

$f(R)$ gravity cosmology in scalar degree of freedom

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Abstract. The models of $f(R)$ gravity belong to an important class of modified gravity models where the late time cosmic accelerated expansion is considered as the manifestation of the large scale modification of the force of gravity. $f(R)$ gravity models can be expressed in terms of a scalar degree of freedom by explicit redefinition of model's variable. Here we report about the study of the features of cosmological parameters and hence the cosmological evolution using the scalar degree of freedom of the $f(R) = \xi R^n$ gravity model in the Friedmann-Lemaître-Robertson-Walker (FLRW) background.

1. Introduction

Although the General Relativity (GR) is a very well-tested and widely accepted theory of gravity, the explanation of the late time accelerated expansion of Universe [1] is out of scope within its ambit. However, because of all the successful test performed so far on GR for the low scales, attempts have been made to modify GR in order to account the late time behaviour of the cosmic expansion as a consequence of the large scale modification of gravity [2,3]. The simplest class of modified gravity theories is the $f(R)$ gravity theories in which Einstein gravity is modified by replacing the Ricci curvature scalar R by an arbitrary curvature function $f(R)$. In the special case when $f(R) \rightarrow R$, $f(R)$ gravity converge to GR without a cosmological constant [2,3]. In last few years the $f(R)$ gravity models have been studied extensively in the cosmological and astrophysical aspects because of a number of very interesting results [4].

It is interesting that a new scalar degree of freedom (sometimes referred as *scalaron*) appears in $f(R)$ theories of gravity due to redefinition of model's variable. The conformal transformation of the metric to the Einstein frame makes it explicit in the action [2,4]. In $f(R)$ gravity the scalar field is coupled to non-relativistic matter (dark matter, baryons) with a universal coupling constant $-1/\sqrt{6}$, and hence it is convenient work on $f(R)$ gravity in the Einstein frame, in which a canonical scalar field is coupled to non-relativistic matter, as far as the scalar field is concerned [2,4]. In this work we have studied the $f(R) = \xi R^n$ gravity model using its scalar degree of freedom in the Einstein frame in the Friedmann-Lemaître-Robertson-Walker background. The basic aim of this study is to see numerically the features of the scalar field coupled to the $f(R)$ gravity model and hence to see the consequent evolution of the cosmological parameters in the Einstein frame. We have considered this model because it has exact power-law solutions and as a result the singularity is not appear in it as suffered by other $f(R)$ models. This manifests in the scalar degree of freedom as the scalar field potential of this model is well behaved function of the field [5]. In Sec.2 we discuss very briefly the basics of formalism of the $f(R)$ gravity theory in the Jordan frame and then we transform the formalism to the Einstein frame using the conformal transformations to developed field equations in terms scalar degree



of freedom. The numerical analysis of the work based on the formalism developed in the Sec.2 is done in the Sec.3 for our model. Finally in the Sec.4 we concludes the results of our analysis.

2. $f(R)$ gravity formalism in Einstein frame

The $f(R)$ gravity action in the Jordan frame is given by [2, 3]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + \int d^4x \mathcal{L}_m(g_{\mu\nu}, \Psi_m), \quad (1)$$

where $\kappa^2 = 8\pi G$, g is the determinant of the metric $g_{\mu\nu}$, and $\mathcal{L}_m$ is the matter Lagrangian, which is the function of the metric $g_{\mu\nu}$ and the matter fields Ψ_m . The variation of the action (1) with respect to $g_{\mu\nu}$ leads to the equation of motion [2]:

$$f' R_{\mu\nu} - \nabla_{\mu\nu} f' + \left(\square f' - \frac{1}{2} f' \right) g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (2)$$

where (ι) denotes the derivatives with respect to R . The term $\square f'$ does not vanish in modified gravity, which means that there is a propagating scalar degree of freedom, defined as, $\varphi = f'$. In practice, it is usually difficult to invert the definition of the scalar degree of freedom explicitly for a given $f(R)$ and only convenient to determine effective potential in a parametric form. However, using the conformal transformation to the Einstein frame it is possible to express the scalar degree of freedom of a given $f(R)$ in the explicit form. In view this we now switch to the Einstein frame under the following conformal transformation [2]:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad (3)$$

where Ω^2 is the conformal factor and the quantities in the Einstein frame are represented by a tilde over them. If we make a choice, $\Omega^2 = f'$, for $f' > 0$ and introduce a new scalar field ϕ defined by $\kappa\phi \equiv \sqrt{3/2} \ln f'$, then under the transformation (3) the action in the Einstein frame can be expressed as [2]

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + \int d^4x \mathcal{L}_m(F^{-1}(\phi) \tilde{g}_{\mu\nu}, \Psi_m), \quad (4)$$

where $V(\phi) = U/f'^2 = (f'R - f)/(2\kappa^2 f'^2)$ is the potential of the field ϕ . Here the conformal factor is $\Omega^2 = f' = \exp(\sqrt{2/3}\kappa\phi)$, which is field dependent. Taking the variation of the action (4) with respect to the field ϕ , we obtain the field equations without the presence of matter as [2]

$$\ddot{\phi} + 3\tilde{H}\dot{\phi} + V_{,\phi} = 0, \quad \tilde{H}^2 = \frac{\kappa^2}{3} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]. \quad (5)$$

Here the dots over ϕ indicate the derivatives with respect to conformal time (i.e. cosmic time in Einstein frame) $\tilde{t}$. $\tilde{H} = \dot{\tilde{a}}/\tilde{a}$ is the Hubble parameter in the Einstein frame, $\tilde{a}(\tilde{t})$ being the scale factor in the same frame in spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with a comonic time-dependent scale factor $a(t)$ in the Jordan frame. Relations of the cosmic time and scale factor in two frames are [2]: $d\tilde{t} = \sqrt{f'} dt$, $\tilde{a} = \sqrt{f'} a$, for $f' > 0$. We finally define the energy density and the pressure of the scalar field as [2]

$$\tilde{\rho}_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad \text{and} \quad \tilde{P}_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \quad (6)$$

It is evident from the above formalism that in Einstein frame the scalar degree of freedom for a given $f(R)$ model can be studied explicitly and conveniently.

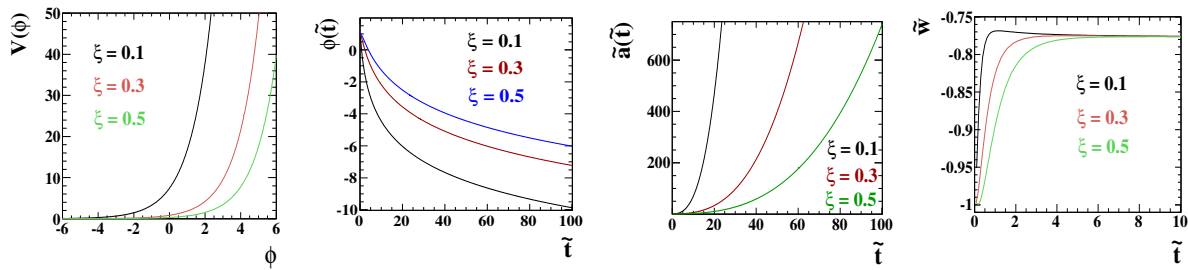


Figure 1. Few numerical results of the work. Here value of κ is taken as one. See [4] for the comprehensive work related to it.

3. Scalar field dynamics of the $f(R) = \xi R^n$ model

Using the formalism of the previous section, in this section we will study numerically different features of the scalar field and consequent cosmological implications of the model, viz.,

$$f(R) = \xi R^n, \quad (7)$$

where ξ and n are model parameters. Here our main emphasis is to see the explicit behavior of scalar degree of freedom possessed by $f(R)$ gravity model and its usefulness to study the cosmological evolution. The scalar field ϕ corresponding to the model (7) can be expressed as

$$\phi = \sqrt{\frac{3}{2}} \frac{1}{\kappa} \ln f' = \sqrt{\frac{3}{2}} \frac{1}{\kappa} \ln(\xi n R^{n-1}). \quad (8)$$

Using this expression for ϕ , the field potential takes the form

$$V(\phi) = \frac{n-1}{2\kappa^2 \xi n^2} \left(\frac{\exp(\sqrt{2/3} \kappa \phi)}{\xi n} \right)^{\frac{2-n}{n-1}}. \quad (9)$$

The first panel of the Fig.1 shows the variation of field potential with respect to the field ϕ for $n = 1.5$ with $\xi = 0.1, 0.3$ and 0.5 . From this panel it is seen that the potential is well behaved function of the field and the potential is small for negative value of the field and it increases very fast as soon as the field ϕ become positive. This process is slowing down when the value of ξ increases, which is more effective for lower value range of ξ . We found that there is no significant difference of the potentials for the different values of n within the range of negative field, but the difference becomes more prominent as soon as the field ϕ acquired the positive values. The effect of n on the value of the potential is similar to that for ξ , but the effect on higher positive value of ϕ is more noticeable in this case than the case for ξ [4].

Using the potential (9), the coupled field equations (5) are solved numerically for different values of n and ξ . The results of the numerical solution are shown in the second panel of the Fig.1 for $n = 1.5$ as an example. As clear from this panel that the value of the field ϕ falls towards to the negative values starting from its initial value as time increases. For the initial period of cosmic time ϕ falls very fast and this tendency slows down as time passage. For higher values of ξ the overall tendency of the ϕ to fall towards the negative values is less in comparison to its lower values. This is similar to the nature of the field potential with respect to ξ . It is found that the field falls faster initially for the smaller value of n as compared to the higher value, but the situation reversed in the latter cosmic times. During the time before the situation is reversed the difference of the field ϕ is not very high, however after this period the difference

become increasingly significant as time elapsed [4]. From these numerical solutions we have calculated the scale factors $\tilde{a}(\tilde{t})$ in Einstein frame for different values of n and ξ . The (cosmic) time evolution of the scale factors obtained from these calculation are shown in the third panel of the Fig.1 for $n = 1.5$ only. It is observed that the $f(R) = \xi R^n$ gravity model leads the very fast expansion of the Universe in the Einstein frame similar to the exponential expansion. The rate of expansion of the Universe slowed down gradually for higher values of ξ in comparison to its lower values. During initial period this process is slow, but become more significant as time passage. On the other hand the expansion is slower for lower value of n in comparison to the case for the higher value of n . In the initial period the difference of the values of the scale factor is very lass, but becomes more prominent in the latter times [4].

Finally we have calculated the equation of state $\tilde{w} = \tilde{\rho}_\phi/\tilde{P}_\phi$ in Einstein frame using equations (6) from the numerical solutions mentioned above, some of results are shown in the last panel of the Fig.1. It is seen that the equation of state $\tilde{w}$ of the scalar field ϕ for the model (7) always less than -0.75 for all cases of our study and for all times. For higher values of ξ , $\tilde{w}$ is smaller than that for smaller value of ξ during period for some initial times, but for latter times $\tilde{w}$ becomes equal for all ξ . For higher value of n , $\tilde{w}$ is always smaller than that for the smaller value of n with a similar trend with respect to time and ξ as mentioned above [4]. As the equation of state is always negative, therefore the model (7) could produced the accelerated expansion of the Universe and it corresponds to dark energy, whose equation of state is $w < -1/3$. If we increase the values of n then the equation of state $\tilde{w}$ in Einstein frame will approach to -1 . This implies that the model (7) with higher values of n (< 2) corresponds to the cosmological constant Λ [4].

4. Conclusion

We have investigated the scalar degree of freedom and it's related cosmological implications of the $f(R)$ gravity model ξR^n with the different values of the model parameters ξ and n . As the scalar degree of freedom of the $f(R)$ gravity model can be treated explicitly in the Einstein frame, so we used this frame for this study by conformal transformations from the Jordan frame. It is found that the field potential is the well behaved function of the field ϕ representing the scalar degree of freedom of the model and increases very fast as ϕ increases, which depends on the model parameters ξ and n such that for the lower value of them such process is faster than higher value. From the numerical simulation of the field equations, we found that ϕ falls towards the negative values as time increases, which is more faster during the initial period of time than late times. The rate of falling of the value of the field is slower for the higher values of the parameter ξ . The study of the time evolution of the scale factor in the Einstein frame shows that the Universe is accelerating very fast for our $f(R)$ gravity model, which depends on the values ξ and n in such a way that accelerating process slows down for higher values of ξ and for lower values of n . The equation of state of the scalar field for the model is always negative and less than $-1/3$, which corresponds to the behaviour of the dark energy that produces the accelerated expansion of the Universe. The equation of state of the field for our model tends to -1 as the value of n increases towards higher values, nearly equal to 2. Thus for such values of n , the scalar field for the model behave as the cosmological constant Λ [4].

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