

Connections between gravitational dynamics and thermodynamics

R-G Cai

State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China

E-mail: cairg@itp.ac.cn

Abstract. We have briefly summarized three aspects of connections between gravitational dynamics and thermodynamics by focusing on three kinds of spacetime horizon: Causal Rindler horizon of spacetime, black hole event horizon, and apparent horizon in a Friedmann-Robertson-Walker (FRW) universe. For the causal Rindler horizon, we have derived the Einstein's field equations from the Clausius relation. For black hole horizon in a static spherically symmetric spacetime, we have shown that at the black hole horizon, the Einstein's field equations can be cast to a form of the first law of thermodynamics. Applying to the Clausius relation to apparent horizon of a FRW universe, one is able to derive the Friedmann equations, not only in Einstein gravity, but also in Lovelock gravity. We have shown that there exists Hawking radiation associated with the apparent horizon in the FRW universe.

1. Introduction

According to Einstein, general relativity describes dynamics of spacetime by use of spacetime metric, while thermodynamics is another subject which describes macroscopic properties of thermal system in terms of energy, pressure, entropy, and temperature, etc. of the system. The first evidence for the deep connections between gravitational dynamics and thermodynamics comes from black hole thermodynamics. Black hole as a fantastic object is predicted by general relativity. Black hole thermodynamics tells us that black hole has a temperature proportional to its surface gravity, and an entropy proportional to its horizon area, and the temperature, entropy, and black hole mass satisfies the first law of thermodynamics. Not only the first law, other three laws of thermodynamics are also obeyed for black hole. In this sense, black hole is nothing but an ordinary thermodynamic system. However, black hole is a special thermal system because its entropy is proportional to its horizon area, while entropy of ordinary thermal system is proportional to its volume. Another feature of black hole thermodynamics is that heat capacity of some black holes may be negative, for example, Schwarzschild black hole.

The geometric feature of black hole temperature and entropy leads one to conjecture that gravity might be an emergent phenomenon, and is a coarse graining description of some underlying microscopic degrees of freedom. In fact, this idea was first proposed by Sakharov in 1967 [1], before black hole thermodynamics was set up. According to Sakharov, spacetime background emerges as a mean field approximation of underlying microscopic degrees of freedom, similar to hydrodynamics or continuum elasticity theory from molecular physics. In recent reviews [2, 3], Padmanabhan has summarized some aspects to understand gravity from thermodynamical viewpoint.



In this paper, we have studied some connections between gravitational dynamics and thermodynamics, in particular, focus on the relation between Einstein's field equations and the first law of thermodynamics. We have investigated the relation by using the three kinds of spacetime horizons: Rindler horizon, black hole horizon, and apparent horizon in a Friedmann-Robertson-Walker (FRW) universe.

The organization of this paper is as follows: In the next Section, we have briefly repeated the derivation process of Einstein's field equations from the first law of thermodynamics made by Jacobson by applying the latter to Rindler horizon of spacetime [4]. In Section 3, we have focused on black hole horizon and shown that the Einstein's field equations can be cast to a form of the first law of thermodynamics, not only in general relation, but also in Horava-Lifshitz gravity [5, 6]. In Section 4, we have focused on apparent horizon of FRW universe, and investigated the relation between Friedmann equations and the first law [7, 8, 9], and shown that there exists Hawking radiation associated with apparent horizon [10].

2. Rindler horizon: From the first law to Einstein's field equations

In the Rindler chart, the Minkowski space can be written as:

$$ds^2 = -x^2 dt^2 + dx^2 + dy^2 + dz^2, \quad (1)$$

where $0 < x < \infty$, $-\infty < t, y, z < \infty$. There is a coordinate singularity at $x = 0$, where the acceleration of the Rindler observers diverges. The locus $x = 0$ in the Rindler chart corresponds to the locus $X^2 - T^2 = 0$ with $X > 0$ in the Cartesian chart of Minkowski space, the latter consists of two null half-planes. The locus $x = 0$ is just the Rindler horizon, and the Rindler observer cannot see any information outside the Rindler wedge.

According to Davies [11] and Unruh [12], an observer, who is with acceleration a in Minkowski space, will detect a temperature given by $T = \frac{a}{2\pi}$, where the so-called geometric units have been taken: $c = \hbar = k_B = 1$. In the Rindler coordinates given in Eq. (1), for an observer with a fixed $x = x_0$, the acceleration $a = 1/x_0$.

Now, consider a certain event P in any spacetime, by equivalent principle, one can introduce a local inertial frame around P with Riemann normal coordinates. One further transforms the local inertial frame to a local Rindler frame by accelerating along an axis with acceleration κ . Then, there is the Unruh temperature associated with the local Rindler horizon as:

$$T = \kappa/2\pi. \quad (2)$$

Suppose the matter in the spacetime is described by the stress-energy tensor T_{ab} . Then the heat flux across the Rindler horizon $\mathcal{H}$ is given by:

$$\delta Q = \int_{\mathcal{H}} T_{ab} \chi^a d\Sigma^b, \quad (3)$$

where χ^a is an approximation boost Killing vector on $\mathcal{H}$. Note that the relations: $\chi^a = -\kappa \lambda k^a$ and $d\Sigma^a = k^a d\lambda dA$, where k^a is the tangent vector to the horizon generators for the fine parameter λ , which vanishes at P , and dA is the area element on a cross section of the horizon. Then, we have:

$$\delta Q = -\kappa \int_{\mathcal{H}} \lambda T_{ab} k^a k^b d\lambda dA. \quad (4)$$

Now, assume that the entropy is proportional to the horizon area, so that the entropy variation associated with a piece of horizon is given by:

$$dS = \eta \delta A, \quad (5)$$

where η is a constant and $\delta\mathcal{A} = \int_{\mathcal{H}} \theta d\lambda dA$. Using the Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b, \quad (6)$$

and assuming the vanishing of expansion θ and shear σ at P , to the leading order of λ , one has $\theta = -\lambda R_{ab}k^a k^b$, which leads to:

$$\delta\mathcal{A} = - \int_{\mathcal{H}} \lambda R_{ab}k^a k^b d\lambda dA. \quad (7)$$

By the Clausius relation $\delta Q = TdS$, one can have:

$$R_{ab} + f g_{ab} = \frac{2\pi}{\eta} T_{ab}, \quad (8)$$

where f is an arbitrary function. To determine the function f , we have employed the conservation law of the stress-energy tensor: $T^{ab}_{;b} = 0$. This leads to $f = -R/2 + \Lambda$, where Λ is a constant, which is nothing but the cosmological constant, as one will see shortly. Put f back to the above equation, one arrives at:

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = \frac{2\pi}{\eta} T_{ab}. \quad (9)$$

One can see immediately from Eq. (9) that it is nothing but the Einstein's field equations, once $G = 1/4\eta$ is identified.

Thus, we have simply repeated the process to derive the Einstein's field equations from the Clausius relation made by Jacobson [4]. The key idea is to demand that this relation holds for all the local Rindler causal horizons through each spacetime point, with δQ and T interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this sense, the Einstein's field equations are nothing but equations of state of spacetime. If this viewpoint is correct, it has significant implication for quantum gravity theory: It may be no more appropriate to canonically quantize the Einstein's field equations than it would be to quantize the wave equation for sound in air.

Further remarks: (i) For $f(R)$ gravity [13] and scalar-tensor gravity [9], it turns out that a non-equilibrium thermodynamic set up has to be employed in order to produce corresponding gravitational field equations. Namely, one needs an entropy production term added to the Clausius relation. (ii) It is further shown that assuming the non-vanishing of the shear for Einstein gravity, the non-equilibrium set up is needed, $dS = \delta Q/T + d_i S$, where the entropy production term is proportional to the squared shear of the horizon [14]. And it leads to a universal ratio of the shear viscosity η to entropy density of the horizon: $\eta/s = 1/4\pi$. (iii) For any diffeomorphism invariant theory, however, it has been shown recently that given Wald's entropy formula, by the Clausius relation, it is possible to derive the gravitational field equations [15, 16]. However, an issue exists that to have the Wald's entropy formula, one has to know first the gravity theory. In this sense, the derivation is not natural and satisfied [17].

3. Black hole horizon: Equivalence between Einstein's field equations and the first law of thermodynamics

We now move to black hole horizon and discuss the relation between the Einstein's field equations and the first law of thermodynamics [5, 6].

Let us first consider the Einstein's field equations:

$$G_{ab} \equiv R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi G T_{ab}, \quad (10)$$

where G_{ab} is the Einstein tensor. Consider a generic static, spherically symmetric spacetime:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + b^2(r)(d\theta^2 + \sin^2\theta d\phi^2), \quad (11)$$

where $f(r)$ and $b(r)$ are two continuous functions of r . Suppose the metric in Eq. (11) describes a black hole with a non-degenerated horizon at r_+ . Then, the horizon has a Hawking temperature given by:

$$T = \frac{1}{4\pi}f'(r_+), \quad (12)$$

where a prime denotes the derivative with respect to r . The Einstein's field equations for the metric in Eq. (11) turn to be:

$$G_t^t = \frac{1}{b^2}(-1 + fb'^2 + b(f'b' + 2fb'')), \quad \text{and} \quad G_r^r = \frac{1}{b^2}(-1 + bf'b' + fb'^2). \quad (13)$$

At the black hole, where $f(r_+) = 0$, G_t^t and G_r^r become identical as:

$$G_t^t = G_r^r = \frac{1}{b^2}(-1 + bf'b')|_{r_+}. \quad (14)$$

The $r - r$ component of the Einstein's field equations at the horizon can be written as:

$$-1 + bf'b' = 8\pi b^2 P, \quad (15)$$

where $P = T_r^r$ is the radial pressure of matter at the horizon. Now, we multiply a displacement dr_+ of the horizon on both sides of the above equation, the resulting equation can be cast as:

$$Td\left(\frac{4\pi b^2}{4G}\right) - d\left(\frac{r_+}{2G}\right) = PdV, \quad (16)$$

where V is the black hole volume and $dV = 4\pi b^2 dr_+$. This equation can be clearly rewritten as:

$$TdS - dE = PdV, \quad (17)$$

with identifications: $S = 4\pi b^2/4G = A/4G$ and $E = r_+/2G$. Note that Eq. (17) is nothing but the first law of thermodynamics. Here, T is the Hawking temperature of the black hole horizon and S is the Bekenstein-Hawking entropy of the black hole. Note that E is the Misner-Sharp energy at the horizon, and is not the ADM mass of the black hole.

Thus, we have shown that at a black hole horizon, the Einstein's field equations can be cast to the form of the first law of thermodynamics, which implies that there exists a deep connection between gravity and thermodynamics. It is further shown that even for Horava-Lifshitz gravity, which is not a diffeomorphism invariant theory, its field equation at black hole horizon also can be cast to the form of the first law of thermodynamics [6]. In addition, it is found that the story goes on for Lovelock gravity [18], BTZ black hole spacetime [19], stationary black holes and evolving spherically symmetric horizons [20]. However, a non-equilibrium thermodynamics setting is needed for $f(R)$ gravity [21].

4. Apparent horizon: Friedmann equation, the first law of thermodynamics and Hawking radiation

Now, we move to apparent horizon in a FRW universe.

4.1. From first law to Friedmann equation

Let us start with an $(n + 1)$ -dimensional FRW metric:

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{n-1}^2 \right), \quad (18)$$

where a is the scale factor and $d\Omega_{n-1}^2$ denotes an $(n - 1)$ -dimensional sphere with unit radius. Without loss of generality, one can take $k = 1, 0$ or -1 , corresponding to a closed, flat or open universe respectively. Einstein's field equations in the FRW metric in Eq. (18) take the form:

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n(n-1)}\rho, \quad \text{and} \quad (19)$$

$$\dot{H} - \frac{k}{a^2} = -\frac{8\pi G}{n-1}(\rho + p), \quad (20)$$

where ρ and p are energy density and pressure of the perfect fluid in the spacetime, $H = \dot{a}/a$ is the Hubble parameter, and the overdot stands for the derivative with respect to cosmic time t . Introducing the physical radius $\tilde{r} = ar$ and the metric in Eq. (18) can be rewritten as:

$$ds^2 = h_{ab} dx^a dx^b + \tilde{r}^2 d\Omega_{n-1}^2, \quad (21)$$

where $x^0 = t$, $x^1 = r$, and $h_{ab} = \text{diag}(-1, a^2/(1 - kr^2))$. By definition of apparent horizon, $h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0$, we have the apparent horizon radius as:

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \quad (22)$$

Now, we have applied the first law of thermodynamics (the Clausius relation) $dE = TdS$, to the apparent horizon of the FRW universe. To do that, we have assumed that there exist temperature and entropy associated with the apparent horizon as:

$$T = \frac{1}{2\pi\tilde{r}_A}, \quad \text{and} \quad S = \frac{A}{4G}, \quad (23)$$

where $A = \tilde{r}_A^{n-1} \Omega_{n-1}$ is the apparent horizon area. Suppose the matter source in the FRW universe is a perfect fluid, its stress-energy tensor is given by: $T_{ab} = (\rho + p)U_a U_b + pg_{ab}$, where U_a is the 4-velocity. Following [22], defining two physical quantities, the energy supply vector Ψ_a , and work density W as:

$$\Psi_a = T_a^b \partial_b \tilde{r} + W \partial_a \tilde{r}, \quad \text{and} \quad W = -\frac{1}{2} T^{ab} h_{ab}, \quad (24)$$

one, then, can calculate the amount of energy across the apparent horizon within the time interval dt as:

$$dE = A(\rho + p)H\tilde{r}_A dt. \quad (25)$$

Using the energy dE together with the temperature T , and entropy S in Eq. (23), the Clausius relation leads to:

$$\dot{H} - \frac{k}{a^2} = -\frac{8\pi G}{n-1}(\rho + p). \quad (26)$$

Clearly, this is nothing but the second Friedmann Eq. (20). Further, using the continuity equation $\dot{\rho} + n(\rho + p) = 0$, and integrating Eq. (26) yields:

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n(n-1)}\rho. \quad (27)$$

Here, an integration constant has been put into the energy density ρ . In fact, the integration constant is just the cosmological constant. This way the cosmological constant appears as an integration constant, whose exact value should be given by initial condition. Thus, the so-called cosmological constant problem does not appear here.

Thus, we have derived the Friedmann equations governing the dynamics of the FRW universe by applying the first law to the apparent horizon of the spacetime with assumption in Eq. (23). In fact, replacing the entropy formula in Eq. (23) by corresponding ones in Gauss-Bonnet gravity and more general Lovelock gravity, in the same way we are also able to derive corresponding Friedmann equations in those gravity theories [7].

The above approach can be used in more general cases [23], in fact. For example, given a relation between entropy and apparent horizon area, we are able to obtain corresponding modified Friedmann equation. (For details, see [24]).

4.2. Equivalence between the Friedmann equation and the first law

Now, we have proved that the Friedmann equation at the apparent horizon can be cast to the form of the first law of thermodynamics [8].

Let us start with the FRW metric in Eq. (18), by definition, $\kappa = \frac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b\tilde{r})$, one has the surface gravity at apparent horizon, given by:

$$\kappa = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \quad (28)$$

In terms of the apparent horizon radius, the first Friedmann Eq. (19) can be expressed as:

$$\frac{1}{\tilde{r}_A^2} = \frac{16\pi G}{n(n-1)}\rho. \quad (29)$$

Taking derivative on both sides of the above equation and using the continuity equation, one has:

$$\frac{1}{\tilde{r}_A^3}d\tilde{r}_A = \frac{8\pi G}{n-1}(\rho + p)Hdt. \quad (30)$$

With identification $T = \kappa/2\pi$, and $S = A/4G$, this equation can be rewritten as:

$$dE = TdS + WdV, \quad (31)$$

where $E = \rho V$, V is the volume of the region inside the apparent horizon, and $W = (\rho - p)$. This is nothing but the form of the so-called unified first law at the apparent horizon of the FRW universe. It is also shown that the form in Eq. (31) also holds for the Gauss-Bonnet gravity and Lovelock gravity. Further, it is found to hold in brane world scenarios [25]. With this, we can obtain the entropy expression for apparent horizon in brane world scenarios. (For a review, see [26]).

It is interesting to note that with the first law given in Eq. (31), starting from some modified Friedmann equation, one is able to get entropy expression of apparent horizon in some quantum corrected gravity. For example, one can obtain the entropy expression associated with the apparent horizon in loop quantum cosmology [24].

4.3. Hawking radiation of apparent horizon

In the process to derive the Friedmann equation, we have assumed that there is a Hawking temperature given in Eq. (23) associated with apparent horizon in a FRW universe. In this subsection, we have proved this by using the quantum tunnelling approach, which is first employed by Parikh and Wilczek [27] to discuss Hawking radiation for black hole horizon.

In terms of the physical radial coordinate $\tilde{r} = ar$, the 4-dimensional FRW metric can be expressed as:

$$ds^2 = -\frac{1 - \tilde{r}^2/\tilde{r}_A^2}{1 - k\tilde{r}^2/a^2} dt^2 - \frac{2H\tilde{r}}{1 - k\tilde{r}^2/a^2} dt d\tilde{r} + \frac{1}{1 - k\tilde{r}^2/a^2} d\tilde{r}^2 + \tilde{r}^2 d\Omega_2^2. \quad (32)$$

In the spherically symmetric metric in Eq. (32), one can define a Kodama vector $K^a = -\epsilon^{ab}\nabla_b\tilde{r} = \sqrt{1 - k\tilde{r}^2/a^2}(\partial/\partial t)^a$. The norm of the Kodama vector is $K^2 = -(1 - \tilde{r}^2/\tilde{r}_A^2)$. One sees that the Kodama vector is time-like, null and spacelike, inside the apparent horizon, at the apparent horizon, and outside the apparent horizon respectively. As a result, the Kodama vector can play the role as a Killing vector does in a de Sitter space in static coordinates.

Within WKB approximation, a particle with mass m in the metric in Eq. (32) satisfies the Hamilton-Jacob equation:

$$g^{\mu\nu}\partial_\mu\mathbf{S}\partial_\nu\mathbf{S} + m^2 = 0, \quad (33)$$

where $\mathbf{S}$ is its action. In the s-wave approximation, one can define the energy, and the radial wave-number of the particle as:

$$\omega = -K^a\partial_a\mathbf{S} = -\sqrt{1 - k\tilde{r}^2/a^2}\partial_t\mathbf{S}, \quad \text{and} \quad k_{\tilde{r}} = (\partial/\partial\tilde{r})^a\partial_a\mathbf{S} = \partial_{\tilde{r}}\mathbf{S}. \quad (34)$$

And the action can be expressed as:

$$\mathbf{S} = -\int \frac{\omega}{\sqrt{1 - k\tilde{r}^2/a^2}} dt + \int k_{\tilde{r}} d\tilde{r}. \quad (35)$$

Then, the Hamilton-Jacob Eq. (33) turns to be:

$$-\frac{\omega^2}{1 - k\tilde{r}^2/a^2} + \frac{2H\tilde{r}\omega}{\sqrt{1 - k\tilde{r}^2/a^2}}k_{\tilde{r}} + (1 - \frac{\tilde{r}^2}{\tilde{r}_A^2})k_{\tilde{r}}^2 + m^2 = 0, \quad (36)$$

which has the solution as:

$$k_{\tilde{r}} = \frac{-H\tilde{r} \pm \sqrt{H^2\tilde{r}^2 + (1 - \tilde{r}^2/\tilde{r}_A^2)[1 - m^2(1 - k\tilde{r}^2/a^2)/\omega^2]}}{(1 - \tilde{r}^2/\tilde{r}_A^2)\sqrt{1 - k\tilde{r}^2/a^2}}\omega, \quad (37)$$

where the plus/minus sign corresponds to an outgoing/incoming mode. Now, we have considered an incoming mode, since the observer is inside the apparent horizon, like the case of particle tunnelling for the cosmological event horizon in de Sitter space [28]. It is obvious that the action $\mathbf{S}$ has a pole at the apparent horizon. Through a contour integral, we have obtained an imaginary part of the action as:

$$\begin{aligned} \text{Im } \mathbf{S} &= -\text{Im} \int \frac{H\tilde{r} + \sqrt{H^2\tilde{r}^2 + (1 - \tilde{r}^2/\tilde{r}_A^2)[1 - m^2(1 - k\tilde{r}^2/a^2)/\omega^2]}}{(1 - \tilde{r}^2/\tilde{r}_A^2)\sqrt{1 - k\tilde{r}^2/a^2}} \omega d\tilde{r} \\ &= \pi\tilde{r}_A\omega. \end{aligned} \quad (38)$$

In the WKB approximation, the emission rate Γ is the square of the tunnelling amplitude (here, the particle tunnels from outside to inside the apparent horizon), and is given by:

$$\Gamma \propto \exp(-2 \text{Im } \mathbf{S}). \quad (39)$$

Combining Eq. (39) with Eq. (38), one can see clearly that the emission rate can be cast in a form of thermal spectrum $\Gamma \sim \exp(-\omega/T)$, with temperature as:

$$T = \frac{1}{2\pi\tilde{r}_A}. \quad (40)$$

Thus, we have finished the proof that an observer inside the apparent horizon will see a thermal spectrum with temperature given by Eq. (40) when particles tunnel from outside the apparent horizon to inside the apparent horizon. This can be explained as Hawking radiation of apparent horizon in the same spirit in the tunnelling approach proposed by Parikh and Wilczek that the Hawking radiation of black hole is expressed as a tunnelling phenomenon. Furthermore, at this level of approximation, the mass of particle does not enter the emission rate. This is just the remarkable feature of thermal spectrum.

5. Conclusion

A surge of evidence shows that there exist deep connections between gravity and thermodynamics. In this paper, we have just mentioned pieces of them. The intrinsic relation between gravity and thermodynamics is required to further study. If gravity is, indeed, not a fundamental interaction in Nature, instead, it is an induced coarse graining description of some microscopic degrees of freedom of spacetime, it is natural to see the deep connection between gravity and thermodynamics, and even further to see the relation between gravity and hydrodynamics. The latter is a very active issue under study recently in the framework of AdS/CFT correspondence.

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