Studying the fidelity of quantum memory based on control-field angular scanning

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Abstract. The fidelity of multimode cavity-assisted quantum storage based on control-field angular scanning is analysed as a function of an input pulse temporal shape, direction of propagation of the control field and spatial structure of the signal field. It is shown by numerical simulation that available range of the angular scanning is reduced with increasing the transverse mode index of the field to be stored. Outside of this range, the output field may contain contributions from undesirable transverse modes due to the cross-talk, which results in reducing efficiency and fidelity.

1. Introduction

In recent years much effort has been directed toward the implementation of quantum memories for light (see the reviews [1–4]). In particular, storage and retrieval of single photons is expected to be necessary for scalable linear-optical quantum computers, efficient quantum repeaters and deterministic single-photon sources. To store and recall optical pulses, such as single-photon wave packets, one can take advantage of an inhomogeneous broadened atomic transition (storage via photon echo) or can use a modulated control field amplitude which matches an input pulse (storage via electromagnetically induced transparency or off-resonant Raman interaction). Recently, another approach has been proposed [5–7] which requires neither inhomogeneous broadening nor temporal modulation of the control field amplitude, but resorts to continuous phase-matching control in an extended resonant medium. When considering off-resonant Raman interaction of a single-photon wave packet and a classical control field in a three-level atomic medium, the phase-matching control can be achieved by modulating the refractive index of the resonant medium [5] or by modulating the direction of propagation of the control field [6, 7]. In particular, a scheme for spatially multimode cavity-assisted quantum storage has been developed in [7]. The angular scanning does not require a direct control of atomic levels and may be realized in materials that do not allow linear Stark or Zeeman effects, while enclosing an atomic ensemble in a cavity makes it possible to achieve high efficiency of quantum storage with optically thin materials and realize the proposed scheme, e.g., in rare-earth-ion-doped crystals. Regarding spatially multimode storage, it is of importance for multiplexing in quantum repeaters and holographic quantum computers.

In the present work, we study the fidelity of the cavity-assisted quantum storage based on control-field angular scanning as a function of an input pulse temporal shape, direction of propagation of the control field and spatial structure of the signal field.

2. The model and basic equations

The idea of the considered quantum storage scheme is illustrated in figure 1. Let the wave vector of the control field be changed in time during the interaction of atoms with the input field to be stored. Then the wave vector of the spin coherence created via off-resonant Raman interaction on the transition $|1\rangle - |2\rangle$ also becomes a function of time so that during storage the single-photon wave packet is mapped into a superposition of (orthogonal) spin waves with different wave vectors. Retrieval is achieved by off-resonant spontaneous Raman scattering of the control field rotated within the same angle range. If direction of rotation during retrieval is the same as that during storage, the output pulse reconstructs the input one, while in the case of the reversed rotation the output pulse becomes a time-reversed replica of the input one. Such storage is possible, provided that the rate of control field angular scanning is sufficiently large, which means that the time interval between creations of orthogonal spin waves is shorter than characteristic time of an input pulse shape.

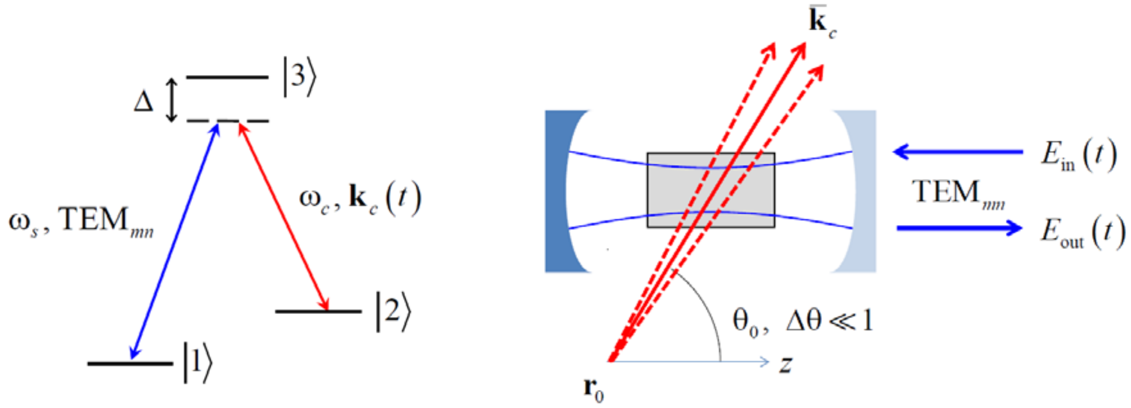

Figure 1. Energy diagram (left) and geometry (right) illustrating cavity-assisted off-resonant Raman interaction between a strong control field with a wave vector $\mathbf{k}_c(t)$, a weak quantum field in a transverse mode TEM$_{mn}$ and a three-level atomic medium. $\mathbf{r}_0$ stands for the point where the phase shift of the control field remains zero during the rotation, $\bar{\mathbf{k}}_c$ corresponds to the average wave vector, and θ_0 is the angle between $\bar{\mathbf{k}}_c$ and the cavity axis *z*. Mirrors are forming a single-ended cavity for the quantum field and fully transmitting for the control field.

The multimode cavity field may be described by a set of the mode functions

$$u_{mnp}(\mathbf{r}) = e^{iq_p z} u_{mn}(\mathbf{r}), \tag{1}$$

where $q_p = 2\pi p / L$, $p \in \mathbb{Z}$, and L is the total cavity length. We restrict the consideration of the cavity field to a single longitudinal mode and suppose that different transverse modes have equal frequencies. The off-resonant interaction between the signal field and atomic system under conditions of rotated controlled field is described by the following equations [7]:

$$\begin{cases} \dfrac{\partial}{\partial t} E_{mn} = -\kappa_{mn} E_{mn} + \sqrt{2\kappa_{mn}}\, E_{mn,\text{in}}(t) + ig\sqrt{N} \displaystyle\sum_{m',n',p} S_{m'n'p} B_{mn,m'n'p}(t), \\ \dfrac{\partial}{\partial t} S_{m'n'p} = -\gamma S_{m'n'p} + ig^* \sqrt{N} \displaystyle\sum_{mn} E_{mn}\, B^*_{mn,m'n'p}(t), \end{cases} \tag{2}$$

where E_{mn} ($E_{mn,\text{in}}$) is the slow-varying cavity (input) field amplitude, S_{mnp} is the slow-varying spin coherence amplitude, $2\kappa_{mn}$ is the cavity decay rate, γ is the spin coherence decay rate, g is the atomic-field coupling constant, and N is the number of atoms. The indexes m,n,p correspond to the mode functions $u_{mnp}(\mathbf{r})$. The coupling between different field modes and spin modes is described by the coefficients

$$B_{mn,m'n'p}(t)=\frac{1}{N}\int d\mathbf{r}\, e^{i\phi(\mathbf{r},t)}e^{iq_p z}u^*_{mn}(\mathbf{r})u_{m'n'}(\mathbf{r}), \tag{3}$$

where integration is performed over the volume of the atomic system, and $\phi(\mathbf{r},t)$ is a phase shift due to the rotation of the control field. The atomic ensemble is supposed to fill the cavity and to have a length $L_z = L$ along the cavity axis z.

In what follows, we assume that the control field propagates in the (x,z) plane at some angle θ_0 to the signal field, its wave vector is rotated within a small angle $\Delta\theta$ during the time interval T, which is the duration of the storage and retrieval process, and the rotation center (phase stationary point) $\mathbf{r}_0$ is located at the center of the sample, where the coordinate system originates. In this case, $\bar{\mathbf{k}}_c = (k_c\sin\theta_0, 0, k_c\cos\theta_0)$ and $\phi(\mathbf{r},t) = k_c(x\cos\theta_0 - z\sin\theta_0)(\Delta\theta/T)t$. Finally, we consider Hermite-Gaussian modes and assume that the origin of the z axis coincides with the beam waist (see [7] for details).

3. Fidelity of quantum storage

We are interested in the fidelity of quantum storage, which is given by

$$F = \eta F', \tag{4}$$

where

$$F' = \frac{\left|\sum_{mn}\int_0^{+\infty} dt \left\langle E^\dagger_{mn,\text{out}}(t)E_{mn,\text{in}}(\pm t)\right\rangle\right|^2}{N_{\text{in}}N_{\text{out}}} \tag{5}$$

is the correlation between the input and output pulse envelopes (the minus sign corresponds to retrieval with time reversal),

$$\begin{aligned} N_{\text{in}} &= \sum_{mn}\int_{-\infty}^{0} dt \left\langle E^\dagger_{mn,\text{in}}(t)E_{mn,\text{in}}(t)\right\rangle, \\ N_{\text{out}} &= \sum_{mn}\int_0^{+\infty} dt \left\langle E^\dagger_{mn,\text{out}}(t)E_{mn,\text{out}}(t)\right\rangle, \end{aligned} \tag{6}$$

and $\eta = N_{\text{out}}/N_{\text{in}}$ is the total storage efficiency. It is assumed that the storage process terminates at the moment $t=0$, while the retrieval process begins at this moment of time. The input-output relation is $E_{mn,\text{out}}(t) = \sqrt{2\kappa_{mn}}E_{mn}(t) - E_{mn,in}(t)$.

Let us start with the fidelity as a function of an input pulse temporal shape by considering storage and retrieval in a specific transverse mode. According to Eqs. (2), at different moments of time the signal field effectively interacts with spin waves having different longitudinal components of wave vectors q_p. The time interval between creations of orthogonal spin waves is equal to [7]

$$\delta = \frac{T}{\Delta\theta}\frac{\lambda_c}{L_z\sin\theta_0}, \tag{7}$$

where λ_c is the wavelength of the control field. Numerical simulations show that a Gaussian pulse with a duration as short as δ can be stored and recalled with a close to unity efficiency [5, 7]. Figure 2 illustrates the fidelity of quantum storage as a function of the pulse duration δ_p (FWHM) for different pulse shapes: Gaussian pulse $E_{\text{in}}(t) \propto \exp\left(-(t-t_0)^2 \sigma^2/2\right)$ of duration $\delta_p \approx 1.67/\sigma$, hyperbolic secant pulse $E_{\text{in}}(t) \propto \text{sech}\left((t-t_0)\sigma\right)$ of duration $\delta_p \approx 1.76/\sigma$, and double-exponential pulse $E_{\text{in}}(t) \propto \exp\left(-|t-t_0|\sigma\right)$ of duration $\delta_p \approx 1.39/\sigma$. Since the reversal switching time δ is equivalent to the inhomogeneous linewidth of the atomic transition, the fidelity reduces when the pulse duration becomes smaller than the switching time. Pulses with more narrow spectral distribution give higher fidelity at the same value of duration. In the case of the bad cavity limit, $2\kappa\delta \gg 1$, the optimal short pulse proves to be double exponential one, which matches the Lorentzian absorption profile.

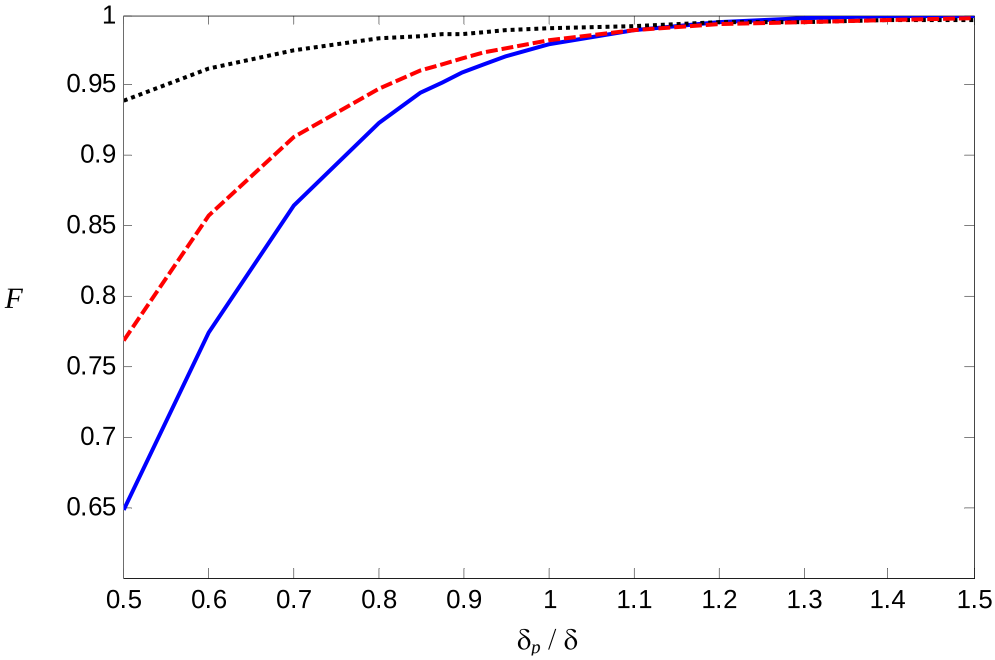

Figure 2. The fidelity of quantum storage as a function of the pulse duration δ_p (FWHM) for different pulse shapes: Gaussian pulse (blue solid line), hyperbolic secant pulse (red dashed line) and double-exponential pulse (black dotted line). The plots are obtained by numerically solving Eqs. (2) for a single TEM$_{mn}$ mode with the following values of parameters $\theta_0 = \pi/2$, $T = 30\delta$, $\gamma = 0$ and $\kappa\delta = 5$.

In a general case, the input field may excite different transverse modes. The spatially multimode storage and retrieval of single photons is possible provided that [7]

$$|\tan\theta_0| \gg \frac{\pi w_0}{L_z}\frac{T}{\delta}\sqrt{m+1}, \tag{8}$$

where w_0 is the waist size, and m is the largest transverse mode index of the field to be stored. Under the condition (8), different transverse modes interact with the atomic system independently and can be stored and recalled in parallel. To satisfy Eq. (8), the angle θ_0 should be large enough, which means that a transverse control field proves to be the ideal one. These features are illustrated in figure 3, which presents the fidelity of quantum storage as a function of the angle between the control field and signal field θ_0 for different transverse modes TEM$_{m0}$. The input pulse is assumed to be a Gaussian one of duration $\delta_p = \delta$. The fidelity decreases when the angle θ_0 approaches the minimum value corresponding to the right hand side of Eq. (8). The larger the mode index m, the smaller available range of the angular scanning. Outside of this range, the output field may contain contributions from undesirable transverse modes due to the cross-talk, which results in reducing efficiency and fidelity.

Figure 4 illustrates storage and retrieval of a single Gaussian pulse in a third order transverse mode TEM_{30}. In the case of a transverse control field, fidelity and efficiency of quantum storage are both close to unity ($\eta = 0.99$, $F = 0.99$). However, they are significantly reduced when the angle is small, e.g., $\eta = 0.63$ and $F = 0.43$ for $\theta_0 = 10^\circ$. In the latter case, the output field can be expressed as a superposition of different transverse modes (71% of TEM_{30}, 13% of TEM_{40}, 8% of TEM_{20}, etc.), which reduces correlation between the input and output pulse shapes.

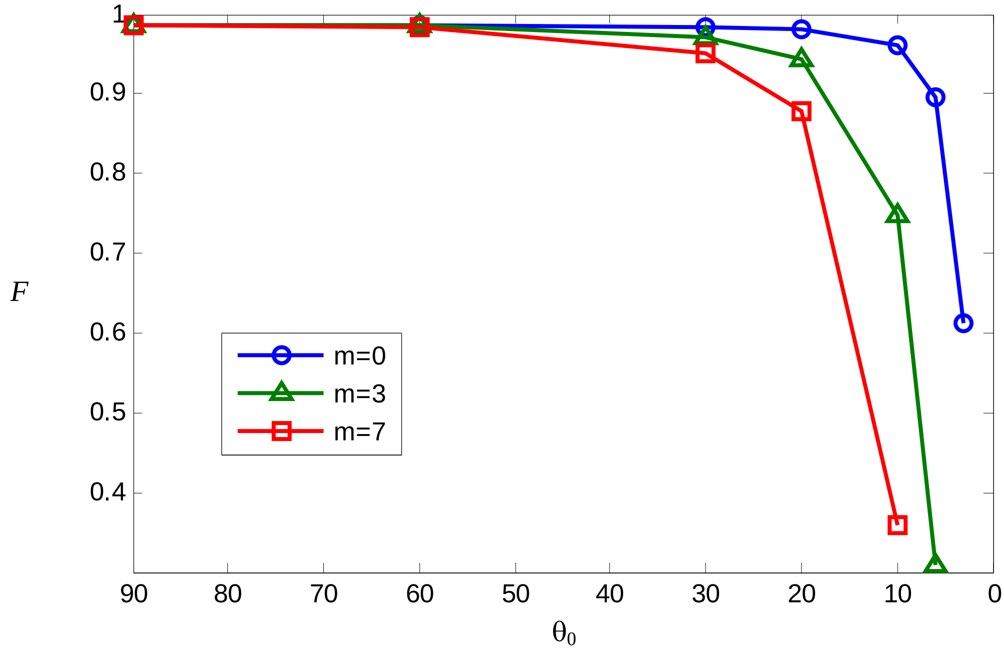

Figure 3. The fidelity of quantum storage as a function of the angle between the control field and signal field for different transverse modes TEM_{m0}. The plot is obtained as a result of numerically solving Eqs. (2) with the following values of parameters: $T = 3\delta$, $\delta_p = \delta$, $\pi w_0 / L_z = 0.1$, $\gamma = 0$ and $\kappa_{mn}\delta = 5$.

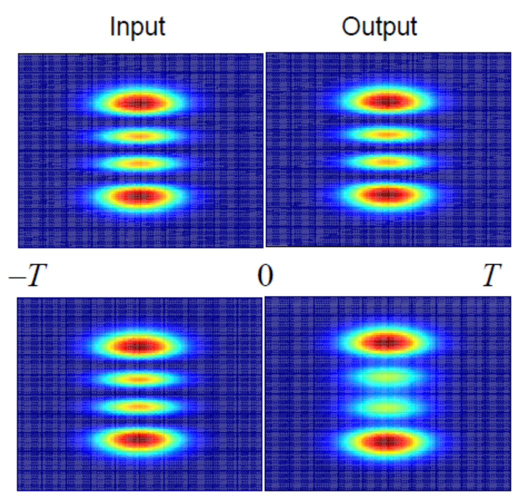

Figure 4. Illustrating storage and retrieval of a single Gaussian pulse in a third order transverse mode TEM_{30}. The left (right) column corresponds to the input (output) pulse. The first line presents storage and retrieval in the case of a transverse control field, $\theta_0 = 90^\circ$, the second line – in the case of $\theta_0 = 10^\circ$. The horizontal axis represents time, while the vertical axis represents the transverse coordinate *x*. The plots are obtained by numerically solving Eqs. (2) with the conditions $T = 10\delta$, $\delta_p = 2\delta$, $\pi w_0 / L_z = 0.1$, $\gamma = 0$ and $\kappa_{mn}\delta = 5$.

4. Conclusion

The fidelity of multimode cavity-assisted quantum storage based on control-field angular scanning, which has been proposed recently in [7], is analysed numerically as a function of an input pulse temporal shape, direction of propagation of the control field and spatial structure of the input field. It is shown that the larger the transverse mode index of the field to be stored, the smaller available range of the angular scanning. Regarding temporal pulse shapes, under conditions of the bad cavity limit double-exponential pulses provide higher fidelity for small values of pulse duration.

Acknowledgments

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References

[1] Lvovsky A I, Sanders B C and Tittel W 2009 *Nat. Photon.* **3** 706
[2] Hammerer K, Sørensen A S and Polzik E S 2010 *Rev. Mod. Phys.* **82** 1041
[3] Tittel W, Afzelius M, Cone R L, Chanelière T, Kröll S, Moiseev S A and Sellars M 2010 *Laser Photon. Rev.* **4** 244
[4] Simon C, Afzelius M, Appel J, Boyer de la Giroday A, Dewhurst S J, Gisin N, Hu C Y, Jelezko F, Kröll S, Müller J H, Nunn J, Polzik E S, Rarity J G, De Riedmatten H, Rosenfeld W, Shields A J, Sköld N, Stevenson R M, Thew R, Walmsley I A, Weber M C, Weinfurter H, Wrachtrup J, and Young R J 2010 *Eur. Phys. J.* D **58** 1
[5] Kalachev A and Kocharovskaya O 2011 *Phys. Rev.* A **83** 053849
[6] Zhang X, Kalachev A and Kocharovskaya O 2013 *Phys. Rev.* A **87** 013811
[7] Kalachev A and Kocharovskaya O 2013 *Phys. Rev.* A **88** 033846