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Higher dimensional cosmological models: An alternative explanation for late time cosmic acceleration

I Pahwa

Department of Physics and Astrophysics, University of Delhi, Delhi 11007, India

E-mail: ipahwa@physics.du.ac.in

Abstract. Einstein's equations have been solved for a homogeneous, anisotropic $1 + 3 + D$ dimensional universe. We have studied the late time acceleration of the universe in this model. The content of the universe is assumed to be such that it exerts zero pressure in visible dimensions and in extra dimensions, the equation of state is given. The parameters of the model are fixed by comparing with observational data on supernovae. We have shown that late time acceleration occurs in the universe with q_0 and z_{tr} being approximately 0.46 and 0.76 respectively.

1. Introduction

Observations of high redshift supernovae (SN) type 1a, large scale structures and cosmic microwave background (CMB) have shown that our universe is in a phase of accelerated expansion. Several theories have been suggested to explain this feature, but a convincing theoretical framework is still lacking. Various models are constructed in extra dimensional context, also including (but not limited to) brane world models [1, 2].

Here, we have considered a simple model, which is used in [3] to solve the horizon problem through extra dimensions in early universe, using an anisotropic fluid residing in $1 + D_1 + D_2$ dimensions, we have adapted the formalism to produce a late time acceleration instead. We have assumed a uniform density matter in the universe, but the pressure exerted by matter in normal dimensions is different from that in extra dimensions.

2. The framework

A spacetime, which has one temporal, three normal spatial dimensions and D extra spatial dimensions, is described by the metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-k_1r^2} + r^2 d\Omega\right) + b^2(t)\left(\frac{dR^2}{1-k_2R^2} + R^2 d\Omega_{D-1}\right). \tag{1}$$

We have assumed spatial curvature of both sub-spaces to be zero, i.e., $k_1 = 0$ and $k_2 = 0$. The parameter D takes integral values, and is to be determined by comparing with observations. Here, $a(t)$ represents the scale factor in the normal 3-dimensions and $b(t)$ denotes the scale factor in the extra dimensions. The content in the universe is assumed to exert pressure, which is different in the normal and extra dimensions.

The energy-momentum tensor is assumed to be of the form: $T^{\mu}_{\nu} = diag\,(-\rho, P_a, P_a, P_a, P_b, \ldots, P_b)$, where ρ is the energy density of the fluid and P_a (P_b) is the pressure exerted in normal (extra) dimensions. For the line element given in Eq. (1) and energy-momentum tensor given above, Einstein's equations can be written as:

$$\frac{\dot a^2}{a^2} + D\frac{\dot a}{a}\frac{\dot b}{b} + \frac{D(D-1)}{6}\frac{\dot b^2}{b^2} = \frac{8\pi G\rho}{3}, \tag{2}$$

$$2\frac{\ddot a}{a} + D\frac{\ddot b}{b} + 2D\frac{\dot a}{a}\frac{\dot b}{b} + \frac{\dot a^2}{a^2} + \frac{D(D-1)}{2}\frac{\dot b^2}{b^2} = -\frac{8\pi G P_a}{3}, \quad and \tag{3}$$

$$(D-1)\frac{\ddot b}{b} + 3\frac{\ddot a}{a} + 3(D-1)\frac{\dot a}{a}\frac{\dot b}{b} + (D-1)(\frac{D}{2}-1)\frac{\dot b^2}{b^2} + 3\frac{\dot a^2}{a^2} = -\frac{8\pi G P_b}{3}. \tag{4}$$

The conservation equation for energy-momentum tensor implies that $T^{\mu}_{\nu;\mu} = 0$, and

$$\frac{d}{dt}(\rho a^3 b^D) + P_a b^D \frac{d}{dt}a^3 + P_b a^3 \frac{d}{dt} b^D = 0. \tag{5}$$

In 3-dimensional standard cosmology, the universe today consists of dark energy, dark matter and baryons. Both dark matter and baryons are very well approximated by pressureless matter. In extra dimensional sub-space, we have assumed a simple form for the equation of state, which is given as $P_b = w_b\rho$ with $w_b = \frac{w}{\bar\rho^{\gamma}}$, where w and γ are parameters of the model, which are determined by comparing with the observational data and $\bar\rho = \frac{\rho}{\rho_c}$, $\rho_c = (3H_0^2)/8\pi G$.

We have now solved Eq. (2) for $\dot a/a$ and bringing it to a form close to the familiar form of the FRW equations, we have:

$$\frac{\dot a^2}{a^2} = \frac{8\,\pi\,G\,\rho}{3} + \frac{D\,(2D+1)}{6}\frac{\dot b^2}{b^2} \mp \frac{D}{2}\frac{\dot b}{b}\sqrt{\frac{D\,(D+2)}{3}\frac{\dot b^2}{b^2} + \frac{32\,\pi\,G\rho}{3}}. \tag{6}$$

This equation would reduce to the standard form in the absence of the last two terms on the right hand side. In other words, the $\dot b/b$ dependent terms act as an effective dark energy source.

3. Cosmological solutions and observational constraints

It will be convenient to express the variables in terms of dimensionless quantities, namely:

$$t \equiv \tau/H_0, \qquad A' \equiv a'/a = \dot a/(a\,H_0), \quad and \quad B' \equiv b'/b = \dot b/(b\,H_0).$$

Here, primes denote derivative with respect to τ. In terms of these variables, and after eliminating $\ddot b$, the equations of motion now read:

$$\begin{aligned} 0 &= (D+2)\,A'' + 3(D+1)\,A'^2 + \frac{D\,(1-D)}{2}\,B'^2 + D\,(D-1)\,A'\,B' + 3\,D\,\Omega_0\,\bar\rho\,w_b, \\ \bar\rho' &= -\bar\rho\,[3\,A' + D\,(1+w_b)\,B'], \quad and \\ B' &= (D-1)^{-1}\left[-3\,A' \pm \sqrt{3\,D^{-1}\left\{(D+2)\,A'^2 + 6\,\frac{(D-1)}{D}\,\bar\rho\Omega_0\right\}}\right]. \end{aligned} \tag{7}$$

We have evolved the equations backward with the 'initial' conditions (at the present epoch $(\tau = 0)$) given by:

$$(\dot a/a)|_{\tau=0}\,H_0 \Rightarrow A'|_{\tau=0} = 1, \quad and \quad \rho/\rho_c|_{\tau=0} = 1 \Rightarrow \bar\rho|_{\tau=0} = 1.$$

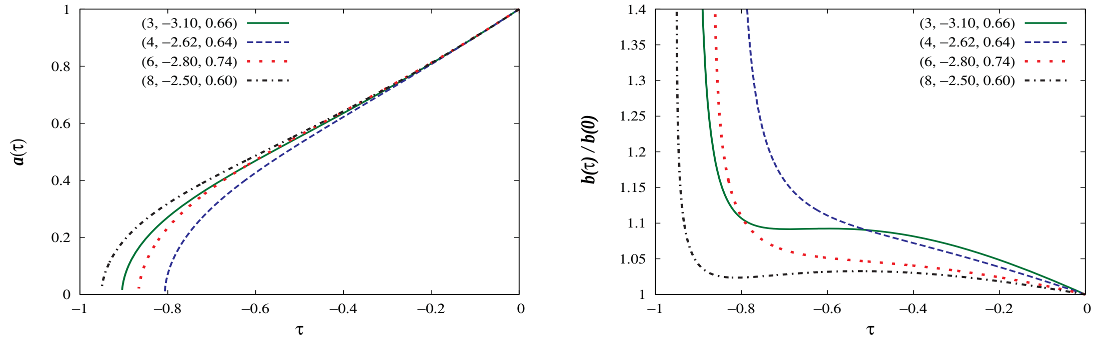

Figure 1. *Left panel*: Behaviour of the scale factors $a(\tau)$, and *Right panel*: $b(\tau)$ with the re-scaled time τ. The numbers in the parentheses refer to (D, w, γ).

Since, Einstein's equations can only determine $A'(\tau)$ and $B'(\tau)$, and not the scale factors themselves, we re-scale the solutions as:

$$a|_{\tau=0} = 1 \quad \Rightarrow \quad A|_{\tau=0} = 0, \quad and \quad b|_{\tau=0} = 1 \quad \Rightarrow \quad B|_{\tau=0} = 0.$$

There exists two solutions, one for each sign in the last of Eq. (7). We have rejected the minus sign branch, as it leads to an accelerated expansion for all times rather than a transition from a decelerated phase to an accelerated one.

Figure 1 represents the solutions for the two scale factors for some particular values of parameters (D, w, γ). (For details of calculation, see [4].) The scale factor $a(\tau)$ for our universe starts from 1 at $\tau = 0$ and decreases monotonically for negative values of τ. And these equations are valid only as long as the universe is matter dominated. As it can be easily seen, the relative evolution in $b(\tau)$ is small.

The epoch of matter-radiation equality $\tau_{eq} \equiv \tau(z = z_{eq})$, has a considerable dependence on the parameter choice. For a given value of D and w, a smaller γ shifts τ_{eq} further into the past, thus, increasing the present-day age of the universe. Similarly, for a given value of D and γ, τ_{eq} shifts further into the past with decrease in w. As we have seen above, the model

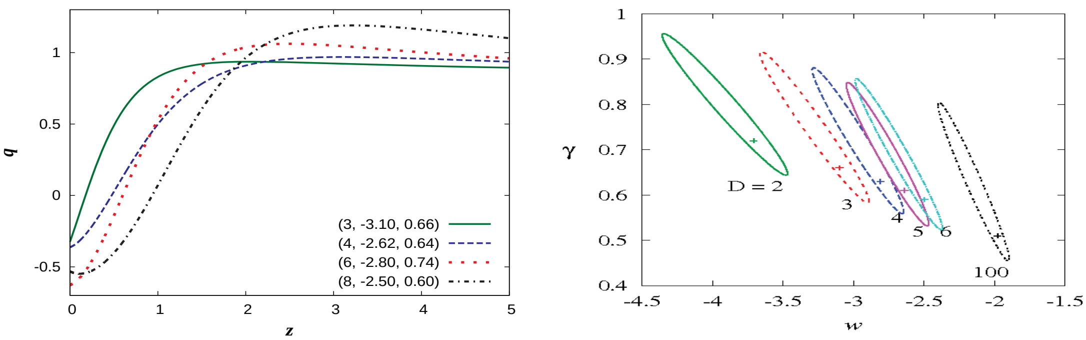

Figure 2. *Left panel*: Evolution of the deceleration parameter q with the redshift z. The numbers in the parentheses refer to (D, w, γ), and *Right panel*: 95% C.L. contours in the $\gamma - w$ plane for different values of D. The points represent the best fit values for each D.

leads to correct late time acceleration for some choice of parameters. We have now looked for it to compare with SN type 1a observational data [5], which lists the distance modulus μ

$\equiv 5 \log d_L + 25$ as well as the redshift for 557 such SN. The luminosity distance $d_L(z)$ is defined through: $d_L(z) = (1+z)\int\limits_0^z \frac{dz'}{H(z')}$.

We have now calculated $d_L(z)$, given our determination of $H(z)$ for a particular choice of parameters. We have defined χ^2-test through $\chi^2(D,\gamma,w) = \sum_{i=0}^{i=n} \frac{[\mu_{obs}(z_i)-\mu_{th}(D,\gamma,w;z_i)]^2}{\sigma_i^2}$, where μ_{th} defines the value expected in our model for a particular choice of parameters, whereas μ_{obs} and σ are the observational value and the associated root-mean-squared error. We have found the best-fit values of the parameters by minimizing the χ^2, which are listed in Table 1. As we have seen, with an increase in D, both $|w|$ and γ decrease. This is based on the fact that the resulting smaller extra-dimensional pressures would now have an enhanced effect in the normal world through the larger effective coupling between $\dot{a}(t)$ and $\dot{b}(t)$.

D	χ^2_{min}	best fit w	best fit γ	q_0	z_{tr}
2	538.92	$-3.71^{+0.25}_{-0.63}$	$0.72^{+0.23}_{-0.08}$	$-0.49^{+0.10}_{-0.26}$	$0.75^{+0.23}_{-0.27}$
3	539.18	$-3.10^{+0.18}_{-0.55}$	$0.66^{+0.25}_{-0.07}$	$-0.47^{+0.09}_{-0.27}$	$0.78^{+0.22}_{-0.32}$
10	539.42	$-2.31^{+0.12}_{-0.38}$	$0.57^{+0.14}_{-0.06}$	$-0.46^{+0.08}_{-0.36}$	$0.79^{+0.19}_{-0.33}$
100	539.74	$-1.98^{+0.07}_{-0.38}$	$0.51^{+0.28}_{-0.05}$	$-0.43^{+0.06}_{-0.30}$	$0.80^{+0.19}_{-0.34}$

Table 1. The error bars correspond to the projections of the 95% C.L. ellipses on the two axes.

Figure 2 (right panel) shows 95% C.L. contours in the $\gamma - w$ plane for a given D. It is easily seen that there is a strong negative correlation between the two parameters. Integral values of γ and positive w are also ruled out. As deduced from Table 1, the position of minima is getting increasingly closer with increase in D arbitrarily, and also shown are the values of q_0 and z_{tr} as determined within our model.

4. Conclusions and discussion

In this work, the effect of the extra dimensions is indirectly seen as the accelerated expansion of the scale factor in normal dimensions. To produce this effect, we have considered matter content as a pressureless gas in the normal dimensions and a monomial equation of state in the extra dimensions. This phenomenon is just produced by a very moderate contraction of the extra dimensions, and in the time interval that $a(\tau)$ has increased by nearly a factor of 3000, $b(\tau)$ has decreased by $\sim 29\%$. The model shows a good agreement with the observational data on SN type 1a for a significantly wide range of parameters as indicated by both a low χ^2 per degree of freedom, but we still need a negative pressure which is limited to the extra dimensions.

Acknowledgements

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