Observational cosmology and the cosmic distance duality relation

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Abstract. We have studied the validity of cosmic distance duality relation between angular diameter and luminosity distances. To test this duality relation, we have used the latest Union2 Supernovae type Ia (SN Ia) data for estimating the luminosity distance. The estimation of angular diameter distance comes from the samples of galaxy clusters (real and mock) and FRIIb radio galaxies. We have parameterized the distance duality relation as a function of redshift in four different ways, and found that the mock data set, which assumes a spherical isothermal β model for the galaxy clusters does not accommodate the distance duality relation, while the real data set which assumes elliptical β model does.

1. Introduction

The reciprocity relation relates the distances between two events (say the source S and the observer O), which are connected by null geodesics. This relation is of fundamental importance in observational cosmology, especially, measurements such as distant type Ia supernovae (SN Ia), cosmic microwave background (CMB), galaxy observations and gravitational lensing [1]. Bassett and Kunz [2] have explored whether this relation can shed some light on the presence of exotic physics, and they have ruled out non-accelerating models of universe (replenishing dust model) by more than 4σ level.

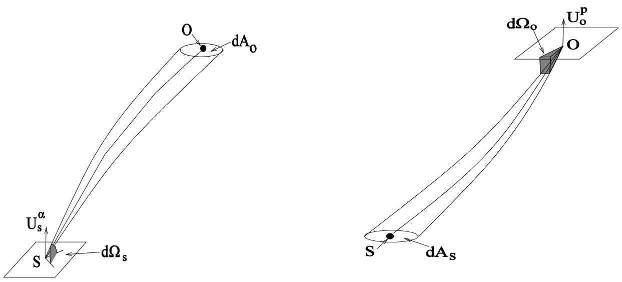

Figure 1. Consider a bundle of null geodesics emanating from the source, subtending a solid angle $d\Omega_s$. This bundle has a cross section dA, and the source angular distance $D(U_s^\alpha)$ is defined as $D(U_s^\alpha) = \left(\frac{dA}{d\Omega_s}\right)^{\frac{1}{2}}$. Similarly, the observer area distance $D(U_s^p)$ is defined as $D(U_s^p) = \left(\frac{dA_s}{d\Omega_o}\right)^{\frac{1}{2}}$.

1.1. *The reciprocity relation*

If the geodesic deviation equation holds and photon travels on null geodesics, it can be shown that (see Figure 1), $D(U_s^\alpha)$ and $D(U_s^p)$ are related as [3, 4]:

$$D(U_s^\alpha) = (1+z)D(U_s^p). \tag{1}$$

This is known as the *reciprocity relation*. Note that this relation holds regardless of the metric or the matter content of the spacetime. Using the reciprocity relation and assuming that the total number of photons are conserved on the cosmic scales (above mentioned observational methods are all based on this fundamental assumption), one can derive a relation between the angular diameter distance and the luminosity distance [4] as:

$$d_L = d_A(1+z)^2. \tag{2}$$

This equation is known as the *distance duality* relation (DD), and it is testable by astronomical observations if d_L and d_A are known for a source. To study the validity of this relation, we have analyzed the following red-shift dependence of DD, given by:

$$\eta(z) \equiv \frac{d_L}{d_A(1+z)^2} \,. \tag{3}$$

Uzan *et al* [5] have used the combined measurements of Sunyaev-Zeldovich effect and X-ray emission data of galaxy clusters and shown that if DD relation does not hold then the angular diameter distance measured from the clusters is $d_A^{cluster}(z) = d_A\,\eta^2$, and hence, the DD relation in Eq. (3) gets modified to:

$$\eta(z) \equiv \frac{d_A^{cluster}(1+z)^2}{d_L} \,. \tag{4}$$

In this paper, our aim is to re-analyze the validity of DD relation in a comprehensive manner by using different data samples and parameterizations [6].

2. Parameterizations, data and method

2.1. η(z) parameterizations

Our parameterizations are inspired by model independent parameterizations for the dark energy equation of state [7]. We are parameterizing $\eta(z)$, whose value stays one when photon number is conserved, gravity is described by a metric theory and photons travel on null geodesics. Any significant violation from the DD relation will hint at the break down of one or more of these assumptions. To model any departure from unity, we have parameterized η with four parametric representations, which are given by:

$$\begin{aligned}
\eta_I(z) &= \eta_0 + \eta_1 z, \\
\eta_{II}(z) &= \eta_2 + \eta_3 \frac{z}{1+z}, \\
\eta_{III}(z) &= \eta_4 + \eta_5 \frac{z}{(1+z)^2}, \quad \textit{and} \\
\eta_{IV}(z) &= \eta_6 - \eta_7 \ln(1+z).
\end{aligned}$$

2.2. Data

For the luminosity distances, we have used latest Union2 SNe Ia data [8], and considered that pair of galaxy cluster/radio galaxy and SN, for which $\Delta z < 0.005$ [9]. Because of this condition, the number of data points are limited to 24, 222 and 12 for the data set I, II and III respectively.

- Data set I: This sample consists of 25 galaxy clusters (isothermal elliptical β model, with concordance model for the cosmological distance-redshift relationship) [10]. The redshift interval for clusters in this sample is $0.02 < z < 0.78$.
- Data set II: This data set contains 578 angular diameter distances of mock clusters. The redshift distribution of this sample is $0.05 < z < 0.76$. (For more details, see [11, 12].)
- Data set III: In this sample, the calculation of angular diameter distance is obtained by using the physical size of extended radio galaxies [13]. This data set contains 20 radio galaxies up to redshift z = 1.8.

2.3. Method

We have performed the χ^2 analysis to fit the parameters of the assumed parameterizations as:

$$\chi^2(p) = \sum_i \frac{(\eta_{th}(z_i, p) - \eta_{obs}(z_i))^2}{\sigma(z_i)^2} , \tag{5}$$

where η_{th} is the assumed form of parameterizations, and η_{obs} is the observed value of η, which is calculated by using d_L and d_A at a particular value of redshift. The unknown parameters are denoted by the variable p.

The η_{obs} for the radio galaxies are obtained by assuming $d_A^{radio} = d_A$ (R A Daly, private communication) in Eq. (3). The angular diameter distance for radio galaxies is given in terms of the dimensionless coordinate distance y, as:

$$d_A(i) = \frac{y(z_i)}{H_0(1 + z_i)} , \tag{6}$$

where $y = a_0 r H_0$, and $a_0 r$ is the coordinate distance [13]. Treating Hubble constant H_0, as a nuisance parameter we have marginalized over it by assuming Gaussian prior. In this analysis, distances are obtained by assuming the flat ΛCDM universe.

3. Results and discussions

We have studied the validity of DD relation using different data sets, assuming four general parameterizations of $\eta(z)$, completely in analogy with the varying equation of state of dark energy $\omega(z)$. Our results are summarized as follows:

- Tables 1, 2 and 3 show the best fit values of the parameters for the four parameterizations for the first, second and third data set respectively.

Table 1. Best fit values for all parameterizations: Data set I

χ^2_ν	Parameters	Parameters
1.217	$\eta_0 = 0.999 \pm 0.144$	$\eta_1 = -0.058 \pm 0.507$
1.216	$\eta_2 = 1.007 \pm 0.170$	$\eta_3 = -0.118 \pm 0.822$
1.216	$\eta_4 = 1.013 \pm 0.205$	$\eta_5 = -0.198 \pm 1.345$
1.216	$\eta_6 = 1.003 \pm 0.157$	$\eta_7 = 0.087 \pm 0.651$

Table 2. Best fit values for all parameterizations: Data set II

χ^2_ν	Parameters	Parameters
1.076	$\eta_0 = 0.973 \pm 0.048$	$\eta_1 = -0.108 \pm 0.159$
1.078	$\eta_2 = 0.971 \pm 0.058$	$\eta_3 = -0.138 \pm 0.267$
1.080	$\eta_4 = 0.972 \pm 0.072$	$\eta_5 = -0.141 \pm 0.453$
1.077	$\eta_6 = 0.972 \pm 0.053$	$\eta_7 = 0.124 \pm 0.208$

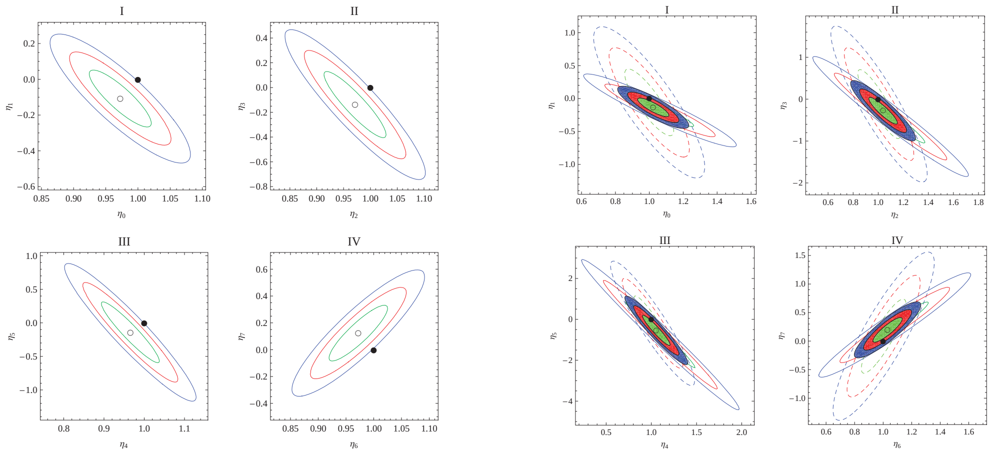

Figure 2. The left panels shows 1σ, 2σ and 3σ contours in $\eta_i - \eta_j$ plane with data set II. In the right panel, dashed, solid and filled contours correspond to data sets I, III and (I + III) respectively. In both the figures, the position of filled circle in the contours indicate the point where $\eta(z = 0) = 1$ and the position of empty circle indicates the best fit value of the parameters.

Table 3. Best fit values for all parameterizations: Data set III

χ^2_ν	Parameters	Parameters
0.944	$\eta_0 = 1.063 \pm 0.198$	$\eta_1 = -0.180 \pm 0.244$
0.971	$\eta_2 = 1.099 \pm 0.274$	$\eta_3 = -0.415 \pm 0.632$
1.021	$\eta_4 = 1.099 \pm 0.385$	$\eta_5 = -0.749 \pm 1.624$
0.958	$\eta_6 = 1.081 \pm 0.235$	$\eta_7 = 0.282 \pm 0.404$

- All the parameterizations have shown degenerate behaviour with the given data sets. The parameterizations are in complete concordance with DD relation within 1σ level for data set I, and within 2σ level for data set III. As expected, the bigger data set of mock galaxy clusters (data set II), which is generated by assuming the spherical isothermal β model for clusters gives tighter constraints on various parameterizations, but most of the parameterizations show significant deviation from DD relation (see figure 2). But the real galaxy cluster data (data set I), which is obtained by assuming isothermal elliptical β model for clusters is in good agreement with DD relation.

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