Halo shapes, initial shear field, and cosmic web

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Abstract. The ellipsoidal collapse model, combined with the excursion set theory, allows one to estimate the shapes of dark matter halos as seen in high-resolution numerical simulations. The same theoretical framework predicts a quasi-universal behaviour for the conditional axis ratio distributions at later times, set by initial conditions and unaltered by non-linear evolution. The formalism for halo shapes is also useful in making the connection with the initial shear field of the cosmic web, which plays a crucial role in the formation of large-scale structures. The author has briefly discussed the basic aspects of the modelling, as well as the implications of a new formula for the constrained eigenvalues of the initial shear field, given the fact that positions are peaks or dips in the corresponding density field – and not random locations. This formula leads to a new generalized excursion set algorithm for peaks in Gaussian random fields. The results highlighted, here, are relevant for a number of applications, especially for weak lensing studies and for devising algorithms to find and classify structures in the cosmic web.

1. Introduction

The gravitational clustering of matter, as seen in large-volume N-body simulations such as the new Horizon Runs [1], show a characteristic anisotropic web-like structure known as the *cosmic web*. Figure 1 exemplifies a snapshot of the matter density field through the Horizon Run 2 at the present epoch, which clearly reveals the typical pattern of halos, filaments and sheets, connected in a network, with voids encompassing the volume in between. A zoom into a smaller region of the large survey volume shows that individual dark matter halos are, indeed, triaxial in shape. This is not surprising, as the statistics of Gaussian random fields implies that spherically symmetric configurations are highly unlikely [2]. However, defining a gravitationally bound object in a simulation is still a matter of debate, and different methods for defining virialized halos give rise to very different statistical shape distributions. This is probably, why most analytic models of structure formation still make the assumption that gravitationally bound objects are spherical, despite dark matter halos are rather elongated, close to prolate in the central parts and rounder in the outskirts (see, for example [3]). In the first part of this paper (Section 2), we have summarized the main results of a new model presented in [4, 5], which allows one to describe the triaxial distribution of halo shapes at any given time, starting from first principles. In particular, a remarkable feature of the model is the ability to explain the universality of the conditional axis ratio distributions at later times. In the second part (Section 3), we have aimed at making the connection with the statistics of the cosmic web. This is

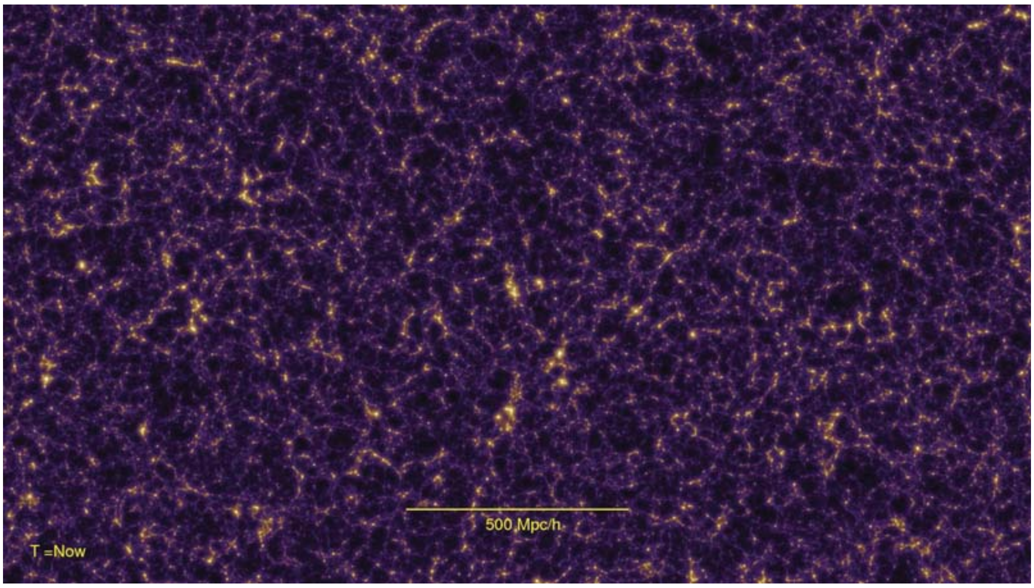

Figure 1. A snapshot of the matter density field from the Horizon Run 2 at the present time [1], which reveals the characteristic *cosmic web* pattern. (Figure courtesy of Juhan Kim.)

achieved by discussing the importance of a new formula for the constrained eigenvalues of the initial shear field presented in [6], which leads to a new generalized excursion set algorithm for peaks [7]. Section 4 concludes with a short highlight of the main findings, and with a description of ongoing and future applications.

2. Halo shapes

The model for triaxial halo shapes developed in [4, 5] has two independent parts: A scheme for how an initially spherical patch evolves and virializes, and a procedure to assign the correct initial shapes to halos of different masses. While the second part is simply given by the statistics of Gaussian random fields, the first one is based on the ellipsoidal collapse framework proposed by [8]. In the ellipsoidal model, the time required to collapse depends not only on the initial over-density $\delta_{\rm i}$ and size $R_{\rm i}$ (or variance $\sigma_{\rm i}^2$) of the patch, but also on the surrounding shear field described by two shape parameters: ellipticity and prolateness (e, p). The essence of the model has been summarized in Figure 2, where 10000 halos of fixed mass $M = 10^{13.5} M_{\odot}$ are evolved from $z = 39$ (left panel) till $z = 0$ (central panel), via a numerical implementation of the ellipsoidal collapse scheme. Comparison with high-resolution simulations, such as those of [3] has shown that results are accurate within the selected mass range (right panel), but significant differences remain at lower and higher masses; reasons for these discrepancies are discussed in detail in [5].

A remarkable feature of the proposed theory is the ability to describe the universality of the conditional distributions of halo shapes, for which simulations also have shown to be quasi-universal (i.e., independent of mass and epoch). This is mainly the reflection of initial conditions, given by the statistics of Gaussian random fields. In particular, if A_1, A_2, A_3 are the scale factors for the principal axes of the ellipsoidal perturbation (ordered as $A_1 \leq A_2 \leq A_3$), and $\delta_{\rm i}$ and e are fixed, then it is easy to show that A_1/A_2 is only a function of prolateness. Hence, for a fixed

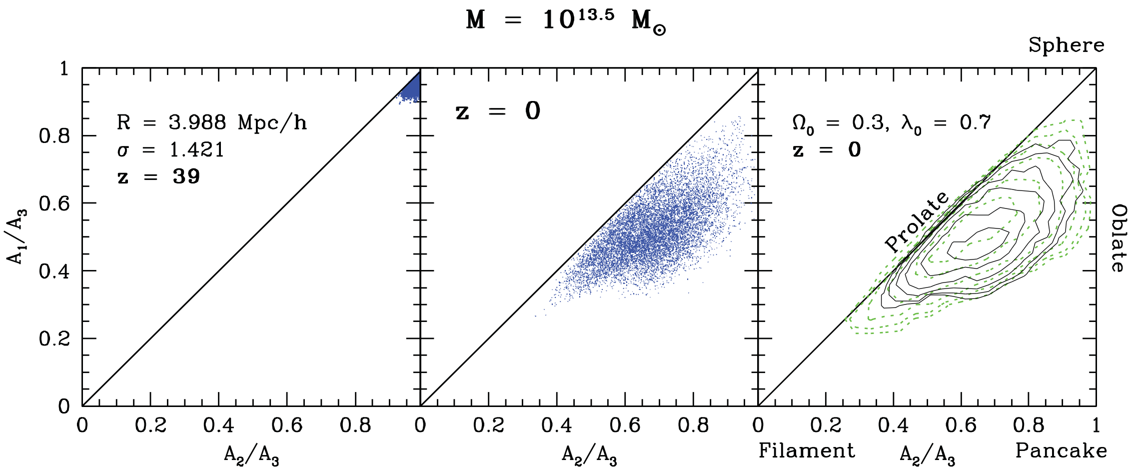

Figure 2. Essence of the halo shape model presented in [4, 5]. Halos of fixed mass $M = 10^{13.5} M_\odot$ and axis ratios A_1/A_3, A_2/A_3 (i.e., $A_1 \le A_2 \le A_3$) are evolved from $z = 39$ (left panel) till $z = 0$ (central panel) with a numerical implementation of the ellipsoidal collapse. The right panel compares the model (solid line) with measurements from numerical simulations [3].

mass, if one defines $A_{12} \equiv A_1/A_2$ and $A_{13} \equiv A_1/A_3$, then,

$$p(A_{12}|A_{13},\delta,\sigma) = \frac{3}{2(1-A_{13})}\left[1 - \frac{(2A_{12}-1-A_{13})^2}{(1-A_{13})^2}\right] \exp\left\{-\frac{5}{8\sigma^2}(2A_{12}-1-A_{13})^2\right\}. \quad (1)$$

At small masses, the exponential term $\to 1$, making this distribution almost universal. This remarkable feature is extremely useful for interpreting various results on halo shapes.

3. Initial shear field

The previous formalism for halo shapes allows one to make the connection with the initial shear field of the cosmic web [6], and with the theory of peaks in Gaussian random fields developed in [9]. In fact, directly related morphological quantities such as e and p are crucial in characterizing the structural properties of halos, voids, sheets and filaments. A central result presented in [6] is a formula which extends Doroshkevich's unconditional relations [2] for the eigenvalues of the linear tidal field, by including correlations between the density and tidal shear tensors. The strength of their correlation is en-capsulated in the coefficient γ, the same factor, which is fundamental in peak theory [9]. In particular, if $\zeta_{\rm i}$ (with $i = 1, 2, 3$) are the constrained eigenvalues of the density matrix, then,

$$p(\zeta_1,\zeta_2,\zeta_3|\gamma) = \frac{15^3}{8\sqrt{5}\pi}\frac{1}{(1-\gamma^2)^3}\exp\left[-\frac{3}{2(1-\gamma^2)}(2K_1^2 - 5K_2)\right]|(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)(\zeta_2-\zeta_3)|,$$

where

$$K_1 = \zeta_1 + \zeta_2 + \zeta_3, \quad \textit{and} \quad (2)$$
$$K_2 = \zeta_1\zeta_2 + \zeta_1\zeta_3 + \zeta_2\zeta_3, \quad (3)$$

and $\zeta_{\rm i} = \lambda_{\rm i} + \gamma\xi_{\rm i}$, with $\lambda_{\rm i}$'s and $\xi_{\rm i}$'s the corresponding unconstrained eigenvalues of the shear tensor and Hessian matrix respectively (see also [6] for more details).

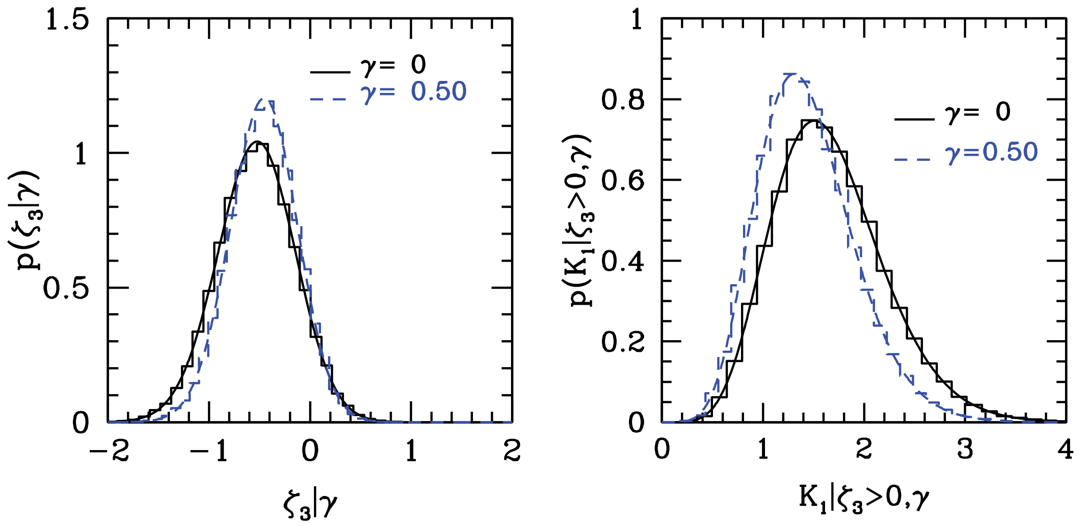

Figure 3. Conditional distributions $p(\zeta_3|\gamma)$ (left) and $p(K_1|\zeta_3 > 0, \gamma)$ (right), from 50000 mock realizations, when $\gamma = 0, 0.5$ (histograms). Curves are theoretical predictions from [6].

Eq. (2) permits to extend several results obtained in [9]; in addition, it naturally suggests a new generalized excursion set algorithm for peaks, which allows one to test the various relations derived in [6] against mock data (this analysis is presented in [7]). An example is shown in Figure 3, where 50000 realizations are used to successfully confirm the analytic expressions for the conditional distributions $p(\zeta_3|\gamma)$ and $p(K_1|\zeta_3 > 0, \gamma)$.

4. Conclusions

The theoretical model for predicting halo shapes described in Section 2 and summarized in Figure 2 has a broad spectrum of applications in cosmology, from weak lensing to the study of early-type galaxies (see [5], for more details). A related aspect is the connection of this theoretical formalism with the initial shear field of the cosmic web, and with the theory of peaks in Gaussian random fields [6, 7, 9]. To this end, in Section 3 we have discussed a new formula for the constrained eigenvalues of the initial shear field presented in [6], which accounts for the fact that halos (voids) may correspond to maxima (minima) of the density field; this leads to a new generalized excursion set algorithm for peaks [7], an example of which is provided in Figure 3. Ongoing and future research directions include applications to the skeleton of the cosmic web, extensions to non-Gaussian fields, structure reconstructions on the basis of tessellations, and the analysis of the properties of halos, voids and filaments within this framework. In addition, the outlined formalism is useful for developing algorithms to find and classify structures in the cosmic web, for shape finders, and to study the environmental dependence of galaxy formation and the role of the peculiar gravity field. Current and future weak lensing and galaxy surveys such as the SDSS-III [10] or Euclid [11] will provide useful data to test these models.

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