Dipole leakage and low CMB multipoles

S Das and T Souradeep

IUCAA, Post Bag 4, Pune University Campus, Ganeshkhind, Pune 411007, India

E-mail: `santanud@iucaa.ernet.in, tarun@iucaa.ernet.in`

Abstract. A number of studies of WMAP-7 have highlighted that the power at the low multipoles in CMB power spectrum is lower than their theoretically predicted value. Angular correlation between the orientations of these low multipoles have also been discovered. While these observations may have cosmological ramification, it is important to investigate possible observational artifacts that can mimic them. The CMB dipole, which is much higher than the quadrupole, can get leaked to the higher multipoles due to the non-circular beam of the CMB experiment. In this paper, an analytical method has been developed and simulations are carried out to study the effect of the non-circular beam on power leakage from the dipole. It has been shown that the small, but non-negligible power from the dipole can get transferred to the quadrupole and the higher multipoles due to the non-circular beam. Simulations have also been carried out for Planck scan strategy, and comparative results between WMAP and Planck have been presented.

1. Introduction

The standard model of cosmology emerging from recent observations is a remarkable success of theoretical physics. It can explain the cosmological observations up to an extremely high precision using a handful set of parameters. However, there are some effects that seem to be anomalous in the standard cosmological model. One of such observational facts is the power anomaly at the low multipoles of the CMB power spectrum. It has been seen that the power at the low multipoles is lower than their theoretical predictions, and there are possibly correlation between the orientations of the low multipoles. Many researchers have tried to explain the phenomenon [1, 2], but it is not satisfactorily explained. In this paper, we have analyzed the effect of the non-circular beam as origin of CMB anomalies at low multipoles.

In most of the CMB experiments such as WMAP, the beam shape of the detectors is non-circular about the pointing direction. However, in the data analysis techniques, the beam is assumed to be circularly symmetric. The CMB dipole is much stronger than the quadrupole. Therefore, due to the non-circularity of the beam, some power from dipole may leak to the quadrupole and immediate higher multipoles. However, assuming a circular beam in the data analysis technique, this leakage of the power from dipole will not be accounted for. Therefore, the contribution of this effect will contaminate the resultant map, generated by this inadequate data analysis technique. Different effects of non-circular beam have been discussed by different authors in [3, 4]. However, this particular effect of leakage from the dominant dipole has not been analyzed yet.

In this paper, analytical methods have been developed to calculate the amount of power leakage from dipole, and simulations with the actual scan pattern of WMAP have been carried out showing the order of power leakage for different WMAP beams. Our results show that

the amount may not be sufficient to explain the anomalies, but the power transfer does have a measurable effect on the quadrupole. In anticipation, a similar simulation has also been carried out with Planck scan pattern. As the data for Planck beams are not available publicly, we have provided only upper limits to the beam non-circularity parameters beyond which the dipole leakage would cause detectable effect in the power spectrum.

2. Analytical description of the beam convolution

This section describes the formalism employed for measuring the power leakage from dipole to the quadrupole and higher multipoles during scanning the sky with a non-circular beam. The measured sky temperature in a CMB experiment is a convolution of the true sky temperature with the beam function. If the measured temperature along γ_i is expressed by $\tilde{T}(\gamma_i)$, whereas the sky temperature along γ is $T(\gamma)$ then:

$$\tilde{T}(\gamma_i) = \int B(\gamma_i, \gamma)T(\gamma)d\Omega_\gamma + T^N(\gamma_i) \ . \tag{1}$$

Here, T^N is the noise in the scan procedure. Since we have dealt with low multipoles, the noise is ignored in our analysis. The beam function $B(\gamma_i, \gamma)$ represents the sensitivity of the telescope around the pointing direction γ_i. This is a two point function and can be expanded in terms of spherical harmonics as:

$$B(\gamma_i, \gamma) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} b_{lm}(\gamma_i)Y_l^m(\gamma) \ . \tag{2}$$

Since we intend to measure the power leakage from dipole to the quadrupole, and it is convenient to consider only the sky dipole map and check the amount of power leakage from the dipole to higher multipole due to the non-circular beam. A dipole map $T(\gamma)$ can be written as a sum of all the spherical harmonics with $l = 1$, i.e., $T(\gamma) = \sum_{m=-1}^{1} a_{1m}Y_1^m(\gamma)$. However, its always possible to choose a coordinate system such that $a_{1,1}$ and $a_{1,-1}$ modes vanish and the sky temperature can be expressed only as $T(\gamma) = T_0 Y_1^0(\gamma)$, where $T_0 = a_{1,0}$ is a constant. In such a case, the measured sky temperature along the γ_i direction can be expressed as:

$$
\begin{aligned}
\tilde{T}(\gamma_i) &= \int B(\gamma_i, \gamma)T(\gamma)d\Omega_\gamma = \int \left[\sum_{l=0}^{\infty} \sum_{m=-l}^{l} b_{lm}(\gamma_i)Y_l^m(\gamma) \right] T(\gamma)d\Omega_\gamma, \\
&= T_0 \sum_{l=0}^{\infty} \sum_{m=-l}^{l} b_{lm}(\gamma_i) \int Y_l^m(\gamma)Y_1^0(\gamma)d\Omega_\gamma = T_0 b_{10}(\gamma_i) \ .
\end{aligned}
\tag{3}
$$

The above expression can not be directly used for measuring the sky temperature, because it contains the beam harmonic coefficient $b_{10}(\gamma_i)$, which is a function of γ_i. It is convenient to orient the beam along some fixed direction of the sky, say, along the $\hat{z}$ direction, and consider the multipole $b_{lm}(\hat{z})$ to characterise the beam [5]. The spherical harmonic coefficients of the beam $b_{lm}(\gamma_i)$ at any particular direction γ_i can be obtained by using Wigner-D functions as:

$$
\begin{aligned}
b_{10}(\gamma_i) &= \sum_{m'=-l}^{l} b_{1m'}(z)D_{0m'}^1, \\
&= b_{1,-1}(z)D_{0,-1}^1(\varphi_i, \theta_i, \rho_i) + b_{1,0}(z)D_{0,0}^1(\varphi_i, \theta_i, \rho_i) + b_{1,1}(z)D_{0,1}^1(\varphi_i, \theta_i, \rho_i), \\
&= b_{1,-1}(z)d_{0,-1}^1(\theta_i)e^{i\rho_i} + b_{1,0}(z)d_{0,0}^1(\theta_i) + b_{1,1}(z)d_{0,1}^1(\theta_i)e^{-i\rho_i} \ .
\end{aligned}
\tag{4}
$$

Substituting explicit expressions in terms of trigonometric functions for $d^1_{0,0}(\theta_i)$, $d^1_{0,-1}(\theta_i)$ and $d^1_{0,1}(\theta_i)$, in the above, and using Eq. (3), the expression for the scanned temperature can be written as:

$$\tilde{T}(\gamma_i) = T_0 b_{10}(z)\cos(\theta_i) + \sqrt{2}T_0\sin(\theta_i)\Big[b_r(z)\cos(\rho_i) + b_i(z)\sin(\rho_i)\Big] . \tag{5}$$

Here, ρ_i is the orientation of the semi-major axis of the beam at the ith scan point. The functions $b_i(z)$ and $b_r(z)$ can be defined as follows: In Eq. (4), $b_{1,1}$ and $b_{1,-1}$ are complex quantities. Since the beam is real, the beam spherical harmonic coefficients should satisfy the relation $b^*_{1,1} = -b_{1,-1}$. Here, b_r and b_i have been defined as $b_{1,1} = b_r + ib_i$, i.e., the real and the imaginary part of $b_{1,1}$.

From Eq. (5), we can calculate the power that gets leaked to the quadrupole or the higher multipoles. It can be seen that the term with $b_{10}(z)$ will not contribute to any power leakage from dipole. Therefore, the dipole to quadrupole power transfer is caused only by the terms multiplied with $b_r(z)$ or $b_i(z)$. Hence, if the experimental beam is designed in such a way that the $b_r(z)$ or $b_i(z)$ components of the beam are completely negligible, then it is possible to completely eliminate the dipole to higher multipole power transfers.

3. Dipole leakage from WMAP scan pattern

The WMAP satellite follows a unique scan pattern, in which pixels near the two poles are scanned for large number of times, whereas, those near the equator are scanned for least number of times. The satellite scans the sky temperature in five different frequency bands, named as K, Ka, Q, V and W in a differential measurements of a pair of horns. Amongst them, Q and V bands have two detectors each, and W band has four detectors. Each of these detectors has a pair of horns, both of which are about 70.5^o off the symmetry axis. It has a fast spin about the symmetry axis with the spin period of around 2.2 minutes. Along with this fast spin, the spacecraft has a slow precession, 22.5^o about the Sun-WMAP line. This precession period is about 1 hour. The earth-sun vector rotates 360° in a year.

The simulation has been carried with a dipole map, similar to the known CMB dipole. We have assumed that the shape of the two beams for a pair of horns of a detector are almost same, i.e., the parameters b_r, b_i and b_{10} are same for both the beams of a detector and the angle between them is 140 degree.

All the simulations have been carried out on Healpix map with $N_{side} = 256$ resolution. The simulation gives us three independent maps from the three independent components of Eq. (5). The figures from the three maps are shown in the Figure 1. All these maps are shown in ecliptic coordinate system. These maps can be multiplied with b_{10}, b_r and b_i, and then summed up to get the final scanned map from a detector and hence, the amount of power transfer can be calculated. The values of the beam spherical harmonic coefficients, i.e., b_r, b_i and b_{10} along with the amount of power leakage are listed in the Table 1.

The analysis shows that the temperature leaked into the quadrupole from the dipole is less than $2\mu K$ compared to the CMB quadrupole measured by WMAP of $\sim 18\mu K$. Although, the dipole power leakage is small, the amount of power leakage is sufficient to suggest a deeper analysis of the WMAP data for this effect.

4. Dipole leakage from Planck scan pattern

The Planck satellite has only one beam for each detector and thus, instead of the differential measurement, it measures the actual temperature of the sky. The Planck satellite beam is approximately 85° off symmetry axis and the precession angle for the satellite is around 7.5^o. The precession rate is taken as one revolution per six months and the spin rate as $180^o/min$.

Table 1. The dipole coefficients b_r and b_i (real and imaginary parts of $b_{1,1}$) of the beam spherical harmonics for different WMAP beams estimated from the publicly available beam maps [6]. The quadrupole and octapole temperature are calculated from the simulation considering the dipole temperature as 3.358 mK.

	b_r	b_i	T_d/T_q	$T_q(\mu K)$	T_d/T_{oc}	$T_{oc}(\mu K)$
K_1	-1.45×10^{-4}	2.37×10^{-5}	1837.9	1.82	11250.6	0.30
Ka_1	-9.42×10^{-5}	-6.03×10^{-5}	2568.3	1.31	16493.7	0.20
Q_1	8.66×10^{-5}	-1.62×10^{-4}	2748.1	1.22	14226.4	0.24
V_2	7.40×10^{-5}	8.11×10^{-5}	3256.4	1.03	19674.1	0.17
W_4	6.55×10^{-6}	2.23×10^{-5}	33048.57	0.10	133621.8	0.025

An analysis similar to that of WMAP has been carried out for Planck satellite also. Three independent maps are calculated from the Planck scan strategy, which are shown in Figure 2. As the data of the Planck beam map is not publicly available, and therefore, the amount of power leakage expected from the dipole can not be computed for the Planck scan pattern. But, the analysis shows that if the b_i and b_r are of the order of 5×10^{-4} or less, then it will cause a temperature leakage less than $1 \mu K$. Therefore, that amount of leakage may be safely ignored for beams satisfying the above limits.
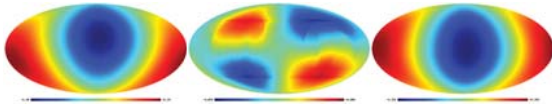

Figure 1. Simulated maps of $\cos \theta$, $\sin \theta \cos \rho$ and $\sin \theta \sin \rho$ components from WMAP scan pattern in the ecliptic coordinate system.
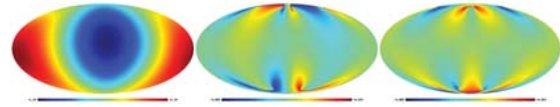
Figure 2. Simulated maps of $\cos \theta$, $\sin \theta \cos \rho$ and $\sin \theta \sin \rho$ components from Planck scan pattern in ecliptic coordinate system.

5. Conclusion

An analytical formalism has been developed to use in simulations of scan strategy to estimate the leakage of power in the dipole anisotropy to the quadrupole and higher multipoles. It has also been shown that the power leakage only depends on the two spherical harmonic coefficients of the satellite beam (b_i and b_r), and therefore, if the beam has been designed in such a way that these two parameters of the beam are small enough, then power leakage will be negligible. For WMAP, the amount of the power leakage is found to be small but not insignificant compared to the low value of quadrupole measured. The simulations also show that the amount of power leakage depends on the scan pattern. For identical beam shapes, the amount of power leakage is more in WMAP scan strategy compared to Planck scan strategy.

References

[1] Moss A, Scott D and Sigurdson K 2011 *JCAP* **1** 001
[2] Feng B and Zhang X 2003 *Phys. Lett. B* **570** 145
[3] Fosalba P, Dore O and Bouchet F R 2002 *Phys. Rev. D* **65** 063003
[4] Mitra S, Sengupta A S and Souradeep T 2004 *Phys. Rev. D* **70** 103002
[5] Souradeep T and Ratra B 2001 *Ap. J.* **560** 28
[6] The beam maps can be found on: *www.lambda.gsfc.nasa.gov/product/map/dr3/beam_profiles_get.cfm*