gtrsim 1$. So a large black hole is never formed in our slowly varying solution.

5. Conclusions and outlook

We have an example of a situation in which the gauge theory has a smooth time-evolution across a region whose dual bulk region has a singularity. An explicit calculation of the final state is possible in principle, but made difficult in practice, because we do not have a complete AdS/CFT dictionary in the stringy regime. Of the possible outcomes, a thermal distribution at late times would imply a Gaussian spectrum while formation of a black hole would source a non-Gaussianity.

Acknowledgements

The author thanks his collaborators Adel Awad, Sumit Das, Jae-Hyuk Oh and Sandip Trivedi, and also Robert Brandenberger, Shiraz Minwalla and K Narayan, with whom he has discussed in various stages. He also thanks the organizers of ICGC-2011 for putting together a very successful conference, and the Huffaker Foundation for a Huffaker Travel Support Grant that made my participation in the conference possible. This work is partially supported by the NSF-PHY-0855614.

References

[1] Maldacena J M 1998 *Adv. Theor. Math. Phys.* **2** 231, 1999 *Int. J. Theor. Phys.* **38** 1113 [arXiv:hep-th/9711200], Witten E 1998 *Adv. Theor. Math. Phys.* **2** 253 [arXiv:hep-th/9802150], Gubser S S, Klebanov I R and Polyakov A M 1998 *Phys. Lett. B* **428** 105 [arXiv:hep-th/9802109], For a standard review, Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 *Phys. Report* **323** 183 [arXiv:hep-th/9905111]

[2] Hertog T and Horowitz G T 2004 *JHEP* **0407** 073 [hep-th/0406134], Craps B, Sethi S and Verlinde E P 2005 *JHEP* **0510** 005 [hep-th/0506180], Das S R and Michelson J 2005 *Phys. Rev. D* **72** 086005 [hep-th/0508068], Das S R and Michelson J 2006 *Phys. Rev. D* **73** 126006 [hep-th/0602099], Das S R, Michelson J, Narayan K and Trivedi S P 2006 *Phys. Rev. D* **74** 026002 [hep-th/0602107], Das S R, Michelson J, Narayan K and Trivedi S P 2007 *Phys. Rev. D* **75** 026002 [hep-th/0610053], Awad A, Das S R, Narayan K and Trivedi S P 2008 *Phys. Rev. D* **77** 046008 [arXiv:0711.2994[hep-th]], Awad A,

Das S R, Nampuri S, Narayan K and Trivedi S P 2009 *Phys. Rev. D* **79** 046004 [arXiv:0807.1517[hep-th]], Turok N, Craps B and Hertog T [arXiv:0711.1824[hep-th]], Craps B, Hertog T and Turok N [arXiv:0712.4180[hep-th]]

[3] Das S R *String Cosmology* ed Erdmenger J 231, Das S R 2009 *PoS ISFTG* 006

[4] Awad A, Das S R, Ghosh A, Oh J H and Trivedi S P 2009 *Phys. Rev. D* **80** 126011 [arXiv:0906.3275[hep-th]]

[5] Bhattacharyya S and Minwalla S 2009 *JHEP* **0909** 034 [arXiv:0904.0464[hep-th]]