

Cosmological parameter estimation using Particle Swarm Optimization

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Abstract. Constraining parameters of a theoretical model from observational data is an important exercise in cosmology. There are many theoretically motivated models, which demand greater number of cosmological parameters than the standard model of cosmology uses, and make the problem of parameter estimation challenging. It is a common practice to employ Bayesian formalism for parameter estimation for which, in general, likelihood surface is probed. For the standard cosmological model with six parameters, likelihood surface is quite smooth and does not have local maxima, and sampling based methods like Markov Chain Monte Carlo (MCMC) method are quite successful. However, when there are a large number of parameters or the likelihood surface is not smooth, other methods may be more effective. In this paper, we have demonstrated application of another method inspired from artificial intelligence, called Particle Swarm Optimization (PSO) for estimating cosmological parameters from Cosmic Microwave Background (CMB) data taken from the WMAP satellite.

1. Introduction

Cosmic Microwave Background (CMB) observations have entered into a new era with space based telescopes like Wilkinson Microwave Background Anisotropy Probe (WMAP) and Planck, and it has become possible to rule out many theoretical models on the basis of data of very high quality. In the framework of Bayesian formalism, this is done by asking the question that which of the theoretical models, as is represented by a set of parameters, is best fit to the observational data or how the probability distributions of various parameters look like [1, 3, 2]. Bayesian formalism of parameter estimation also needs the likelihood function $\mathcal{L}(\theta)$ or the probability $P(d|\theta)$ of realizing the data d , given a set of parameters θ , and is directly related to the probability distribution $P(\theta|d)$ (posterior) of parameters, given the data using Bayes theorem [4, 5], we have:

$$P(\theta|d) \propto P(d|\theta)P(\theta). \quad (1)$$

When we are mainly interested in finding the set of parameters for which the likelihood function is maximum, the problem of parameter estimation can be considered an optimization problem, where we have tried to find the vector θ_0 for which the function $\mathcal{L}(\theta)$ has the highest value. The simplest way to find the global maximum of the optimization function would be to set up a grid in the multi-dimensional parameter space and compare the value of the function at each grid point. Grid based search is a prohibitively expensive exercise when dimensionality of the search space is large, i.e., computational cost grows exponentially with the number of search dimensions. There are many classic methods, which use the value of the function and



its derivative at some initial guess point, and then iteratively find the maximum point. These methods are quite effective in lower dimensions but become intractable in higher dimensions. Apart from that, they are good mostly for finding the local maxima. To overcome the limitation of finding only the local maxima, stochastic methods are generally employed, many of which are inspired from artificial intelligence techniques like Artificial Neural Network (ANN), Genetic Algorithms (GA), and Swarm Intelligence (SI). Here, we have demonstrated the application of Particle Swarm Optimization (PSO) for cosmological parameter estimation from WMAP-7 data.

The plan of this paper is as follows: In Section 2, we have given a short introduction of PSO, and the details of which can be found in our earlier paper [6]. In Section 3, we have presented our results, and in Section 4 we have summarized and concluded.

2. Particle Swarm Optimization

Proposed by Kennedy and Eberhart [7], PSO is a population based search procedure inspired from group search, in which the members of a group use their own learning experience and that of other members of the group for exploration, as is observed in bird flocks and fish schools. Although, applications of PSO are quite common in engineering and artificial intelligence problems, it has been applied successfully in astrophysics problems relatively recently [8].

In PSO, a team of particles (computational agents) is launched in the multi-dimensional search space following certain rules for updating positions and velocities. The velocity $V^i(t)$ and position $X^i(t)$ of a particle labelled by the index i at time t is updated in the following way:

$$V^i(t+1) = wV^i(t) + c_1\xi_1(X_{\text{Pbest}}^i - X^i(t)) + c_2\xi_2(X_{\text{Gbest}} - X^i(t)), \quad (2)$$

and

$$X^i(t+1) = X^i(t) + V^i(t+1), \quad (3)$$

where c_1 and c_2 are two acceleration coefficients, w is the inertia weight, and ξ_1 and ξ_2 are uniform random members in the range $[0, 1]$ (c_1, c_2 and w are also called *design parameters*). The n -dimensional vector X_{Pbest}^i (personal best) and X_{Gbest} (global best) are the points in the search space, at which the particle i has found the highest value so far, and the point at which the highest value has been found by any of the particles respectively, i.e., X_{Gbest} is one of the X_{Pbest} . Apart from setting the values of the design parameters in any PSO implementation, we need to decide on schemes for Eq. (1), setting up the initial boundary conditions given in Eq. (2), and topping criteria given in Eq. (3). In order to stop particles from leaving the search region, we also need to fix the maximum velocity of particles. In our exercise, we have set the initial values of the positions and velocities of particles randomly in the allowed range and use reflecting boundary conditions. We have stopped the search when we have found that the improvement in the value of the fitness function remains smaller than a given number for some fixed number of steps. The details of our PSO implementation can be found in [6].

3. Results

We have considered a six parameter cosmological model $(\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda, A_s, n_s, \tau)$, where $\Omega_b, \Omega_c, \Omega_\Lambda$ are the energy densities of baryons, cold dark matter and cosmological constants respectively, h is the Hubble parameter, and A_s, n_s and τ are the amplitude, spectral index of primordial power spectrum and reionization optical depths respectively. The range over which we have carried out the search and the best fit parameters we have obtained are given in Table 1.

The procedure we have followed in PSO has three main steps: (i) We have computed theoretical angular power spectrum C_l for a set of cosmological parameters using publicly available code CAMB [10], (ii) We have computed the likelihood function using theoretical angular power spectrum and WMAP-7 data. We have used the likelihood code provided by

Cosmological parameters from PSO						
Variable		PSO best fit			WMAP best fit[9]	
	Range	Gbest	Mean	std. dev.	ML	Mean
$\Omega_b h^2$	(0.01,0.04)	0.022044	0.022043	0.000490	0.02227	$0.02249^{+0.00056}_{-0.00057}$
$\Omega_c h^2$	(0.01,0.20)	0.112111	0.112129	0.005489	0.1116	0.1120 ± 0.0056
Ω_Λ	(0.50,0.75)	0.722980	0.720416	0.030279	0.729	$0.727^{+0.030}_{-0.029}$
n_s	(0.50,1.50)	0.963800	0.963794	0.012470	0.966	0.967 ± 0.014
$A_s/10^{-9}$	(1.0,4.0)	2.445123	2.446698	0.111354	2.42	2.43 ± 0.11
τ	(0.01,0.11)	0.084124	0.084124	0.013444	0.0865	0.088 ± 0.015

Table 1. The first and the second columns in this table show the PSO fitting parameters and their range respectively, and third, fourth and fifth columns show the best fit (**Gbest**), average values and the errors (which are computed by fitting the sampled function) on fitted parameters respectively. In the last two columns, we have given the maximum likelihood (ML) and the average values of the cosmological parameters as reported by the WMAP-7 team respectively. We can see that the values of parameters which we get using PSO are not very different from the values reported by the WMAP-7 team.

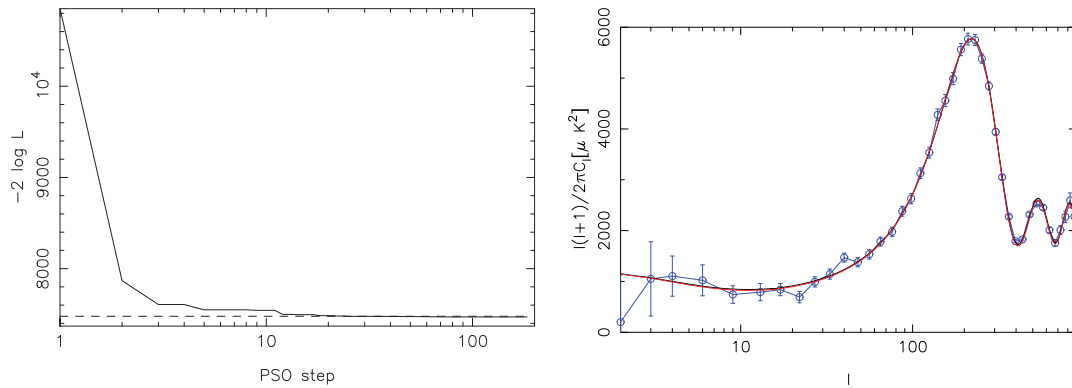


Figure 1. The left and right panel in this figure show the change in the fitness function ($\chi_{\text{eff}}^2 = -2\log \mathcal{L}$) and the best fit angular power spectrum we have obtained from PSO. We can see that in the beginning, the fitness function (solid line in the left panel) changes rapidly but later on when the **Gbest** approaches close to the global maximum, and there is hardly any change. For reference, we have shown the value of $-2\log \mathcal{L}$ for WMAP-7 data using dashed line. The red black and red lines in the right panel show the angular power spectrum for the best fit cosmological parameter as reported by the WMAP-7 team, and for the parameters we have obtained using PSO respectively. We have also shown the binned power spectrum with error bars from the WMAP-7 data. Note that the small difference (see Table 1) between the PSO best fit parameters and the WMAP-7 best fit parameters leads to a difference $\Delta\chi_{\text{eff}}^2 \approx 2$ (it is smaller for PSO)

WMAP-7 team for computing the likelihood function [11], and (iii) We have computed the new set of points in the parameter space following the PSO algorithm (see [6]).

The main results of our work are summarized in Table 1, in the form of the best fit (**Gbest**) values of the cosmological parameters. We have reported the value of the **Gbest** as well as the average location of the PSO particles at the last step, which matches well with the **Gbest** value.

Since PSO is an optimization method, it is designed to find the point at which the fitness (likelihood) function has the maximum value rather than sampling the fitness function, and therefore, sampling done by PSO is not good enough to do statistics as in the case of MCMC. In order to get some idea of the errors on various estimated parameters, we have fitted a multi-dimensional paraboloid to the sampled points close to the **Gbest** and identified the fitting coefficients with Hessian matrix, and got error from the inverse of it, i.e., covariance matrix (for details see [6]).

Left panel in Figure 1 shows the change in the $\chi^2_{\text{eff}} = -2\log \mathcal{L}$ as PSO progresses. It is clear that in the beginning, the improvement in the value of the fitness function is large, and it decreases as **Gbest** reaches close to the global maximum. Once **Gbest** remains close to the global maximum for a given number of steps, we can stop the search. In the right panel of the Figure 1, we have shown the best fit angular power spectrum for the cosmological parameters we have obtained using PSO and for their standard values as reported by the WMAP-7 team. We have also shown the binned power spectrum from the WMAP-7 data with error bars in the right panel.

4. Conclusions

Sampling based methods like MCMC are quite common in parameter estimation from observational data, but their use is mostly limited to situations when dimensionality of the search space is low and the likelihood surface is smooth. Here, we have demonstrated that PSO, which is a population based search procedure may have advantages in problems, in which the dimensionality of the search space is large and/or the fitness function has multiple local maxima. Based on a very simple algorithm and having a few design parameters, PSO is quite easy to programme and parallelize on a multi-processor system.

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