Thermodynamic origin of interaction in the cosmological tachyonic field

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Abstract. We propose a model with thermodynamic origin of interaction among the components of tachyonic scalar field in the universe. The components are taken to exist in the state of non-equilibrium initially but undergoing a transition towards the equilibrium state. We show that this transition requires interaction among the components. During such transition phase, the transfer of energy among the components is governed by the second law of thermodynamics with local violation of conservation of energy for individual components. It is further shown that in our proposed mechanism the interaction itself generates an increase of entropy in an evolving universe, and thus it might indicate a possible solution to the well-known entropy problem.

1. Introduction

Our universe is currently in a phase of accelerated expansion as interpreted through the cosmological observations over the past decade such as SNe type Ia [1, 2], Cosmic Microwave Background Radiation(CMBR) [3], Baryon Acoustic Oscillations(BAO) [4] in galaxy surveys etc. It is understood that nearly 73% of the total content in the universe is composed of an exotic component with negative pressure which, often termed as dark energy, is held responsible for this accelerated expansion at the present epoch. The Lagrangian $L = -V(\phi)\sqrt{1-\partial_i\phi\partial^i\phi}$ for the proposed tachyonic scalar field ϕ taken from [5] (arising from string theory) provides with the corresponding action as

$$\mathcal{A} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} - V(\phi)\sqrt{1-\partial^i\phi\partial_i\phi}\right) \tag{1}$$

and the energy-momentum tensor $T^{ik} = \frac{\partial L}{\partial(\partial_i\phi)}\partial^k\phi - g^{ik}L$ yields the energy density and pressure (for spatially homogeneous field approximation) as

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}; \qquad P = -V(\phi)\sqrt{1-\dot{\phi}^2} \tag{2}$$

where an overdot denotes the derivative with respect to time (as in the rest of this paper). Here, we consider ρ and P as composed of two components [6] which fit in so well that if one component is matter with $w_m = 0$, then the other one turns out to be the cosmological constant

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with $w_\lambda = -1$. Thus, we obtain these components from (2), respectively, as

$$\rho_m = \frac{V(\phi)\dot{\phi}^2}{\sqrt{1-\dot{\phi}^2}}; \qquad P_m = 0 \tag{3}$$

and

$$\rho_\lambda = V(\phi)\sqrt{1-\dot{\phi}^2}; \quad P_\lambda = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{4}$$

2. Interaction between components

Next, we study the mutual interaction of components of tachyonic scalar field. The interacting dark energy models have been recently proposed by several authors [7–10]. Under these circumstances, the cosmological constant is no longer a true constant and the rate of its decline is ascertained by the complementary evolution of dark matter energy densities. It is obvious that during interaction the overall conservation of energy is kept intact. The components are in non-equilibrium (thermal, mechanical or else) and to achieve the equilibrium the components interact mutually. The transfer of energy is governed by the second law of thermodynamics. If the components are in equilibrium then due to interaction perturbation tries to restore or achieve new stable state (Le Châtelier-Braun principle) [11, 12]. The individual equations of energy conservation for dark matter ($w_m = 0$) and the cosmological constant ($w_\lambda = -1$) are respectively given as

$$\dot{\rho_m} = -3H\rho_m + Q; \quad \dot{\rho_\lambda} = -Q. \tag{5}$$

Considering the dimensional compatibility it is natural choice that the interaction strength Q should be function of the Hubble parameter and energy density. Due to the lack of information regarding the exact nature of dark matter and dark energy (as the cosmological constant or else) we could not fixed the exact form of interaction strength. Thus, with this motivation we present the form of interaction term, heuristically, as the function of time rate of change in energy density. Thus, the simple assumption about the interaction strength is that Q should be small and positive. If it had large and negative value then dark energy would have dominated the expansion practically from the outset and galaxies could not have formed at the desired epochs. Thus,

$$Q = \alpha\dot{\rho_m}. \tag{6}$$

where α is the proportionality constant. The interaction strength would, in general, depend on temperature (however, here we take it as independent of temperature). From (5) with form of Q as in (6), we have the following functional form of the energy densities of the components of tachyonic field

$$\rho_m = \rho_m^0\left(\frac{a}{a_0}\right)^{-3/(1-\alpha)}; \quad \rho_\lambda = \rho_\lambda^0 - \alpha\rho_m^0\left[\left(\frac{a}{a_0}\right)^{-3/1-\alpha} - 1\right] \tag{7}$$

where ρ_m^0 and ρ_λ^0 are the values of energy density of dark matter and dark energy respectively at present epoch scale factor a_0. It can be seen that in the absence of interaction ($\alpha = 0$) we have $\rho_m = \rho_m^0(a/a_0)^{-3}$ and $\rho_\lambda = \rho_\lambda^0$ = constant, as expected in the standard approach with a truly constant cosmological constant.

3. Constraints on interaction strength from observations

The difference of the squares of the normalized Hubble parameter $E^2(x)$ at two different redshifts x_i and x_j for the interaction term $Q = \alpha\dot{\rho_m}$ is given as

$$E^2(x_i) - E^2(x_j) = \triangle E^2(x_i, x_j) = (1-\alpha)\Omega_m^0\left[x_i^{3/1-\alpha} - x_j^{3/1-\alpha}\right]. \tag{8}$$

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Measuring the values of $\triangle E^2(x_i, x_j)$ from the redshift observations and Ω_m^0 as the concordance value from CMBR, BAO etc. data analysis, we calculate α for the (x_i, x_j) pair of redshifts corresponding to interaction form Q. We emphasize that the choice of such pair is desirable over a large range of redshift observations, since we must determine the interaction strength over a long period of cosmic evolution. This is required so as to constrain the structure formation both at high- and low-z epochs. The precise measurements of H_0 will be helpful to break the degeneracy among the cosmological parameters [13]. Here, we take a set of values of $H(z)$ [14,15] as: at $z = 0.1$, 0.4, 1.3, and 1.75, the values of $H(z) = 69 \pm 12$, 95 ± 17, 168 ± 17 and 202 ± 40 km s^{-1} Mpc^{-1} respectively. Further calculations may be done by taking the present values $H_0 = 73.8 \pm 2.4 \approx 73.8$ km s^{-1} Mpc^{-1} [16] and the density parameter of matter component $\Omega_m^0 = 0.272$ [17]. Thus, choosing the four epochs as $x_1 = 1.1$, $x_2 = 1.4$, $x_3 = 2.3$ and $x_4 = 2.75$ for these redshifts, the squared normalized Hubble parameter may be calculated in a straightforward manner as given below:

$E^2(x_1) = 0.874$, $E^2(x_2) = 1.657$, $E^2(x_3) = 5.182$, and $E^2(x_4) = 7.492$. and thus $\Delta E^2(x_1, x_2) = -0.783$, $\Delta E^2(x_1, x_3) = -4.308$, $\Delta E^2(x_1, x_4) = -6.618$, $\Delta E^2(x_2, x_3) = -3.525$, $\Delta E^2(x_2, x_4) = -5.835$ and $\Delta E^2(x_3, x_4) = -2.310$.

Using these data we can proceed to determine six values of the proportionality constant α of the interaction form $Q = \alpha \dot{\rho_m}$ from (8). The solution of these six equations helps in estimation of α.

4. Entropy of the Universe

The entropy of the FRW universe includes two terms– one is the entropy of the apparent horizon and the second is entropy contributed by fluids enclosed by the horizon. First, let us consider entropy of the apparent horizon. The entropy of the event horizon is proportional to its surface area A [18] and is given as

$$S_A = \frac{k_B}{4} \frac{A}{l_{pl}^2}. \tag{9}$$

where $A = 4\pi r_A^2$ is the area of the apparent horizon and $r_A = \left(\sqrt{H^2 + \frac{k}{a^2}}\right)^{-1}$ is radius of the horizon. Also, l_{pl} and k_B are Planck's length and Boltzmann's constant respectively. Using the Friedmann equations $H = \frac{8\pi G}{3}[\rho_m + \rho_\lambda]$ (for $k = 0$ universe) we have the following expression for the apparent horizon

$$A = \frac{3}{2G}[\rho_m^0 x^{3/1-\alpha} + \rho_\lambda^0 - \alpha \rho_\lambda^0 (x^{3/1-\alpha} - 1)]. \tag{10}$$

Hence, the entropy of apparent horizon is

$$S_A = \frac{3k_B}{8Gl_{pl}^2}[\rho_m^0 x^{3/1-\alpha} + \rho_\lambda^0 - \alpha \rho_\lambda^0 (x^{3/1-\alpha} - 1)]. \tag{11}$$

The entropy of fluids enclosed by the apparent horizon can be calculated as follows. If there are N dust particles and each particle contributes an entropy equal to k_B, the total entropy of fluid inside the apparent horizon will be equal to $S_m = k_B.N$ with $N = \left(\frac{4\pi}{3} r_A^3\right) n$ where n is the number density of the dust particles. The conservation law of the number density gives $n = n_0 \left(\frac{a_0}{a}\right)^3 = n_0 x^3$. Thus, the entropy contributed by fluids turns out to be

$$S_m = k_B \frac{4\pi}{3} r_A^3 n_0 x^3 \tag{12}$$

whereas the total entropy of universe becomes equal to

$$S_A + S_m > 0. \tag{13}$$

The interaction between components inside the apparent horizon also produces entropy and must contribute to the observed large amount of entropy in the universe.

5. Conclusion

The cosmic tachyonic scalar field may be decomposed into several components with the assumption that the field is spatially homogeneous. Here we take two components, one is pressure-less dust matter and other is the cosmological constant (because its equation of state is -1). The non-equilibrium state of two components has a tendency to undergo a transition into the equilibrium state and to achieve this state, these components enter into mutual interaction. During interaction the conservation of energy of individual components gets violated while the total energy of field is held conserved. The thermodynamic laws responsible for the transfer of energy between components are obeyed with the heuristic choice of interaction strength Q. Normalized Hubble parameter at different redshifts provided by the observations helps to constrain the interaction strength Q with the calculation of α. The entropy of universe appears as a sum of entropy of apparent horizon and entropy of fluid inside apparent horizon. We assume the fluid inside the apparent horizon is dust having finite number of particles.

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