Journal of Physics: Conference Series **481** (2014) 012001 doi:10.1088/1742-6596/481/1/012001

Partons at small x

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Abstract. We give a brief summary of the work on partons at small Bjorken x pursued by us at Gauhati University in recent years. We use both conventional DGLAP method and not so conventional self-similarity inspired method. In DGLAP approach, we solve the QCD evolution equations approximately at small x by two methods- Lagrange's and method of characteristics. We apply them to both spin dependent and spin independent structure functions. In the second approach, we use the notion of self-similarity of fractal geometry to construct Transverse Momentum Dependent Parton Distributions (TMD's), integrated Parton Densities (PDF's) and determine them from HERA collider data. We then use them to compute fractions of momentum carried by quarks and gluons inside the proton and compare with QCD expectations. In view of topical importance of multi parton interactions at LHC at TeV scale, we also outline the construction of Double Parton Distributions (dPDF's) with self-similarity at small x_1, x_2.

1. Introduction

In deep inelastic e-p scattering, x denotes the fraction of momentum carried by partons in a nucleon. Small x ($x \ll 1$) partons were coined "wee" partons by Feynman more than forty years ago in pre QCD era, in late 1960's and early 1970's. Since then the subject attracted immense interest both theoretically and experimentally. Both in the experiments at lepton hadron collider "HERA" at DESY, Hamburg and the present LHC at CERN, Geneva, the subject has been attracting keen attention. [1-3]. As noted in the abstract, the paper summaries the specific aspects of small x partons. In section 2, we summarise the DGLAP based models while the section 3 is devoted to self-similarity based model of parton at small x.

2. DGLAP based models

In this approach, the Q^2 evolution of the parton distribution functions (PDF) or the structure functions are determined by a set of integro-differential equation in variable $t = \log \frac{Q^2}{\Lambda^2}$ known as DGLAP equation [4-6] named after the authors. In more recent years, several alternative evolution equations have also been suggested like BFKL [7], GLR [8], BK [9], and JIMWLK [10]. Each of them has varied degree of theoretical refinement compared to DGLAP. But at phenomenological level, DGLAP appears to be the most popular one both at HERA and LHC regimes.

As early as 1987 [11], a programme of obtaining approximate analytical solutions of these equations was initiated, focussing on small x in later years [12].

Neglecting the quark part and assuming factorization of x and Q^2, as

$$G(x, Q^2) = g(x).h(Q^2) \tag{1}$$

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Sometimes back we obtained [13] the following gluon distributions:

$$G(x,t) = G(x,t_0)\left(\frac{t}{t_0}\right)^{p(x)} \tag{2}$$

with

$$p(x) = \frac{36}{25}\left\{\left(\frac{11}{12} - \frac{n_f}{18}\right) + ln(1-x) + I_g(x)\right\} \tag{3}$$

where the integral $I_g(x)$ is explicit function of x.

Using the standard approximate relation between the gluon and the longitudinal structure function $F_L(x,Q^2)$ [14], we also computed $F_L(x,Q^2)$ and results came close to data.

A new relation between the gluon and the slope of the structure function [15] derived by us

$$\frac{\partial F_2(x,Q^2)}{\partial \log Q^2} = \frac{5\alpha_s}{3\pi} G\left(\frac{4}{3}x\right) \tag{4}$$

was an improvement over the similar relation by Prytz [16] earlier.

DGLAP equations are differential equations in variable $t = \log\frac{Q^2}{\Lambda^2}$. Using a Taylor series approximation at small x, it was later shown [17] that these equations can be re-expressed as differential equations in two variables x and t of the form :

$$Q(x,t)\frac{\partial F_2(x,t)}{\partial t} + P(x,t)\frac{\partial F_2(x,t)}{\partial x} = R(x,t) \tag{5}$$

where $Q(x,t), P(x,t)$ and $R(x,t)$ are calculable.

There are two standard methods of solutions of such equations: (a) Lagrange's method [18] and (b) method of characteristics. [19] Our analysis with the Lagrange's method indicates that the solutions are not unique. While earlier study suggests [17,20],

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)\left(\frac{t}{t_0}\right), \tag{6}$$

a general analysis in later years [21] indicates that equation(6) is a particular case of the most general solution at small x.

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{n(x,t)}, \qquad n > 0 \tag{7}$$

Our results achieved by the method of characteristics, on the other hand, suggests that the gluon density at small x [22] is given by

$$G(x,t) = G(x)\left(\frac{t}{t_0}\right)^{l(x,t)} \tag{8}$$

where, $l(x,t)$ is explicitly calculable. More recently, tensor structure function $b_1^d(x,Q^2)$ of the deuteron has been computed [23] using the method of characteristics and considering both NLO and NNLO effects using the relation

$$b_1^d(x,Q^2) = -\frac{3}{2}A_{zz}^d F_1^d(x,Q^2) \tag{9}$$

where the asymmetry factor A_{zz}^d is taken from HERMES data [24]. The results in ref [23] indicates that the predicted behaviour of b_1^d is in good agreement with experiment within the limits of uncertainty. A comparative analysis of the polarised structure function $g_1^{NS}(x,Q^2)$ and the unpolarised one $F_2^{NS}(x,Q^2)$ with the two methods have also been recently reported [25,26] besides a similar one by a new method: the method of successive approximation [27].

Table 1. Values of Q^2 for different values of allowed lower and upper bounds of quark and gluon momentum fractions [38]

Lower bounds of $\langle\widehat{x}\rangle_q$	Upper bounds of $\langle\widehat{x}\rangle_g$	Q^2 in GeV^2 $a=1$	$a=3.1418$
$\frac{9}{25}$	$\frac{16}{25}$	5.97×10^{7}	4.48×10^{1}
$\frac{3}{7}$	$\frac{4}{7}$	3.077×10^{9}	1.57×10^{2}
$\frac{15}{31}$	$\frac{16}{31}$	7.37×10^{10}	4.32×10^{2}
$\frac{1}{2}$	$\frac{1}{2}$	1.86×10^{11}	5.80×10^{2}
$\frac{9}{17}$	$\frac{8}{17}$	1.01×10^{12}	9.94×10^{2}
$\frac{9}{13}$	$\frac{4}{13}$	1.17×10^{16}	1.95×10^{4}
$\frac{3}{4}$	$\frac{1}{4}$	3.22×10^{17}	5.62×10^{4}
$\frac{15}{19}$	$\frac{4}{19}$	3.11×10^{18}	1.16×10^{5}
$\frac{9}{11}$	$\frac{1}{11}$	1.62×10^{19}	1.95×10^{5}
1	0	5.58×10^{23}	5.43×10^{6}

3. Self-similarity based models

Self-similarity is an inherent property of fractals [28]. Since 1980's, the notion of fractals has found its application in high energy physics through the self-similar nature of hadron multi-particle distribution [29,30]. Relevance of these ideas in deep-inelastic scattering was first noted by Dremin and Levtchenko [31]. Later, Lastovicka [32] suggested how fractality can be used to construct PDF's and structure function at small x. Choosing the relevant scaling variables as $\frac{1}{x}$ and $(1+Q^2/Q_0^2)$, following form of PDF was obtained :

$$q_i(x,Q^2)=\frac{e^{D_0^i}\,Q_0^2\,x^{-D_2}}{1+D_3+D_1 ln\frac{1}{x}}\left[\left(\frac{1}{x}\right)^{D_1 ln\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{D_3+1}-1\right] \tag{10}$$

where the parameters $D_0^i,\ Q_0^2,\ D_1,\ D_2,\ D_3$ are model parameters fitted from HERA data [33].

Later, an alternative parameters of the PDF was suggested [34] and applied to Ultra High Energy Neutrino-Nucleon collision [35].

The model of equation(10) can be used to calculate the fraction of momentum carried by quarks and gluons using the momentum sum rule [36] and assuming its validity for $0<x<1$. A recent analysis [37] indicates that in such a model, fraction of momentum of quarks $\langle x\rangle_q$ increases with Q^2 while that of gluons $\langle x\rangle_g$ decreases. At $Q^2=45\ \mathrm{GeV}^2$, on the other hand $\langle x\rangle_q\sim1$ suggests the saturation of the momentum sum rule. In a more recent work [38] $\langle x\rangle_q$ and $\langle x\rangle_g$ were re-estimated for partons within the limited x-range $6.2\times10^{-7}\le x\le10^{-2}$ [32]. It leads to lower bounds $\langle x\rangle_q$ and upper bounds $\langle x\rangle_g$ in the model [Table1]. In the table1, $a=1$ and $a=3.1418$ corresponds to integrally and fractionally charged partons. In this case, the momentum sum rule saturates at an energy scale beyond the reach of even ultra high energy neutrinos.

Let us now discuss the implication of the models at LHC. At hadron collider like LHC, each hadron is described as a collection of essentially free elementary constituents. The interactions between constituents belonging to different colliding hadrons are the seeds of multi-partonic interaction (MPI). Due to the composite nature of each hadron, it is possible to have multiple parton hard scattering: i.e. events in which two or more distinct hard parton scattering occur simultaneously in a single hadron-hadron collision. In such a case, double parton scattering (DPS) plays an important role. Experimentally, such DPS leads to four jet events at hadron colliders. CMS collaboration and ATLAS collaboration are already studying such processes [39].

In QCD, to study such DPS, one needs double PDFs (dPDFs) of each hadron and their evolution. Unlike conventional PDFs, DGLAP equations are generalised to double DGLAP [40] equations to study evolution of dPDFs.

Recently, we have suggested a model of dPDFs based on self-similarity at small x_1 and x_2 [41]. The model contains total thirteen parameters to be fitted from the data at LHC. It is also shown that the constructed dPDF does not factorise into the single PDFs in conformation with QCD expectation. It also satisfies the condition that at the kinematic boundary $x_1 + x_2 = 1$ (where x_1 and x_2 are the longitudinal fractional momentum of two interacting partons), the dPDF vanishes and its simplest form under plausible assumptions is

$$D_{ij}(x_1, x_2) \sim \left(\frac{1}{x_1}\right)^{D_1 \log \frac{1}{x_2}} \left(\frac{1-(x_1+x_2)}{x_1+x_2}\right)^{D_2} \tag{11}$$

where D_1 and D_2 are the model parameters to be fitted. Further work is under progress [42] to make the self-similarity based models compatible with DGLAP evolution as well as Froissart saturation.

4. Acknowledgments

I thank the organisers of the NC-HEPC, Gauhati University for inviting me to deliver the talk. I also thank the collaborators who contributed at several stages of the developments of the present work spending more than two decades. Finally I acknowledge the help of Ms Baishali Saikia in preparing the manuscript. This talk is dedicated to the memory of Prof S. A. S. Ahmed(1948-2012).

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