

Characteristics of potential well for nondegenerate electronic gas in a equilibrium dusty plasma

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Abstract. Calculated of the potential well for electrons of the dust-electron plasma depending on the temperature, size of the dust particles, the distance between them and the kind of material of which they consist. The shape of the potential well is significantly different from the rectangular approximation.

1. Introduction

Dusty plasmas in which the concentration of ionized gas atoms is negligible compared to the concentration of electrons, called dust-electron plasma [1-6]. Such a plasma is formed during combustion of various fuels at atmospheric pressure, plasma chemical reactors, MHD generator channels and several processes for obtaining the functional coating with the gas discharge [7,8]. At the temperature of the working gas from 1000 K to 2000 K, solid or liquid dust particles are positively charged as a result of electron emission from the surface and therefore the gas phase contains free electrons. The presence of charged dust particles leads to an uneven distribution of electric potential and the formation of potential wells for electrons.

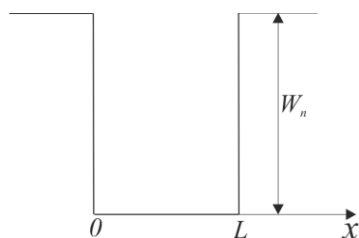


Fig. 1.

Distribution of potential and concentration of electrons inside and around the dust particles depend on the parameters of the potential well. Usually in the approximate calculations believe that the potential well is as shown in Fig. 1, where L – characteristic linear dimension of the solid particle, W_n – the depth of the potential well. It is assumed that in the areas $x < 0$ and $x > L$ free electron concentration is equal to zero.

We show that this approach has some major drawbacks. Static equilibrium condition of the electron gas can be written as [9]

$$\mu - q\phi = \text{const}, \quad (1)$$

where μ - Fermi energy, q – the absolute value of the electron charge, ϕ - the electric potential.

We write the Gauss theorem to the electrostatic field

$$\iint_S E_\psi dS = \frac{Q}{\epsilon \epsilon_0}, \quad (2)$$

where ε - относительная диэлектрическая проницаемость, ε_0 - relative dielectric permittivity, ε_0 - electric constant, $E_\psi = E \cos \alpha$ - the projection of the electric field on the external normal, Q - charge inside the surface S . If the S take the surface of a dust particle in the form of a sphere of radius R , then equation (2) takes the form

$$4\pi R^2 \varepsilon_0 E(R) = Q, \quad (3)$$

If the electrons are only inside the particles in $r \leq R$, space charge Q is zero. Then from (3) that $E(R)=0$.

From (1) we find

$$\frac{d\mu}{dr} - q \frac{d\phi}{dr} = 0, \quad (4)$$

The Fermi energy [8] of a non-degenerate electron gas is

$$\mu = \theta \cdot \ln \left[\frac{n_e}{2} \left(\frac{h^2}{2\pi m \theta} \right)^{3/2} \right], \quad (5)$$

Substituting this expression in (4) with the communication potential of the field intensity gives

$$\theta \frac{dn_e}{dr} + n_e q E = 0.$$

Taking into account the equation of state for non-degenerate electron gas

$$p = n_e \theta, \quad (6)$$

where $\theta = kT$ - statistical temperature, k - Boltzmann's constant, T - absolute temperature, we find

$$\frac{dp}{dr} + n_e e E = 0. \quad (7)$$

The value $\frac{dp}{dr}$ is the power of pressure, acting on unit volume of the electron gas, and $F = n_e q E$ - the force per unit volume of the electron gas with the electric field. If in the area $r < R$ concentration of electrons is constant and equal n_{e0} , and when $r \geq R$ it is zero, when $r = R$ we have $\frac{dp}{dr} = \infty$ and force $n_e q E(R)$ equal to zero. Thus, equation (7) does not hold. Therefore, the assumption of a constant electron density in the area $r < R$ and the absence of electrons in the area $r > R$ is wrong. To resolve this issue, a closer look at the parameters of the potential well of the potential distribution and the concentration of electrons inside a dust particle in its vicinity.

2. Theoretical background

Consider this issue in the case of spherical particles. Let the inner radius of the solid particle is equal to R_1 , outer radius R , in the region of $R_1 \leq r \leq R$ is equal to the hole concentration n_i and the electron gas in the conduction band is not degenerate. We assume that the distance between dust particles is $2l$. We approximately assume that the distribution of the parameters of a spherically symmetric.

We find the distribution of the potential $\phi(r)$ and the electron density $n_e(r)$ in the region $0 \leq r \leq l$ under the following conditions:

$$n_i(r) = \begin{cases} 0, & \text{if } r < R_1 \text{ and } r > R \\ \text{const}, & \text{if } R_1 \leq r \leq R \end{cases}. \quad (8)$$

$$\phi'(0)=0, \phi'(l)=0. \quad (9)$$

Gauss's theorem (2) for this case, we write as

$$\oint_S \varepsilon_0 E_\psi dS = \iiint_V (n_i - n_e) q dV, \quad (10)$$

where V - the volume inside the surface S .

In view of the Boltzmann distribution for the electron gas

$$n_e = n_{e0} e^{\frac{q\phi}{kT}}, \quad (11)$$

from (10) is obtained by Poisson-Boltzmann equation

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = \frac{q}{\epsilon \epsilon_0} \left(n_{e0} e^{\frac{q\phi}{kT}} - n_i \right). \quad (12)$$

By introducing the dimensionless variables

$$x = \frac{r}{R}, \quad \phi = \frac{q\phi}{kT}, \quad \bar{n}_i = \frac{n_i}{n_{e0}}, \quad a^2 = \frac{q^2 R^2 n_{e0}}{kT \epsilon \epsilon_0}. \quad (13)$$

from (12) we obtain

$$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{d\phi}{dx} \right) - a^2 (e^\phi - \bar{n}_i) = 0.$$

Consider the case where $|\phi| \ll 1$. We expand the amount of e^ϕ Taylor series and only the first two terms, i.e. $e^\phi = 1 + \phi$. With this expression we obtain

$$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{d\phi}{dx} \right) = a^2 (1 + \phi - \bar{n}_i). \quad (14)$$

The general solution of (14) is a function

$$\phi(x) = \bar{n}_i - 1 + \frac{Ae^{ax} + Be^{-ax}}{x}. \quad (15)$$

In the area $0 \leq x \leq \frac{R_1}{R}$ potential distribution under condition $\phi_1(l)=0$ and $\bar{n}_i = 0$ has the form

$$\phi_1(x) = -1 + \frac{e^{ax} - e^{-ax}}{2ax}. \quad (16)$$

In the area $\frac{R_1}{R} \leq x \leq 1$ potential distribution can be written as

$$\phi_2(x) = \bar{n}_i - 1 + \frac{C_3 e^{ax} + C_4 e^{-ax}}{x}, \quad (17)$$

where $C_3 = \frac{1}{2a} - \frac{R_1}{2aR} \bar{n}_i e^{-\frac{aR_1}{R}} (a + \frac{R}{R_1})$, $C_4 = -\frac{1}{2a} - \frac{R_1}{2aR} \bar{n}_i e^{\frac{aR_1}{R}} (a - \frac{R}{R_1})$.

The potential distribution $\phi_3(x)$ in the area $1 \leq x \leq \frac{l}{R}$:

$$\phi_3(x) = -1 + \frac{C_5 e^{ax} + C_6 e^{-ax}}{x}, \quad (18)$$

Constant C_5 and C_6 are determined from the conditions $\phi_2(1) = \phi_3(1)$, $\phi_2'(1) = \phi_3'(1)$. The condition $\phi'(l)=0$ is used to determine the electron density n_{e0} at $x = 0$, which is included in all previous formulas.

To establish the shape of the potential well we use the following expression that determines the distribution of the potential energy of the electrons from the W coordinate r :

$$W(r) = \phi(l) - \phi(r), \quad (19)$$

3. Results

The results of calculation using formula (19) in the case of spherical particles at $T=1000$ K, $R_1 = 2 \cdot 10^{-7}$ m, $R = 10^{-6}$ m, $n_i = 10^{18} \text{ m}^{-3}$, $l = 10^{-5}$ m shown in Fig. 2. As can be seen, in $r > R$, W value increases due to escape of electrons from dust particles in the surrounding area.

In the particular case of $R_l=0$, the resulting formulas are simplified. In particular, for potential distribution, formulas

$$\phi(x) = \frac{e^a(a-1) - \left(\frac{a\lambda-1}{a\lambda+1}\right)e^{a(2\lambda-1)}(a+1)}{2a\left(e^a(a-1) - (a+1)\left(\frac{a\lambda-1}{a\lambda+1}\right)e^{a(2\lambda-1)} + 1 + \left(\frac{a\lambda-1}{a\lambda+1}\right)e^{2a\lambda}\right)} \left(\frac{e^{ax} - e^{-ax}}{x} - 2a\right), \quad \text{at } 0 \leq x \leq 1,$$

$$\phi(x) = -1 + \frac{e^a(a-1) + e^{-a}(a+1)}{2a\left(e^a(a-1) - (a+1)\left(\frac{a\lambda-1}{a\lambda+1}\right)e^{a(2\lambda-1)} + 1 + \left(\frac{a\lambda-1}{a\lambda+1}\right)e^{2a\lambda}\right)} \frac{e^{ax} + \left(\frac{a\lambda-1}{a\lambda+1}\right)e^{a(2\lambda-x)}}{x}, \quad \text{at } 1 \leq x \leq \lambda,$$

where $\lambda = l/R$.

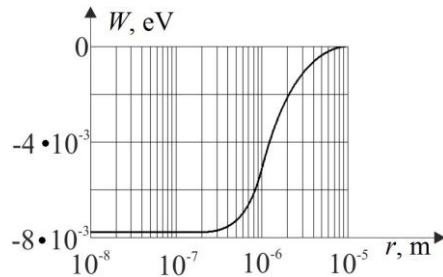


Fig. 2.

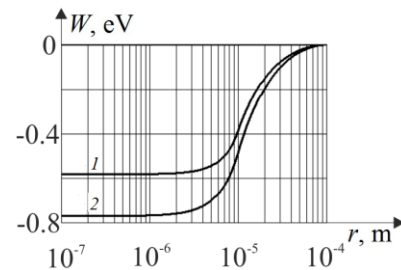


Fig. 3.

The calculation results for the case when the ball at $T=1000$ K (curve 1) and at $T=2000$ K (curve 2), $R = 10^{-5}$ m, $n_i = 10^{18} \text{ m}^{-3}$, $l = 10^{-4}$ m presented in Fig. 3. As shown, the parameters of the potential wells are temperature dependent. With increasing temperature, the well depth increases. This effect is associated with an increase in electron gas pressure and as a result, high yield of electrons beyond the dust particles with increasing temperature. Thus, it was found that in general, the potential well is not rectangular but has a more complex geometry.

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