Warm dust-acoustic shocks

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Abstract. We investigate the formation of shock waves in a warm dusty plasma with equilibrium drift. We show that the natural decay rate of dust charge plays an important role in determining the properties of the shock front in such a plasma.

1. Introduction

We now know that dust grains form an integral part of space and astrophysical plasmas [1–4] and play an important role in determining the dynamical behavior of these systems. This realization led many investigations which reveals the importances of dust grains on various wave modes and their nonlinear counterparts [5,6]. Rao et. al. [7], are among the first few authors, to study the *dust-acoustic* mode. Numerous investigations have been made which involved the study of evolution of various nonlinear solitary waves [8, 9], shock waves [10, 11], as well as double layers [12, 13]. In this paper, we investigate the dust-acoustic shock waves in a warm dusty plasma.

2. Basic equations of the model

Below are the equations of our 1-D nonlinear model of warm dusty plasma,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{1}$$

$$m_d n_d \left(\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x}\right) = e z_d n_d \frac{\partial \phi}{\partial x} - \frac{\partial p_d}{\partial x}, \tag{2}$$

$$\frac{\partial p_d}{\partial t} + u_d \frac{\partial p_d}{\partial x} + \gamma p_d \frac{\partial u_d}{\partial x} = 0, \tag{3}$$

$$\varepsilon_0 \frac{\partial^2 \phi}{\partial x^2} = z_d n_d + n_e - n_i, \tag{4}$$

where $n_{i,e,d}$ are the ion, electron, and dust densities and the other symbols have their usual meanings. While the dust particles are massive and inertial, the electrons and ions are assumed to be Maxwellian, so that the electron and ion densities are solely described by the plasma potential ϕ.

Using a perturbation scheme $f = f_0 + \tilde{f}$, where f_0 is the equilibrium and $\tilde{f}$ is the perturbed parts of any physical quantity f, we can approximate the Poisson equation as,

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} \simeq \alpha\tilde{\phi} + \beta\tilde{\phi}^2 + \frac{e}{\varepsilon_0} z_d n_d, \tag{5}$$

where

$$\alpha = \frac{e^2}{\varepsilon_0 k_B}\left(\frac{n_{e0}}{T_{e0}} + \frac{n_{i0}}{T_{i0}}\right), \quad \beta = \frac{e^2}{2\varepsilon_0 k_B^2}\left(\frac{n_{e0}}{T_{e0}^2} - \frac{n_{i0}}{T_{i0}^2}\right). \tag{6}$$

The closure of the model will be provided by the dust-charging equation, which is given by,

$$\frac{\partial q_d}{\partial t} = I_d, \tag{7}$$

where q_d is the dust-charge and I_d is the charging current. The dust-charging equation in presence of Maxwellian electron and ions can be simplified as,

$$\frac{\partial \tilde{z}_d}{\partial t} + \eta \tilde{z}_d = p_1 \tilde{\phi} + p_2 \tilde{\phi}^2, \tag{8}$$

where the terms $p_{1,2}$ are given by,

$$p_1 = \frac{|I_{e0}|}{k_B}\left(\frac{1}{T_{io}} + \frac{1}{T_{e0}}\right), \quad p_2 = e\frac{|I_{e0}|}{k_B^2}\left(\frac{1}{T_{eo}^2} - \frac{1}{T_{i0}^2}\right) \tag{9}$$

and η is the natural decay rate of dust-charge [14, 15].

3. Nonlinear perturbation and shock wave solution

We now employ the reductive perturbation to our set of equations, Eqs.(1-3,5,8) by introducing a space and time stretching as $\xi = \epsilon(x - v_0 t), \tau = \epsilon^2 t$, followed by an expansion of the perturbed plasma parameters, $(f = n_d, u_d p_d, \phi), f = \sum_{j=0}^{\infty} \epsilon^j f^{(j)}$, where ϵ is the small expansion parameter. Note that in these expansions, $\phi^{(0)} = 0$. Using this scheme and applying to our model equations, to the lowest order in ϵ, we get the following relations,

$$n_d^{(1)} = -\frac{(\alpha + p/\eta)}{e z_{d0}}\phi^{(1)}, \qquad u_d^{(1)} = -V\frac{(\alpha + p/\eta)}{e n_{d0} z_{d0}}\phi^{(1)}, \tag{10}$$

$$p_d^{(1)} = -\gamma p_{d0}\frac{(\alpha + p/\eta)}{e n_{d0} z_{d0}}\phi^{(1)}, \qquad z_d^{(1)} = \frac{p_1}{\eta}\phi^{(1)}, \tag{11}$$

where $p = e n_{d0} p_1/\varepsilon_0$ and $V = v_0 - u_{d0}$ is the relative velocity of the comoving frame of the perturbation with respecting to the equilibrium drift u_{d0}, that the dust particles might have. The above equations requires a compatibility condition that,

$$V^2 = v_{\rm th}^2 + \frac{\omega_d^2}{(\alpha + p/\eta)}, \tag{12}$$

where $\omega_d = \sqrt{n_{d0} e^2 z_{d0}^2/(\varepsilon_0 m_d)}$ is the dust plasma frequency and $v_{\rm th} = \sqrt{\gamma p_{d0}/(m_d n_{d0})}$ is the dust thermal velocity.

Continuing to the next higher order in ϵ, we get the following equations,

$$\begin{aligned}
\frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial}{\partial \xi}\left(n_d^{(1)} u_d^{(1)}\right) &= V\frac{\partial n_d^{(2)}}{\partial \xi} - n_{d0}\frac{\partial u_d^{(2)}}{\partial \xi}, &(13)\\
n_{d0}\left(\frac{\partial u_d^{(1)}}{\partial \tau} + u_d^{(1)}\frac{\partial u_d^{(1)}}{\partial \xi}\right) &= V\left(n_d^{(1)}\frac{\partial u_d^{(1)}}{\partial \xi} + n_{d0}\frac{\partial u_d^{(2)}}{\partial \xi}\right) + \frac{e z_{d0}}{m_d}\left(n_{d0}\frac{\partial \phi^{(2)}}{\partial \xi} + n_d^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi}\right)\\
&\quad + \frac{e n_{d0}}{m_d} z_d^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} - \frac{1}{m_d}\frac{\partial p_d^{(2)}}{\partial x}, &(14)\\
-\frac{\partial p_d^{(1)}}{\partial \tau} &= u_d^{(1)}\frac{\partial p_d^{(1)}}{\partial \xi} + \gamma p_d^{(1)}\frac{\partial u_d^{(1)}}{\partial \xi} + p_{d0}\frac{\partial p_d^{(2)}}{\partial \xi} - V\frac{\partial p_d^{(2)}}{\partial \xi}, &(15)\\
0 &= z n_d^{(2)} + n_d^{(1)} z_d^{(1)} + n_{d0} z_d^{(2)} - \beta\left(\phi^{(1)}\right)^2 - \alpha\phi^{(2)}, &(16)\\
0 &= \eta z_d^{(2)} - p_2\left(\phi^{(1)}\right)^2 - p_1\phi^{(2)} - v_0\frac{\partial z_d^{(1)}}{\partial \xi}. &(17)
\end{aligned}$$

Eliminating the second order quantities from Eqs.(13-17) and using the results for the first order quantities in terms of $\phi^{(1)}$, we finally derive the nonlinear shock wave equation, known as the Burger equation [16] as,

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = 0, \tag{18}$$

where the coefficients are given by,

$$\begin{aligned}
A &= \frac{(\alpha + p/\eta)}{2V n_{d0}}\left[2V^2 + v_{\rm th}^2(\gamma - 1)\right] + \frac{\left(V^2 - v_{\rm th}^2\right)}{V z_{d0}}\left[\frac{1}{(\alpha + p/\eta)}\left(\frac{p_2 n_{d0}}{\eta} + \varepsilon_0\alpha\right) + \frac{p_1}{\eta}\right]\\
&\quad - \frac{e}{2m_d V}\left[\frac{p_1 n_{d0}}{\eta(\alpha + p/\eta)} + z_{d0}\right], &(19)\\
B &= \frac{v_0 n_{d0}(V^2 - v_{\rm th}^2)}{2(\alpha + p/\eta)V\eta^2 z_{d0}}. &(20)
\end{aligned}$$

The coefficient A is the coefficient of nonlinearity and B represents dissipation. Although, our model does not include any physical dissipation, the dust-charge fluctuation is manifested as dissipation in our model, which drives the shock front.

4. Concluding remarks

We note that the natural decay rate of dust charge η plays an important role in determining the behaviour of the shock front. Taking an asymptotic limit of η, on the dissipation coefficient B, we get,

$$\begin{aligned}
B &\simeq \frac{v_0 \varepsilon z_{d0}}{2 m_d p_1\sqrt{\omega_d^2\alpha^{-1} + v_{\rm th}^2}}, &&\text{for } \eta \simeq 0,\\
B &\simeq 0, &&\text{for } \eta \gg 1,
\end{aligned} \tag{21}$$

which reaches a constant value for small η and become zero for large η. This essentially means that when the natural decay rate of dust charge becomes zero, the shock front still survives owing to the dust-charge fluctuation whereas when η becomes large, the dust-charge dissipates very quickly, which destroys the shock front. It is interesting to look at these asymptotic values when the dust is essentially cold i.e. for small p_{d0},

$$B \simeq \frac{\varepsilon_0 v_0}{2e z_{d0}}\sqrt{\frac{\omega_d^2}{p\eta}}, \quad \text{for } \eta, p_{d0} \simeq 0, \tag{22}$$

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when B becomes very large for small η.

References

[1] C. K. Goertz, Phys. Scr. 27, (1989) 271.
[2] T. G. Northrop and T. J. Birmingham, planet. Space Sci. 38, (1990) 319.
[3] F. Melandso, T. K. Aslakson, and O. Havnes, Planet. Space Sci. 41, (1993) 321.
[4] N. D' Angelo, Planet. Space Sci. 42, (1994) 507.
[5] C. B. Dwivedi and B. P. Pandey, Phys. Plasmas 2, (1995) 4134.
[6] N. N. Rao and P. K. Shukla, Planet. Space Sci. 42, (1995) 221.
[7] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. 38, (1993) 543.
[8] F. Sayed and A. A. Mamun, Phys. Plasmas 14, (2007) 014502.
[9] M. A. Mirza, S. Mahmood, N. Jehan and N. Ali, Phys. Sci. 75, (2007) 755.
[10] B. Eliasson and P. K. Shukla, Phys. Rev. E 69, (2004) 067401.
[11] P. K. Shukla and A. A. Mamun, New J. Phys. 5, (2003) 173.
[12] M. A. Raadu, Phys. Rep. 78, (1997) 25.
[13] G. C. Das and K. Devi, Astrophys. And Space Sci. 330, (2010) 79.
[14] G. C. Das, Balen Choudhury, and M. P. Bora, Phys. Plasmas 17, (2010) 123707.
[15] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E 48(5), (1993) 3930.
[16] J. M. Burger, Proc. R. Neth. Acad. Soc. 32, (1929) 414.