

New frontiers in neutrino physics

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Abstract. The origin of neutrino masses and mixings is an important topic in high energy physics. In the present work, two mixing schemes namely Bi-maximal (BL) and Tri-bimaximal (TBM) mixings which are derived from $\mu-\tau$ symmetric neutrino mass matrix, are examined after taking corrections from charged lepton sector. While tri-bimaximal mixing with appropriate charged lepton correction, is in good agreement with latest observations, the Bi-maximal mixing after taking charged lepton correction, fails to agree completely with observation. A new scheme called strict Bi-large (BL) mixing scheme is introduced in place of BM mixing scheme. The BL scheme works very well in complete agreement with data. Some of the unsettled issues in neutrino physics are outlined for further investigations.

1. Introduction

Recent observations [1] on neutrino oscillation indicate large values of $\sin^2 \theta_{13}$ i.e., $\theta_{13} \sim \theta_c / \sqrt{2}$. The question being asked is: Is it accidental or manifestation of some underlying quark-lepton complementarity (QLC) relations? Further, the recent data [2] also shows certain indication of a mild deviation of θ_{23} mixing from the maximal $\tan^2 \theta_{23} = 1.0$ i.e., $d_{23} = 0.5 - \sin^2 \theta_{23} \sim 0.1$. Significant deviation of θ_{12} from tri-bimaximal mixing ($\tan^2 \theta_{12} = 0.5$) is an important issue in search for the nature of U_{PMNS} matrix. We have also witnessed the first glimpse on the CP violating Dirac phase from the global fit, and a possibility of theoretical predictions for δ_{CP} is an exciting field of study. Since the leptonic mixing angles are relatively large, it is important to explore more detailed searches on large CP-violation effect in the leptonic sector.

2. CKM matrix in quark physics

We first describe the quark mixing matrix known as CKM matrix [3], for possible comparison with the corresponding leptonic mixing matrix known as PMNS matrix [4]. This connection is illustrated by a simple process like β decay. The β -decay process is represented as $n \rightarrow p + e^- + \bar{\nu}_e$. In term of quarks, it is replaced by $d \rightarrow u + e^- + \bar{\nu}_e$.

The correspondence between the mass-eigenstates and weak-flavour eigenstates, leads to the quark mixing matrix known as CKM matrix, and this is illustrated as follows,

$$\left. \begin{aligned} \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix} &\Rightarrow \begin{pmatrix} u & c & t \\ d' & s' & b' \end{pmatrix} \\ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} &= U_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \end{aligned} \right\} \quad (1)$$



where the CKM matrix is given by a 3×3 unitary matrix,

$$\mathbf{U}_{\text{CKM}} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}. \quad (2)$$

Parametrization [3] of CKM matrix can be done by the product of three rotational matrices: $\theta_{23}, \theta_{13}, \theta_{12}$ leading to $U_{\text{CKM}} = R_{23}R_{13}R_{12}$.

The final form of U_{CKM} is given by

$$\mathbf{U}_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (3)$$

In term of Wolfenstein's parametrization [3], CKM matrix takes the form,

$$\mathbf{U}_{\text{CKM}} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (4)$$

where, $\lambda \approx 0.22, |A| \approx 0.9 \pm 0.18, \eta \ll 0, \tan \theta_{12} = |\frac{V_{us}}{V_{ud}}|, \tan \theta_{23} = |\frac{V_{cb}}{V_{tb}}|, \sin \theta_{13} = |V_{ub}|; s_{13} \approx 0, \sin \theta_{12} \approx \lambda \approx 0.22; \theta_{12} = \theta^c = 12.6^\circ \pm 0.1^\circ$.

3. Neutrino mixing from neutrino oscillation

In order to explain the neutrino mixings derived from neutrino oscillation phenomenon, we hypothesise the existence of mass eigenstate as well as flavour eigenstate in leptonic sector in analogy with quark sector. This is illustrated as follows,

$$\begin{pmatrix} e^- & \mu^- & \tau^- \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \Rightarrow \begin{pmatrix} e^- & \mu^- & \tau^- \\ \nu_e & \nu_\mu & \nu_\tau \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

which leads to the PMNS mixing matrix,

$$\mathbf{U}_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = R_{23}R_{13}R_{12}. \quad (5)$$

Though the parametrization is through three rotations like CKM matrix, the values of three angles and phase are different in both cases. In case of PMNS matrix, the three angles are relatively large. They are extracted from neutrino oscillation experiments. Probability of neutrino oscillation from $\nu_e \rightarrow \nu_\mu$ is given by

$$P(t)_{\nu_e \rightarrow \nu_\mu} = \sin^2(2\theta_{12}) \sin^2 \left[1.267 \Delta m_{21}^2 \frac{L}{E} \right]$$

where, θ_{12} is the solar mixing angle, L is in metre, E in MeV , Δm_{21}^2 in eV^2 . In the structure of U_{PMNS} , unlike CKM matrix, there are two complex Majorana phases represented by

$$P = \begin{pmatrix} e^{i\theta_1} & 0 & 0 \\ 0 & e^{i\theta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

if neutrino masses are Majorana type. Majorana fermions are self-conjugate particles and neutrino could satisfy this condition. Thus $U_{\text{PMNS}} = R_{23}R_{13}R_{12} \cdot P$. The structure of U_{PMNS} takes the following form,

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P \quad (7)$$

In order to construct the PMNS mixing matrix, we have to get the diagonalizing matrices from neutrino mass matrix as well as charged lepton mass matrix in analogy to the construction of CKM matrix. Two conventions [5-8] of diagonalization are adopted. We follow a convention which gives the following relations: $U_{\text{CKM}} = U_{uL}^\dagger U_{dL}$, $m_e = U_{eL} m_e^{\text{diag}} U_{eR}^\dagger$, $U_{\text{PMNS}} = U_e^\dagger U_\nu$ and $m_\nu = U_\nu^* m_\nu^{\text{diag}} U_\nu^\dagger$. The neutrino mass matrix is expressible in the basis where charged lepton mass matrix is diagonal, $m'_\nu = (U_{eL}^\dagger m_\nu U_{eL})$.

4. Extracting U_{PMNS} matrix

The usual ways of getting neutrino mass matrix is from left-right symmetric GUT models such as SO(10) GUT, through celebrated seesaw formula [9], $m_{LL} = m_{LR} m_{RR}^{-1} m_{LR}^\dagger$. Another possible source of neutrino mass matrix is from certain discrete symmetries like $\mu - \tau$ symmetry [10]. The $\mu - \tau$ symmetric mass matrix leads to the following PMNS mixing matrix where solar mixing angle is arbitrary,

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ \frac{-s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & \frac{-c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (8)$$

When $s_{12} = \frac{1}{\sqrt{2}}$, $\tan^2 \theta_{12} = 1.0$ leads to Bimaximal [11] mixing matrix (BM)

$$U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (9)$$

When $s_{12} = \frac{1}{\sqrt{3}}$, $\tan^2 \theta_{12} = 0.5$ leads to tri-bimaximal [12] mixing matrix (TBM)

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{1}{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (10)$$

The validity of both BM as well as TBM will be examined with appropriate charged lepton corrections. This is the central issue of the present investigation.

5. Charged lepton correction to BM mixing with CP phase

To rescue the Bimaximal mixing (BM), we follow the procedure of taking charged lepton correction in two ways [13] for introducing the Dirac CP Phase. Thus

A. Dirac CP Phase through Charged lepton mixing matrix (U_{el}) :

$$U_{el} = \begin{pmatrix} \tilde{c}_{12} & \tilde{s}_{12}e^{-i\phi} & 0 \\ -\tilde{s}_{12}e^{i\phi} & \tilde{c}_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11)$$

and this leads to neutrino mixing matrix ($U_{\text{PMNS}} = U_{eL}^\dagger U_{BM}$),

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{c}_{12} + \frac{\tilde{s}_{12}}{\sqrt{2}}e^{i\phi}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12} - \frac{\tilde{s}_{12}}{\sqrt{2}}e^{i\phi}) & -\frac{\tilde{s}_{12}}{\sqrt{2}}e^{i\phi} \\ -\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}e^{-i\phi}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}e^{-i\phi}) & \frac{\tilde{c}_{12}}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (12)$$

B. Dirac CP phase through neutrino mass sector:

$$U_{\text{PMNS}} = U_{eL}^\dagger R_{23} \text{Diag}(e^{i\delta}, 1, e^{-i\delta}) R_{12}. \quad (13)$$

Using $\theta_{23} = \frac{\pi}{4}$ and $\theta_{12} = \frac{\pi}{4}$ the complete PMNS mixing matrix is given by:

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{c}_{12}e^{i\delta} + \frac{\tilde{s}_{12}}{\sqrt{2}}) & \frac{1}{\sqrt{2}}(\tilde{c}_{12}e^{i\delta} - \frac{\tilde{s}_{12}}{\sqrt{2}}) & -\frac{\tilde{s}_{12}}{\sqrt{2}}e^{-i\delta} \\ -\frac{1}{2}(\tilde{c}_{12} - \sqrt{2}\tilde{s}_{12}e^{i\delta}) & \frac{1}{2}(\tilde{c}_{12} + \sqrt{2}\tilde{s}_{12}e^{i\delta}) & \frac{\tilde{c}_{12}}{\sqrt{2}}e^{-i\delta} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}e^{-i\delta} \end{pmatrix}. \quad (14)$$

Amethod : (15)

$$J_{\text{CP}}^{BM} = \frac{1}{4\sqrt{2}} \sin \tilde{\theta}_{12} \cos \tilde{\theta}_{12} \sin \phi \quad (16)$$

(17)

$$\tan^2 \theta_{12} = \frac{2 - \tilde{s}_{12}^2 - 2\sqrt{2}\tilde{c}_{12}\tilde{s}_{12} \cos \phi}{2 - \tilde{s}_{12}^2 + 2\sqrt{2}\tilde{c}_{12}\tilde{s}_{12} \cos \phi} \quad (18)$$

Bmethod : (19)

$$J_{\text{CP}}^{BM} = \frac{1}{4\sqrt{2}} \sin \tilde{\theta}_{12} \cos \tilde{\theta}_{12} \sin \delta \quad (20)$$

(21)

$$\tan^2 \theta_{12} = \frac{2 - \tilde{s}_{12}^2 - 2\sqrt{2}\tilde{c}_{12}\tilde{s}_{12} \cos \delta}{2 - \tilde{s}_{12}^2 + 2\sqrt{2}\tilde{c}_{12}\tilde{s}_{12} \cos \delta} \quad (22)$$

(23)

Both methods give the same expressions. For $\delta \neq 0$ the prediction on $\tan^2 \theta_{12}$ cannot accomodate global data. However the prediction on atmospheric angle agrees with data [13].

Using $\sin \tilde{\theta}_{12} = \sqrt{2} \sin \theta_{13}$ along with the approximation $\cos \tilde{\theta}_{12} \approx 1$ gives

$$J_{\text{CP}}^{BM} \approx \frac{1}{4} \sin \theta_{13} \sin \delta.$$

6. Charged lepton correction to Tri-bimaximal mixing with and without Dirac CP phase

In order to rescue the TBM we suggest the following charged leptonic mixing matrix satisfying unitarity condition

$$\tilde{U}_{el} = \begin{pmatrix} 1 - \frac{\lambda^2}{4} & -\frac{\lambda}{2} & -\frac{\lambda}{2} \\ \frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\ \frac{\lambda}{2} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8} \end{pmatrix} \quad (24)$$

Using $U_{\text{PMNS}} = \tilde{U}_{el}^\dagger U_{\text{TBM}}$ we get the complete PMNS mixing matrix,

$$U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}}(1 + \lambda) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}}(1 - \lambda) & -\frac{1}{\sqrt{3}}(1 + \frac{\lambda}{2}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \end{pmatrix}. \quad (25)$$

However in order to achieve $\tan^2 \theta_{23} < 1$, we modify the charged lepton mixing matrix \tilde{U}_{el} as

$$U_{el}^\dagger = \tilde{R}_{23}^\dagger \tilde{U}_{el}^\dagger$$

where \tilde{R}_{23} is given by

$$\tilde{R}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \tilde{c}_{23} & \tilde{s}_{23} \\ 0 & -\tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}, S_{23}^2 = A\lambda^2 \approx 0.041. \quad (26)$$

This leads to the following predictions

$$\tan^2 \theta_{12} = 0.5 \quad (27)$$

$$\tan^2 \theta_{23} = \left(\frac{1 - \tan \tilde{\theta}_{23}}{1 + \tan \tilde{\theta}_{23}} \right)^2 < 1 \quad (28)$$

$$\sin \theta_{13} = \frac{\lambda}{\sqrt{2}} \approx 0.146. \quad (29)$$

In order to incorporate Dirac type CP effect in TBM scheme, we use [13,14]

$$U_{TBC} = \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{2}}e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + \lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2}e^{i\delta}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}}(1 - \lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1 + \frac{\lambda}{2}e^{i\delta}) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda^2}{4}) \end{pmatrix} \quad (30)$$

where the complete PMNS mixing matrix is given by $U_{\text{PMNS}} = \tilde{R}_{23}^\dagger U_{TBC}$ with phase. Here the prediction on $\tan^2 \theta_{12}, \tan^2 \theta_{23}, \sin^2 \theta_{13}$ are free from CP phase δ . And we also obtain

$$J_{\text{CP}}^{TB} \approx \frac{1}{3\sqrt{2}} \sin \theta_{13} \sin \delta.$$

The desired deviation of atmospheric angle from maximal value is obtained. However the deviation of solar angle from TBM value is not possible here to explain the data.

6.1. A new approach with the changing of the position of phase in TBM case

Following the suggestion from Fritzsch and Xing [15], U_{13} is replaced by U'_{13} , resulting a new TBM matrix U'_{TBM} ,

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \quad (31)$$

$$U'_{13} = \begin{pmatrix} c_{13}e^{i\delta} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13}e^{-i\delta} \end{pmatrix} \quad (32)$$

$$U'_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}}e^{i\delta} & \frac{1}{\sqrt{3}}e^{i\delta} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}e^{-i\delta} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}e^{-i\delta} \end{pmatrix} \quad (33)$$

After taking CKM type charged lepton correction [16] we get

$$U_{\text{PMNS}} = U_l^\dagger U'_{\text{TBM}} = U_{\text{CKM}}^\dagger U'_{\text{TBM}}.$$

which predicts the best fit value $\tan^2 \theta_{12} = 0.47$ and $J_{\text{CP}} = 0.0350$ for $\cos \delta \approx 0.147$. The prediction on atmospheric angle is in good agreement [16] with observation. If $\cos \delta = 1$ we get only $\tan^2 \theta_{12} \approx 0.232$ which is very less as compared to data. The predictions of other two angles are unaffected by the phases i.e, $\sin^2 \theta_{13} = 0.024$ and $\tan^2 \theta_{23} = 0.80$. The CP violating parameter J_{CP} is also predicted with right order [16].

7. Strict Bi-large mixing with charged lepton correction

The problem in Bi-maximal mixing to produce correct solar mixing, is rectified with the ansatz of strict Bi-Large mixing [17] having the following choices,

$$\sin \theta_{13} = \lambda_c, \quad \sin \theta_{12} = 3\lambda_c, \quad \sin \theta_{23} = 3\lambda_c \quad . \quad (34)$$

where $\lambda_c \approx 0.22$. Strict BL mixing matrix thus takes the form [17],

$$U_{BL} = \begin{pmatrix} \frac{3}{4}(1 - \frac{\lambda_c^2}{2}) & \frac{\sqrt{7}}{4}(1 - \frac{\lambda_c^2}{2}) & \lambda_c \\ -\frac{3\sqrt{7}}{16}(1 + \lambda_c) & \frac{9}{16}(1 - \frac{7}{9}\lambda_c) & \frac{\sqrt{7}}{4}(1 - \frac{\lambda_c^2}{2}) \\ \frac{7}{16}(1 - \frac{9}{7}\lambda_c) & -\frac{3\sqrt{7}}{16}(1 + \lambda_c) & \frac{3}{4}(1 - \frac{\lambda_c^2}{2}) \end{pmatrix} \quad (35)$$

After inclusion of CKM-type charged lepton correction,

$$U_{eL} = \begin{pmatrix} 1 - \frac{\lambda_c^2}{2} & \lambda_c e^{i\delta_{cp}} & 0 \\ -\lambda_c e^{-i\delta_{cp}} & 1 - \frac{\lambda_c^2}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (36)$$

we get the following predictions,

$$\sin^2 \theta_{13} = 0.0245, \sin^2 \theta_{12} = 0.3209, \sin^2 \theta_{23} = 0.4533,$$

$$\delta_{\text{CP}} = 0.2515\pi, J_{\text{CP}}^{BL} \approx 0.0216$$

The predictions [17] of BL mixings on solar, atmospheric and Chooz mixing angles, are in agreement with recent data. Bilarge Mixing can be explained from certain underlying discrete symmetry in string theory.

8. Conclusions

To summarise, we have described the analogy between the CKM and PMNS matrix through the existence of both mass eigenstates and flavour eigenstates. The parametrizations in both cases are presented in terms of three rotations. In case of PMNS mixing matrix, the Bimaximal mixing matrix is nearly ruled out [13,17] even after charged lepton correction is included. Bilarge mixing matrix [17] or similar modification may be introduced to meet the challenges. Tri-bimaximal mixing is very much consistent with charged lepton correction, with or without phase contribution [13,16].

Some of the pending major issues on neutrino physics are - nature of neutrino mass: Dirac or Majorana nature [18]; pattern of absolute neutrino masses: normal, inverted or quasi-degenerate type [19]; possible existence of sterile neutrinos: a fourth state of neutrino flavour etc. The connection between the neutrinos and the Dark side [20] of the universe will continue to be an exciting research area [21].

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