Journal of Physics: Conference Series **481** (2014) 012008 doi:10.1088/1742-6596/481/1/012008

Deflection of light due to rotating mass - a comparison among the results of different approaches

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Abstract. It is known that light gets deflected due to mass. Value of deflection of light for static body is not same as it is in the case of rotating body. The deflection for a static body entirely depends on the gravitational mass where in case of a rotating body new terms will be included due to rotation. The bending angle of light is also not same in the equatorial plane and non equatorial plane for rotating Kerr body. The light bending angle is also direction of motion dependent i.e. if the motion of the light ray is in the direction of rotation, bending angle is greater than the static case and if the ray is in the opposite direction of rotation, the bending angle is smaller than the static case in equatorial plane. There are two approaches to obtain the bending angle, null geodesic of photon and change of effective refractive index. In this paper a comparison will be made among the results of different approaches.

1. Introduction

General relativity, a theory of gravitation was developed by Einstein from 1907 and finally formulated it in 1915. Two very important consequences of general relativity are bending of light ray in in presence of gravitational field and rotating mass drag the space time around them, a phenomenon called frame dragging. The exact solution of Einstein's field equation for a static, uncharged body was found by Schwarzschild in 1915, for an uncharged rotating body was solved by Kerr in 1963 and for a rotating charged body was found by Newman which is known as Kerr-Newman metric [1, 2].

So there are three factors namely mass, angular momentum and charge which can influence the curvature of space time. As a direct consequence of it, the amount of deflection of light ray would also be influenced by these factors. Many works have been done on the deflection of light due to static, rotating and charged rotating body by using two approaches, null geodesic of photon approach and effective refractive index of material medium approach. In this paper a comparison will be made among the results of different approaches to obtain the bending angle for static and rotating mass.

2. Bending of light due to static mass using null geodesic approach

In the year of 1911, a prediction was published by Einstein for bending angle of light ray from a distance source passing close to the sun at minimum distance R would get deflected with an angle.

$$\alpha = \frac{4GM}{c^2R}, \tag{1}$$

where G is gravitational constant, M is the mass of sun and c is the speed of light.

The observational verification of his prediction was made in the year 1919 during total solar eclipse. For the sun the angle of deflection of light was 1.75 sec. [1, 2]. The light deflection due to a static charged body was obtained by Virbhadra, Narasimher & Chitre up to second order term [4]

$$\alpha = \frac{4GM}{c^2 r_0} + \frac{4G^2M^2}{C^4 r_0^2}\left(\frac{15\pi}{16} - 2\right) + \frac{2}{r_0^2}\left\{2\frac{GM}{c^2}\left[\left(\frac{GM}{c^2}\right)^2 + q^2\right]^{\frac{1}{2}} - \frac{q^2\pi}{8}\right\}, \tag{2}$$

where q is the charge of the body, the bending angle for static body would be obtained by setting q equals zero

$$\alpha = \frac{4GM}{c^2 r_0} + \frac{4G^2M^2}{C^4 r_0^2}\left(\frac{15\pi}{16} - 2\right) + \frac{4G^2M^2}{C^4 r_0^2} \tag{3}$$

Higher order terms obtained by Keeton & Petter [5]

$$\alpha = 4h + \left(-4 + \frac{15\pi}{4}\right)h^2 + \left(\frac{122}{3} - \frac{15\pi}{2}\right)h^3 + \left(-130 + \frac{3465\pi}{64}\right)h^4 + \left(\frac{7783}{10} - \frac{3465\pi}{16}\right)h^5 + \left(-\frac{21397}{6} + \frac{310695\pi}{256}\right)h^6 + \mathrm{O}(h)^7 \tag{4}$$

Where,

$$h = \frac{m_0}{r_0}$$

r_0 =closest approach and

$$m_0 = \frac{GM}{c^2}$$

In terms of impact parameter b,

$$\alpha(b) = 4\left(\frac{m_0}{b}\right) + \frac{15\pi}{4}\left(\frac{m_0}{b}\right)^2 + \frac{128}{3}\left(\frac{m_0}{b}\right)^3 + \frac{3465\pi}{64}\left(\frac{m_0}{b}\right)^4 + \frac{3584}{5}\left(\frac{m_0}{b}\right)^5 + \frac{255255\pi}{256}\left(\frac{m_0}{b}\right)^6 + \mathrm{O}\left(\frac{m_0}{b}\right)^7 \tag{5}$$

An analytical perturbation frame work was developed by Iyer & Petter [6] to obtain the bending angle of light due static black hole which is

$$\alpha = 4h + \left(-4 + \frac{15\pi}{4}\right)h^2 + \left(\frac{122}{3} - \frac{15\pi}{2}\right)h^3 + \left(-130 + \frac{3465\pi}{64}\right)h^4 + \left(\frac{7783}{10} - \frac{3465\pi}{16}\right)h^5 + \left(-\frac{21397}{6} + \frac{310695\pi}{256}\right)h^6 + \mathrm{O}(h)^7 \tag{6}$$

Equation (6) is similar with equation (4)

2.1. Bending of light due to static mass using material medium approach

Fishback & Freeman obtained the bending angle of light up to first order term for static body using effective refractive index of material medium approach [3]

$$(\varphi)^1 = \frac{2(1+\gamma)GM}{bc^2} \tag{7}$$

For $\gamma=1$ this expression is similar to Einsteins expression of bending angle. More exact expression for gravitational deflection of light had been given by Sen (2010) [9] using the material medium approach and the deflection angle for static mass was obtained as

$$\Delta\phi = 2\frac{r_0^2}{r_g(r_0-r_g)}\int_0^{\frac{r_g}{r_0}} x\left\{1-\left[\frac{r_0^2}{r_g(r_0-r_g)}\right]^2 x^2(1-x)^2\right\}^{-\frac{1}{2}} dx \tag{8}$$

Where $x = r_g / r$ and r_0 =closest approach and for weak field this expression reduce to standard form

$$\Delta\varphi = \frac{4GM}{c^2 r_0} \tag{9}$$

It can be said that the results are similar for both null geodesic of photon and effective refractive index of material medium approach.

3. Bending angle of light due to rotating mass.

In general relativity the space time around a body is described by Kerr metric. According to this metric a rotating body drag the space-time around it and every object coming close to the rotating body entrained to participate in its rotation. This effect is called frame dragging. The rotational frame dragging effect was first derived from the theory of general relativity by two Austrian physicists Josef Lense and Hans Thirring [1,2], which was known as Lense-Thirring effect. The deflection produced in presence of a rotating black hole explicitly depends on direction of light. Compared to the zero spin Schwarzschild case, the bending angle was greater for direct orbit and smaller for retrograde orbits. Exact bending angle for rotating black hole in the equatorial plane obtained by Iyer & Hansen is [7, 8].

$$\begin{aligned}\alpha(b) = {} & 4\left(\frac{m_0}{b}\right)+\left(\frac{15\pi}{4}-4sa\right)\left(\frac{m_0}{b}\right)^2+\left(\frac{128}{3}-10\pi sa+4a^2\right)\left(\frac{m_0}{b}\right)^3+ \\ & \left(\frac{3465\pi}{64}-192sa+\frac{285\pi}{16}a^2-4sa^3\right)\left(\frac{m_0}{b}\right)^4\end{aligned} \tag{10}$$

where s is +1 for direct and -1 for retrograde motion and

$$a = \frac{j}{Mc}$$

and j is the angular momentum.

Azami, Keeton & Petter [10, 11] have shown that for off equatorial light ray bending angle has two components. One is in the equatorial plane and another is perpendicular to the equatorial plane. Equatorial component is

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$$\alpha(b) = 4\left(\frac{m_0}{b}\right) + \left(\frac{15\pi}{4} - 4sa\right)\left(\frac{m_0}{b}\right)^2 + \left(\frac{128}{3} - 10\pi sa + 4a^2\right)\left(\frac{m_0}{b}\right)^3 + \left(\frac{3465\pi}{64} - 192sa + \frac{285\pi}{16}a^2 - 4sa^3\right)\left(\frac{m_0}{b}\right)^4 \tag{11}$$

This expression in equation (11) is similar to the expression obtained by Iyer and Hansen [7] shown in equation (10). Both expression reduce to Schwarzschild series if a is set to zero.

4. Conclusions

From above discussion three points are clear that whatever is the method (null geodesic of photon or effective refractive index of material medium) the expression of light deflection is same in case of static gravitational mass. Equatorial component of quasi-equatorial bending angle is similar to equatorial bending angle and for zero spin it reduces to the bending angle expression for static body. Spin does not have any contribution to the first order term.

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