

# High-velocity collision of particles around a rapidly rotating black hole

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**Abstract.** We have derived a general formula for the centre-of-mass (CM) energy for the near-horizon collision of two general geodesic particles around a Kerr black hole. We have found that if the angular momentum of the particle satisfies the critical condition, the CM energy can be arbitrarily high. We have then applied the formula to the collision of a particle orbiting an innermost stable circular orbit (ISCO) and another generic particle near the horizon, and found that the CM energy is arbitrarily high if we take the maximal limit of the black hole spin. In view of the astrophysical significance of the ISCO, this implies that particles can collide around a rapidly rotating black hole with a very high CM energy without any artificial fine-tuning. We have next applied the formula to the collision of general inclined geodesic particles and shown that in the direct collision scenario, the collision with an arbitrarily high CM energy can occur near the horizon of maximally rotating black holes, not only at the equator but also on a belt centred at the equator between two latitudes. This is also true in the scenario through the collision of a last stable orbit particle. This strongly suggests that if signals due to high-energy collision are to be observed, such signals will be generated primarily on this belt.

## 1. Introduction

Many black hole candidates have been observed in X-ray binaries and galactic centres. In the former case, electromagnetic radiation emitted from accretion disks around black holes of mass  $\sim 10 M_{\odot}$  is observed. In the latter case, the central objects are believed to be supermassive black holes of mass  $\sim 10^6 - 10^9 M_{\odot}$ . However, no direct signature of black hole horizons has been observed yet. In the near future, the observation with the resolution of the black hole horizon scale might catch the direct image of the black hole horizon, which is called a *black hole shadow*, in terms of the millimetre electromagnetic waves. Moreover, when gravitational radiation from black holes is observed, it will directly reveal the spacetime geometry in the vicinity of black hole horizons.

Astrophysical black holes are generally rotating and uniquely described by a Kerr metric, in which the line element is given by:

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mar \sin^2 \theta}{\rho^2} d\phi dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2 \sin^2 \theta}{\rho^2}\right) \sin^2 \theta d\phi^2, \quad (1)$$

where  $\rho^2 = r^2 + a^2 \cos^2 \theta$  and  $\Delta = r^2 - 2Mr + a^2$ . If  $0 \leq |a| \leq M$ ,  $\Delta$  vanishes at  $r = r_{\pm} = M \pm \sqrt{M^2 - a^2}$ . The horizon radius is given by  $r_H = r_+$ . The angular velocity



of the horizon is given by:

$$\Omega_H = \frac{a}{r_H^2 + a^2} = \frac{a}{2M(M + \sqrt{M^2 - a^2})}. \quad (2)$$

We define the non-dimensional Kerr parameter  $a_* = a/M$ . We can assume  $a \geq 0$  without loss of generality.

Bañados *et al* [1] have proposed a scenario, where rapidly rotating black holes may act as particle accelerators. More precisely, they have considered the following scenario: Let us drop two particles of the same rest mass at rest at infinity on the equatorial plane of a rotating black hole and assume that the two particles collide near the horizon. If the maximum spin limit  $a_* \rightarrow 1$  is taken and the angular momentum of either particle is fine-tuned, then the centre-of-mass (CM) energy, which is the invariant measure of the collision energy, can be arbitrarily high. This article is based on our recent papers [2, 3].

## 2. The collision of general geodesic particles

The CM energy of particles 1 and 2 at the same spacetime point is defined by:

$$E_{\text{cm}}^2 = -(p_1 + p_2)^a (p_1 + p_2)_a = m_1^2 + m_2^2 - 2g_{ab}p_1^a p_2^b, \quad (3)$$

where  $p_i^a$  and  $m_i$  are the four-momentum and the rest mass of particle  $i$  ( $= 1, 2$ ) respectively. This is a scalar quantity, therefore coordinate-independent and in principle observable. This is the energy of the two colliding particles observed by a local observer, which is at rest in the CM frame of the two particles.

The general geodesic equations are integrable with four conserved quantities [4]. The conserved quantities are the rest mass  $m$ , the energy  $E$ , the angular momentum  $L$ , and the Carter constant  $\mathcal{Q}$ . The geodesic orbits are given by the following set of first-order ordinary differential equations:

$$\rho^2 \dot{t} = -a(aE \sin^2 \theta - L) + (r^2 + a^2)P/\Delta, \quad (4)$$

$$\rho^2 \dot{\phi} = -(aE - L/\sin^2 \theta) + aP/\Delta, \quad (5)$$

$$\rho^2 \dot{r} = \sigma_r \sqrt{R}, \quad \text{and} \quad (6)$$

$$\rho^2 \dot{\theta} = \sigma_\theta \sqrt{\Theta}, \quad (7)$$

where dot  $(\dot{\phantom{x}}) = d/d\lambda$ ,  $\sigma_r = \pm 1$ ,  $\sigma_\theta = \pm 1$ ,

$$P = P(r) = (r^2 + a^2)E - aL, \quad (8)$$

$$R = R(r) = P^2 - \Delta[m^2 r^2 + (L - aE)^2 + \mathcal{Q}], \quad \text{and} \quad (9)$$

$$\Theta = \Theta(\theta) = \mathcal{Q} - \cos^2 \theta [a^2(m^2 - E^2) + L^2/\sin^2 \theta]. \quad (10)$$

The condition  $\dot{t} > 0$  must be satisfied for physical particle orbits, which is called the ‘forward-in-time’ condition. In the near-horizon limit, this reduces to:

$$L \leq \Omega_H^{-1} E \equiv L_c. \quad (11)$$

We have considered the collision of two particles outside the event horizon. Then, we have shown that the CM energy is bounded except in the limit to the horizon. For the near-horizon collision, the CM energy is given by:

$$E_{\text{cm}}^2 = m_1^2 + m_2^2 + \frac{1}{r_H^2 + a^2 \cos^2 \theta} \left[ (m_1^2 r_H^2 + \mathcal{K}_1) \frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1} + (m_2^2 r_H^2 + \mathcal{K}_2) \frac{E_1 - \Omega_H L_1}{E_2 - \Omega_H L_2} \right. \\ \left. - \frac{2(L_1 - a \sin^2 \theta E_1)(L_2 - a \sin^2 \theta E_2)}{\sin^2 \theta} - 2\sigma_{1\theta} \sqrt{\Theta_1} \sigma_{2\theta} \sqrt{\Theta_2} \right], \quad (12)$$

where  $\mathcal{K}_i \equiv \mathcal{Q}_i + (L_i - aE_i)^2$ . Therefore, if the angular momentum of either of the particles is fine-tuned so that  $L_i \rightarrow \Omega_H^{-1} E_i$ , the CM energy  $E_{\text{cm}}$  can be arbitrarily high. We have called particles with  $L = \Omega_H^{-1} E$  as *critical particles*.

We have defined the effective potential as follows:

$$\frac{1}{2}\dot{r}^2 + \frac{r^4}{\rho^4}V(r) = 0, \quad (13)$$

where

$$V(r) = -\frac{R(r)}{2r^4}. \quad (14)$$

It is useful to classify critical particles in terms of the behaviour of the effective potential at the horizon. The classification is summarized in Table 1.

**Table 1.** The classification of critical particles in terms of  $V(r_H)$ ,  $V'(r_H)$  and  $V''(r_H)$ .

Class	$V(r)$ at $r = r_H$	BH spin	Scenario
I	$V = V' = 0, V'' < 0$	$ a  = M$	Direct collision
II	$V = V' = V'' = 0$	$ a  = M$	LSO (ISCO) collision
III	$V = V' = 0, V'' > 0$	$ a  = M$	Multiple scattering
IV	$V = 0, V' > 0$	$0 <  a  < M$	Multiple scattering

### 3. The collision of an ISCO particle

The Kerr black hole has an innermost stable circular orbit (ISCO) and its generalization to non-equatorial orbits, which is called a last stable orbit (LSO). The ISCO is important in astrophysical contexts. The inner edge of the standard accretion disk is usually given by the ISCO. In the extreme-mass-ratio-inspirals (EMRIs), the transition of the adiabatic inspiral phase to the plunge phase occurs at the ISCO. We have seen that the fine-tuning is naturally realised by an ISCO particle as:  $a_* \rightarrow 1$ ,  $r_{\text{ISCO}} \rightarrow r_H$  and  $L \rightarrow \Omega_H^{-1} E$ .

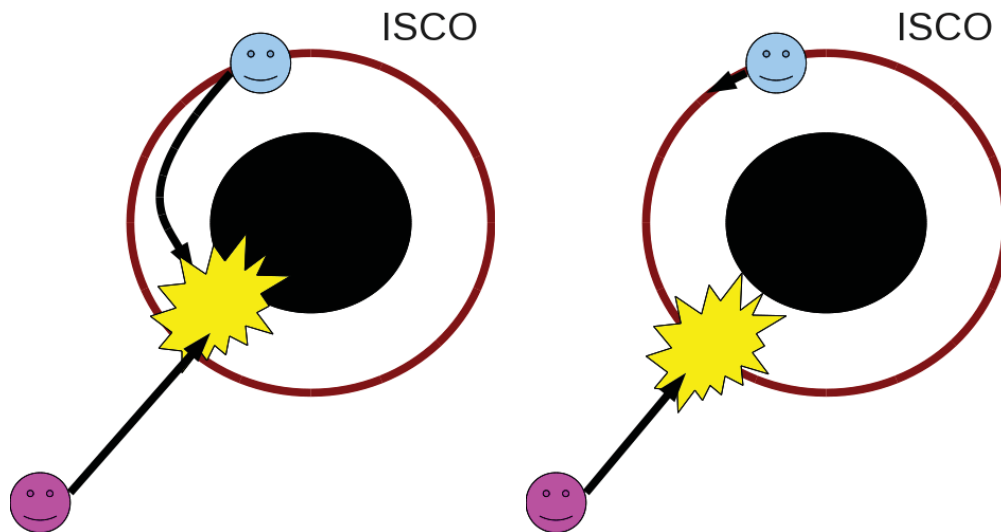
We have considered the collision of an ISCO particle with another generic particle, and two distinct kinds of collision of an ISCO particle. The one is the near-horizon collision, where the ISCO particle plunges to the vicinity of the horizon and collide with a counterpart particle there. The other is the on-ISCO collision, where the ISCO particle collide with a counterpart at the ISCO radius. The schematic figures for these two kinds of collision are shown in Figure 1. For the near-horizon collision of an ISCO particle with a generic counterpart, we obtain:

$$\frac{E_{\text{cm}}}{2m_0} \approx \frac{1}{2^{1/2}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[4]{1 - a_*^2}}, \quad (15)$$

for  $a_* \approx 1$ . For the collision of an ISCO particle with a generic counterpart at the ISCO radius, we obtain:

$$\frac{E_{\text{cm}}}{2m_0} \approx \frac{1}{2^{1/6}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[6]{1 - a_*^2}}, \quad (16)$$

for  $a_* \approx 1$ .



**Figure 1.** *Left panel:* The near-horizon collision of an ISCO particle plunging from the ISCO. *Right panel:* The collision of an ISCO particle at the radius of the ISCO

#### 4. High-velocity collision apart from the equatorial plane

It is intriguing whether the high-velocity collision is restricted to that on the equatorial plane or not. Here, we have concentrated ourselves on the direct collision scenario and the ISCO collision scenario, for which the high-velocity collision with the critical particles is of classes I and II respectively, because the physical relevance of the other cases is not clear. This implies that we can assume  $V = V' = 0$  and  $V'' \leq 0$  at  $r = r_H$ . Consequently, the black hole must be maximally rotating and  $\theta$  is restricted. The allowed region is the belt centred at the equator between latitudes  $\pm 42.94^\circ$ .

#### 5. Summary

The CM energy of two colliding geodesic particles near the horizon can be arbitrarily high in the maximal rotation limit with the fine-tuning of the angular momentum. The required fine-tuning of the angular momentum is naturally realised by a particle orbiting the ISCO or LSO. The high-velocity collision can occur, not only at the equator but also at the latitude between  $\pm 42.94^\circ$  near the horizon in the maximal rotation limit of the black hole.

As future prospects, we would like to raise the investigation of the effects of gravitational radiation reaction and self-force, and the theoretical predictions to astrophysical observation.

#### References

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