Comments on the perturbation of Cornell potential in a QCD potential model

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Abstract. We find in the analysis that the linear part of the Cornell potential can be treated as perturbation for a set of larger values of α_s in the range $0.4 \leq \alpha_s \leq 0.75$ with a constant shift within the range of $-0.4\,\mathrm{GeV} \leq c \leq -1\,\mathrm{GeV}$. Moreover with the same range of constant shift in the Potential, we expect better results with Coulombic part as perturbation for $\alpha_s \leq 0.4$.

1. Introduction

In the potential models, the effective potential between a quark and antiquark can be taken as the Coulomb-plus-linear potential,

$$V(r) = -\frac{4\alpha_s}{r} + br + c. \tag{1}$$

This potential has received a great deal of attention in particle physics, more precisely in the context of meson spectroscopy where it is used to describe systems of quark and antiquark bound states.However, it has been found to be questionable about the numbers of free parameters(α_s, b, c) and numbers of findings in any potential model. The success of a phenomenological model depends on reducing the free model parameters to obtain more precise values with proper arguments and analysis.

In this letter, we put forward the comments on linear part of the Potential as perturbation with Coulombic part as Parent [1, 2] as well as Coulombic part as perturbation with linear as parent [3] in a potential model and attempt to put some constraints on the model parameters.

2. The method of perturbation

It is well known that one cannot solve the Schrdinger equation in quantum mechanics with the QCD potential(equation (1)) except for some simple models. Perturbation theory has been helpful since the earliest applications of quantum mechanics in this regard. In fact, perturbation theory is probably one of the approximate methods that most appeals to intuition [4].

The advantage of taking Cornell Potential for study is that it leads naturally to two choices of "parent" Hamiltonian, one based on the Coulomb part and the other on the linear term, which can be usefully compared. It is expected that a critical role is played by r_0 where the Potential $V(r) = 0$. Aitchison and Dudek in Reference [5] put an argument that if the size of a state measured by $\langle r \rangle < r_0$, then the Coulomb part as the "Parent" will perform better

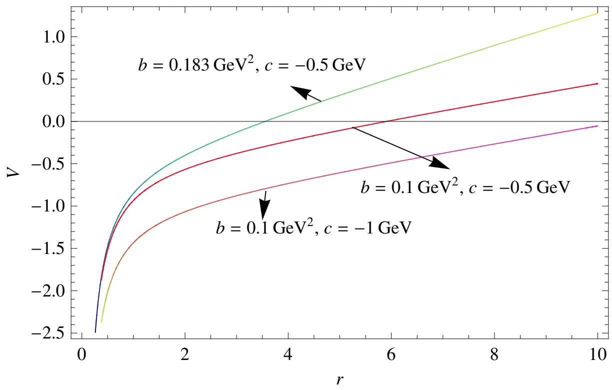

Figure 1: Variation of $V(r)$ with the variation of b, c and α_s.

and if not so the linear part as "parent" will perform better. The Aitchison's work also showed that with Coulombic part as perturbation(VIPT), bottomonium spectra are well explained than charmonium where as charmonium states are well explained with linear part as parent. It becomes noteworthy in this context that the critical distance r_0 is not a constant and can be enhanced by reducing b and c or by increasing α_s. In Figure 1, we show the variation of $V(r)$ with the variation of model parameters.

3. The QCD potential model

For completeness and proper reference we put the last modified version of our model wave function with Coulombic part as parent as [6, 7]

$$\psi_{\text{rel+conf}}(r) = \frac{N'}{\sqrt{\pi a_0^3}} e^{\frac{-r}{a_0}} \left(C' - \frac{\mu b a_0 r^2}{2} \right) \left(\frac{r}{a_0} \right)^{-\epsilon} \tag{2}$$

where N' is the normalisation constant. All other terms involved in equation 2 are explained in reference [1, 2, 6] with a correction for ϵ [7]

$$\epsilon = 1 - \sqrt{1 - \left(\frac{4}{3} \alpha_s \right)^2}. \tag{3}$$

4. Constraints from two points of view

In this section, we discuss the constrains of the model parameters. The values of α_s and the constant shift$c$ of the potential $V(r)$ are expected to fit from mass spectroscopy of hadrons to study its other properties. A narrow range of the free parameters in a potential model measures its success and applicability as well. Here we have tried to show some constrains on the free parameters α_s and c from two points of view.

4.1. From the convergence point of view

In Reference [1, 2], it is shown from the momentum transform of equation(2) (with $C' = 1$) that confinement can be treated as perturbation provided

$$\frac{(4 - \epsilon)(3 - \epsilon)\mu b a_0^3}{2(1 + a_0^2 Q^2)} \ll 1. \tag{4}$$

Table 1: Q_0^2 (in GeV^2) from Ref. [1]

Mesons	$\alpha_s = 0.65$	$\alpha_s = 0.6$	$\alpha_s = 0.5$
D	3.5	4.1	5.5
D_s	3.5	4.2	5.8
B	1.4	1.7	2.3
B_s	3.9	4.4	5.5
B_c	0	0.4	2.0

Table 2: Q_0^2 (in GeV^2) from equation(5)

Mesons	$\alpha_s = 0.5$	$\alpha_s = 0.4$
D	0.0264	0.0362
D_s	0.0304	0.0490
B	0.0286	0.0412
B_s	0.0300	0.0530
B_c	0	0.0162

Table 3: Values of r_1 and r_0 for $c = -0.5\,\mathrm{GeV}$

mesons	$\alpha_s = 0.65$		$\alpha_s = 0.5$	
	r_1	r_0	r_1	r_0
$D(c\bar{u}/cd)$	6.26	3.94	8.14	3.71
$D(c\bar{s})$	4.70	3.94	6.11	3.71
$B(\bar{b}u/\bar{b}d)$	5.50	3.94	7.15	3.71
$B_s(\bar{b}s)$	3.93	3.94	5.1	3.71
$B_c(\bar{b}c)$	1.46	3.94	1.90	3.71

Table 4: Values of r_1 and r_0 for $c = -1\,\mathrm{GeV}$

mesons	$\alpha_s = 0.65$		$\alpha_s = 0.5$		$\alpha_s = 0.4$	
	r_1	r_0	r_1	r_0	r_1	r_0
$D(c\bar{u}/cd)$	6.26	6.22	8.14	6.06	10.18	5.95
$D(c\bar{s})$	4.70	6.22	6.11	6.06	7.63	5.95
$B(\bar{b}u/\bar{b}d)$	5.50	6.22	7.15	6.06	8.93	5.95
$B_s(\bar{b}s)$	3.93	6.22	5.1	6.06	6.39	5.95
$B_c(\bar{b}c)$	1.46	6.22	1.90	6.06	2.38	5.95

The values of Q_0^2 from equation (6) with $b = 0.183\,\mathrm{GeV}^{-2}$ for B and D mesons are shown in Table 1. The standard spectroscopic result $b = 0.183\,\mathrm{GeV}^2$ [8] can be accommodated by a proper choice of c [2, 6], so that the perturbative condition of equation (4) becomes

$$\frac{(4-\epsilon)(3-\epsilon)\mu b a_0^3}{2(1+a_0^2 Q^2)} \ll C'. \tag{5}$$

With this condition, one can impose $b = 0.183\,\mathrm{GeV}^2$ for low Q^2 value. The improved values of Q_0^2 from equation(5) with $b = 0.183\,\mathrm{GeV}^2$, $m_{u/d} = 0.33\,\mathrm{GeV}$,$m_s = 0.483\,\mathrm{GeV}$,$m_c = 1.55\mathrm{GeV}$,$m_b = 4.93\mathrm{GeV}$ for B and D mesons are presented in Table 2.

Thus to incorporate lower value of Q^2 ($Q^2 \leq \Lambda_{QCD}^2$),with linear part as perturbation in the improved version one expects a bound of α_s as $0.4 \leq \alpha_s \leq 0.75$. The results of the References [6, 9] also indicates that with $\alpha_s = 0.6$ one can expect high accuracy towards experimental and other theoretical values.

4.2. From the condition of Aitchison and Dudek

Considering the argument of Aitchison and Dudek [5] $\langle r \rangle < r_0$ to treat the linear part as perturbation, we get

$$\langle r \rangle_{\text{coul}} = \int \psi^* r \psi dr = \frac{3a_0}{2} = r_1\,(\text{say}) \tag{6}$$

and the critical distance r_0 at which $V(r_0) = 0$ can be obtained by the relation

$$br_0^2 + cr_0 - \frac{4\alpha_s}{3} = 0. \tag{7}$$

The variation of r_1 and r_0 with the model parameters can be easily studied from the above relations. From the calculation it seems to be clear that to treat linear part as perturbation with the valid condition $\langle r\rangle < r_0$ one has to choose the value of $c \ll -0.5$ GeV. With $c = -1$GeV the condition is found to be valid for certain value of α_s. In Table 3 and Table 4, we present our result with $c = -0.5$ GeV and $c = -1$ GeV.

4.3. Constrains on α_s

From the above analysis we see that in the perturbation procedure the value of α_s and the model parameter c plays a crucial role in choosing the parent and perturbative terms. From the reality condition and convergence of series demands the value of α_s within the range of $0.4 \leq \alpha_s \leq 0.75$ in the model without putting any further restriction or constraints. However the logarithmic decrease of α_s depends on the QCD energy scale parameter Λ_{QCD} which is a free parameter and has to be measured in the experiments. One well known formula to fix the value of α_s in Quark models is taken as [7, 10]

$$\alpha_s\left(\mu^2\right) = \frac{4\pi}{\left(11 - \frac{2n_f}{3}\right)\ln\left(\frac{\mu^2+M_B^2}{\Lambda_{QCD}^2}\right)} \tag{8}$$

where, n_f is the number of light flavours, μ is renormalistion scale related to the constituent quark masses as $\mu = 2\frac{m_i m_j}{m_i+m_j}$. M_B is the background mass related to the confinement term of the potential as $M_B = 2.24 \times b^{1/2} = 0.95$ GeV. The reality condition of α_s in equation(8) requires that $\Lambda_{QCD} \leq 460\,\mathrm{MeV}$. By fitting the ρ meson mass in equation(8) one easily obtains QCD scale parameter $\Lambda_{QCD} = 413\,\mathrm{MeV}$ [10].

5. Conclusion and comments

In this letter we mainly devote in finding the analytical conditions to treat the linear part of the Cornell potential as perturbation. We find from the convergence point of view that one can consider the confining part of the potential as perturbation with $0.4 \leq \alpha_s \leq 0.75$ and $c = -0.5$GeV. From table.3 and table.4 it seems to be clear that the validity of the condition $\langle r\rangle < r_0$ demands parametrisation of $\langle c\rangle < -0.5\,\mathrm{GeV}$ and $\alpha_s > 0.6$. However with linear part as perturbation, if the value of α_s in the above range is taken to be granted, then with the same potential another possibility of considering the coulombic part as perturbation also arises for a value of $\alpha_s \leq 0.4$. Interestingly, in Reference. [3], it is shown that with $\alpha_s = 0.39$ and $\alpha_s = 0.22$ one can obtain the required values of slope and curvature in the model with coulombic part as perturbation. The results in Reference [3], clearly indicates that with coulombic part as perturbation, one can get improved results with $\alpha_s \leq 0.4$ than $\alpha_s \geq 0.4$.

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