Self-similarity based model of double parton distribution functions at LHC

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Abstract. The notion of self-similarity, pursued earlier for small x physics at HERA, is used to construct a model for double parton distribution functions (dPDFs) at small x_1 and x_2, the longitudinal fractional momenta of two partons. A necessary input to such a theoretical approach is the parton distribution functions (PDFs). In this model, the most general form of dPDF is found to have total thirteen parameters to be fitted from LHC data. We show that the constructed dPDF does not factorize into two single PDFs in conformity with QCD expectation and it vanishes at the kinematic boundary.

1. Introduction

Double Parton Distribution Functions (dPDFs) [1–5] are the simplest distribution functions that occur in the multi-partonic interactions (MPI) and are of great relevance for LHC physics as they represent a background for the search of new physics. They are equally important in the study of double parton scattering cross-sections. In this paper, we use the notion of self-similarity based model of proton structure function $F_2\left(x, Q^2\right)$ [6] to construct a Transverse Momentum Dependent double Parton Density Function (TMD dPDF) and a dPDF at small x_1 and x_2, the longitudinal fractional momenta of two partons. Parton distribution functions (PDFs) [7] play the role of a necessary input to such a theoretical approach. They are among the most important sources of information on hadron structure at the level of quarks and gluons. We also investigate if the constructed dPDF has a self-similar behavior at small x_1 and x_2 while it vanishes at the kinematic boundary $x_1 + x_2 = 1$.

2. Formalism

2.1. Self-similarity based TMD dPDF and dPDF at small x_1, x_2

The self-similarity based model of the nucleon structure function proposed in Ref. [6] had been designed to be valid in the kinematical region $6.2 \times 10^{-7} \leq x \leq 10^{-2}$ and $0.045 \leq Q^2 \leq 120\,\mathrm{GeV}^2$. It, however, did not take into account the large x behavior of the PDF [8] which is not unexpected. Till date, there is no phenomenological justification of self-similarity at large x. But it is not unreasonable to assume that self-similarity does not terminate abruptly at $x \approx 0.01$, but smoothly vanishes at $x = 1$, the valence quark limit of proton with no trace of self-similarity at all. We consider this point of view in our present work.

Unlike in Ref. [6], where there is only one hard scale k_t^2 in the TMD PDF, TMD dPDF has two hard scales $k_t^{2(1)}$ and $k_t^{2(2)}$ of partons carrying fractional momenta x_1 and x_2 of flavors i

and j respectively. Corresponding to the virtuality Q^2 of deep inelastic scattering (DIS), the partons also have hard scales $Q^{2(1)}$ and $Q^{2(2)}$. For simplicity, we assume them to be fixed. Following Ref. [6], the TMD dPDF for partons of flavors i and j will then have the following basic magnification factors:

$$\begin{aligned} M_1 &= \frac{1}{x_1} \\ M_2 &= \frac{1}{x_2} \\ M_3 &= \left(1 + \frac{k_t^{2(1)}}{k_0^2}\right) \\ M_4 &= \left(1 + \frac{k_t^{2(2)}}{k_0^2}\right) \end{aligned} \tag{1}$$

where k_0^2 is the transverse mass cut-off parameter and has been introduced to take care of the dimension.

The TMD dPDF for a pair of partons of flavors i and j is then given as:

$$\begin{aligned} \log f_{ij}\left(x_1, x_2, k_t^{2(1)}, k_t^{2(2)}\right) &= D_1 \log M_1 \log M_2 + D_2 \log M_1 \log M_4 + D_3 \log M_2 \log M_3 \\ &+ D_4 \log M_3 \log M_4 + D_5 \log M_1 \log M_2 \log M_3 \\ &+ D_6 \log M_1 \log M_2 \log M_4 + D_7 \log M_1 \log M_3 \log M_4 \\ &+ D_8 \log M_2 \log M_3 \log M_4 + D_9 \log M_1 \log M_2 \log M_3 \log M_4 \\ &+ D_0^{ij} - \log M^4 \end{aligned} \tag{2}$$

Consequently, the TMD dPDF is integrated over $Q^{2(1)}$ and $Q^{2(2)}$ to obtain the desired dPDF. It is given as

$$D_{ij}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) = \frac{e^{D_0^{ij}}}{M^4} \left(\frac{1}{x_1}\right)^{D_1 \log \frac{1}{x_2}} I\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) \tag{3}$$

where $I\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right)$ is the double integration over $k_t^{2(1)}$ and $k_t^{2(2)}$ and its expression is given by,

$$I\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) = \int_0^{Q^{2(1)}} dk_t^{2(1)} \int_0^{Q^{2(2)}} dk_t^{2(2)} \left(\frac{k_t^{2(1)} + k_0^2}{k_0^2}\right)^A \left(\frac{k_t^{2(2)} + k_0^2}{k_0^2}\right)^B, \tag{4}$$

where

$$\begin{aligned} A &= D_3 \log \frac{1}{x_2} + D_4 \log \left(\frac{k_t^{2(2)} + k_0^2}{k_0^2}\right) + D_5 \log \frac{1}{x_1} \log \frac{1}{x_2} + D_7 \log \frac{1}{x_1} \log \left(\frac{k_t^{2(2)} + k_0^2}{k_0^2}\right) \\ B &= D_2 \log \frac{1}{x_1} + D_6 \log \frac{1}{x_1} \log \frac{1}{x_2} + D_8 \log \frac{1}{x_2} \log \left(\frac{k_t^{2(1)} + k_0^2}{k_0^2}\right) \\ &\quad + D_9 \log \frac{1}{x_1} \log \frac{1}{x_2} \log \left(\frac{k_t^{2(1)} + k_0^2}{k_0^2}\right) \end{aligned}$$

The self-similar dPDF at small x_1 and x_2 obtained in Eq.(3) therefore contains 12 parameters, viz. 9 fractal parameters $(D_1, ..., D_9)$, one normalization constant D_0^{ij}, one mass scale M^4 (which has been introduced in Eq.(2) so as to make the dPDF dimensionless) and one transverse mass cut-off parameter k_0^2. We note that the integration [Eq.(4)] is not factorisable in x_1 and x_2. Thus the usual factorisability assumption [9] that a dPDF can be considered as a product of two single PDFs does not hold good in the present self-similarity based dPDF.

2.2. Self-similarity based TMD dPDF and dPDF at the boundary $x_1 + x_2 = 1$
The standard behavior of single PDF is

$$\lim_{x_1 \to 1} D_i\left(x_1, Q^{2(1)}\right) = 0 \,\text{and}\, \lim_{x_2 \to 1} D_j\left(x_2, Q^{2(2)}\right) = 0 \tag{5}$$

The corresponding behavior of dPDF at the kinematic boundary, on the other hand, is [1]

$$\lim_{x_1+x_2 \to 1} D_{ij}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) = 0 \tag{6}$$

It is to be noted that Eq.(5) and Eq.(6) do not conform with each other indicating again that the simple assumption of factorisability of dPDF into two PDFs fails at the kinematic boundary $x_1 + x_2 = 1$. Thus the present self-similarity based model of dPDF conforms to the QCD expectation of non-factorisability [10].

As mentioned above, the notion of self-similarity for dPDF at large x is not expected to hold. However, a simple parameter-free way of incorporating the kinematic boundary condition [Eq.(6)] is to introduce an additional magnification factor $M_5 = \left(\frac{1}{x_1+x_2} - 1\right)$, which for $x_2 = 0$ and $x_1 \to 0$ ($x_1 = 0$ and $x_2 \to 0$) approaches $M_1(M_2)$ of Eq.(1). The corresponding dPDF thus obtained is

$$\tilde{D}_{ij}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) = \frac{e^{D_0^{ij}}}{M^4}\left(\frac{1}{x_1}\right)^{D_1 \log \frac{1}{x_2}}\left(\frac{1-(x_1+x_2)}{x_1+x_2}\right)^{D_{10}} \tilde{I}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) \tag{7}$$

where $\tilde{I}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right)$ is the double integration over the transverse momenta $k_t^{2(1)}$ and $k_t^{2(2)}$ and has the same expression as that of $I\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right)$ as given in Eq.(4). The final expression of dPDF at the kinematic boundary in this approach thus contains total thirteen parameters (12 parameters as mentioned above and one additional fractal parameter D_{10}) to be determined from LHC data.

3. Graphical analysis of dPDFs

The simplest model of dPDF for small x_1 and x_2 can be obtained if we assume that D_1, $D_{10} \gg D_2, \ldots, D_9$. The dPDF expression of Eq.(3) will then be reduced to

$$D_{ij}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) \simeq \frac{e^{D_0^{ij}}}{M^4}\left(\frac{1}{x_1}\right)^{D_1 \log \frac{1}{x_2}} Q^{2(1)}Q^{2(2)} \tag{8}$$

The dPDF at the kinematic boundary [Eq.(7)] also gets reduced to

$$\tilde{D}_{ij}\left(x_1, x_2, Q^{2(1)}, Q^{2(2)}\right) \simeq \frac{e^{D_0^{ij}}}{M^4}\left(\frac{1}{x_1}\right)^{D_1 \log \frac{1}{x_2}}\left(\frac{1-(x_1+x_2)}{x_1+x_2}\right)^{D_{10}} Q^{2(1)}Q^{2(2)} \tag{9}$$

For qualitative feature of the model, we plot in Figure 1 the dPDF vs x for a few representative values of $x_2 = 0.3$, 0.1 and 0.6 using Eq.(8) (dashed lines) and Eq.(9) (solid lines). It shows the qualitative difference between the self-similarity based model of Ref.[6] and the smooth extrapolation for large x.

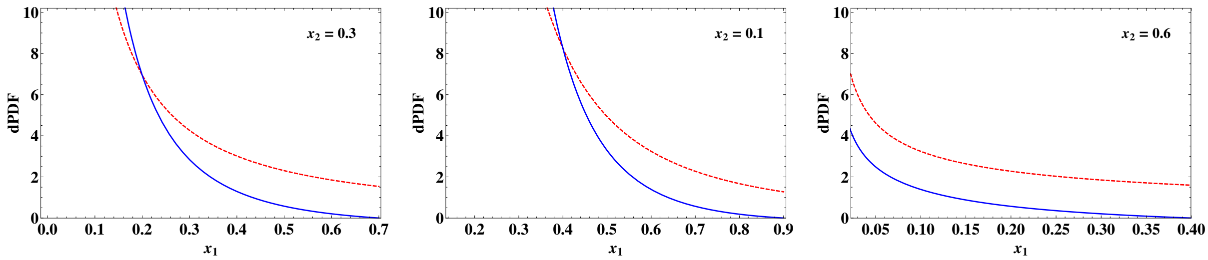

Figure 1. Double Parton Distribution Functions (dPDFs) shown as a function of x.

4. Conclusions

In this paper, we have introduced the self-similarity based formalism of dPDF at small x_1, x_2. We have found that the constructed dPDF does not factorize into two single PDFs in conformity with the QCD expectation. We have also suggested a smooth continuous form of dPDF which has both the expected self-similar behavior at small x_1 and x_2 and vanishes at the kinematic boundary. This is achieved by introducing an additional factor in the defining TMD dPDF.

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