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Electron rest mass and line spectrum of atoms in the photonic crystal medium

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Abstract. We study a recently predicted novel-type quantum electrodynamical effect that takes place in the photonic crystal medium. It consists in the fact that the modification of the interaction of a charged particle with its own radiation field in the photonic crystal medium results in the change in its mass. Consequences and possible applications of this effect are discussed. It is shown that, as a consequence of the effect, spectrum of the light emitted by atoms may be profoundly modified if, instead of being in free space, atoms are enclosed in air voids of a photonic crystal. Spectral lines of atoms in the photonic crystal medium are shifted in frequency, and this shift may be comparable to the atomic transition frequencies in free space.

1. Introduction

There has been a growing interest in the study of photonic crystals (PC) that are a new type of optical material with a periodic dielectric structure. A remarkable feature of photonic crystals is that its periodic dielectric function yields a photonic band structure with photonic band gaps. It has been shown that the gap edge has novel effects on optical behavior of atoms in photonic crystals such as the appearance of photon-atom bound states [1], spectral splitting [2], coherent control of spontaneous emission [3] etc. Strong suppression or enhancement of light emission which take place in PC is expected to modify the Lamb shift in atoms. Earlier studies give rise to controversial results for the Lamb shift. The isotropic dispersion model [1] predicts an anomalous Lamb shift, while the anisotropic model [4] suggests that the Lamb shift should be much smaller than that in vacuum. The calculation of the radiation using the formalism of local density of states (LDOS) resolved the dispute and the prediction of the giant Lamb shift appears to be correct [5]. Thus, the interaction of a bound electron in a photonic crystal with its own radiation field may significantly differs from that in vacuum. However, this may have a significant effect not only on the Lamb shift. In fact, the electron mass with which we usually deal includes the contribution from the interaction with its own radiation field in vacuum. This means that in a photonic crystal the electron mass should be changed. In Ref.[6] it has been shown that in the photonic crystal medium a quantum electrodynamical effect of a new type takes place. Because of the strong modification of the interaction of a charged particle in the PC medium with its own radiation field, the electromagnetic mass in PC differs from its electromagnetic mass in vacuum, which is included into the observable rest mass of the electron. This in turn gives rise to the fact that in the PC medium the electron rest mass changes its value. The effect is of interest both from the fundamental and the practical points of view. In this work we study the effect of the change in the electron rest mass on the linear spectrum of atoms placed in air voids of a photonic crystal and discuss possibilities of the development of novel-type sources with the line spectrum based on this effect.

2. Influence of an environment on the electromagnetic mass of the electron

In describing the QED effects in atoms placed in PCs, one has to take into account the influence of the PC medium on the propagation of virtual photons that come into play in the process of the self-interaction of the atomic electron. Formally, carrying out the mass renormalization for the electron in the PC medium should result in the subtraction of the self-energy of the free electron modified by this medium from the modified self-energy of the bound electron, and this subtraction was used [1, 4, 5, 7–9] in the studies of the Lamb shift in atoms placed in PCs. However, this way of solving the problem leads to missing some important contributions to energy levels from the self-interaction of atomic electrons. In fact, the renormalization theory prescribes to also add the subtracted term m_{em} to the "bare" mass m_0 of the electron in order to arrive at its physical mass $m_e = m_0 + m_{em}$. Here we mean one of the two approaches to renormalization. It has the merit of a clear physical interpretation, but the second approach, the method of counterterms, is the one normally used in quantum field theory. In the second approach the electron mass m_e in the original Lagrangian is regarded as the physical mass. To cancel the contribution from the self-energy of the free electron an extra term is added to the Lagrangian that is called the mass-renormalization counterterm. The problem consists in the fact that the value of the electromagnetic mass of the electron in the PC medium should differ from that in vacuum, and therefore the result of adding this electromagnetic mass to its "bare" mass will not be the physical mass. Obviously, the change of the value of the electron mass δm_{pc} is the difference between the values of the electromagnetic masses in the PC medium m_{em}^{pc} and the electromagnetic mass m_{em} in vacuum

$$\delta m_{pc} = m_{em}^{pc} - m_{em}. \tag{1}$$

Thus, the influence of the PC medium on the interaction of an electron with its own radiation field results in the change in its mass: $m_e \to m_{pc} = m_e + \delta m_{pc}$. Here we assume that the electron is in an air void of the PC.

The PC medium correction δm_{pc} to the electron rest mass is given by the equation [6]

$$\delta m_{pc} = \frac{\alpha}{\mathbf{p}^2 \pi^2} \left[\sum_n \int\limits_{FBZ} \frac{d^3k}{\omega_{\mathbf{k}n}^2} \sum_{\mathbf{G}} |\mathbf{p} \cdot \mathbf{E}_{\mathbf{k}n}(\mathbf{G})|^2 - \int \frac{d^3k}{2\mathbf{k}^2} \sum_{\lambda=1}^{2} |\mathbf{p} \cdot \varepsilon_\lambda(\mathbf{k})|^2 \right] \tag{2}$$

with $\mathbf{E}_{\mathbf{k}n}(\mathbf{G})$ being the coefficients in the plane-wave expansion $\mathbf{E}_{\mathbf{k}n}(\mathbf{r}) = \sum_{\mathbf{G}} \mathbf{E}_{\mathbf{k}n}(\mathbf{G}) e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}$ of the Bloch eigenfunctions $\mathbf{E}_{\mathbf{k}n}(\mathbf{r})$. Here $\mathbf{G}$ is the reciprocal lattice vector of the photonic crystal ($\mathbf{G} = N_1\mathbf{b}_1 + N_2\mathbf{b}_2 + N_3\mathbf{b}_3$ where $\mathbf{b}_i$ are primitive basis vectors of a reciprocal lattice). Thus, in contrast to the Lamb shift, the PC medium correction to the electron mass does not depend on the position of the electron in a PC's air void. At the same time, this correction depends on the direction unit vector $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$ of the electron momentum.

It should be noted that the mass dependence on orientation of the electron momentum in the PC is not surprising. It is a consequence of anisotropy of the crystal. In solid-state physics, crystal anisotropies result in the fact that the effective mass of an electron depends on direction of the electron momentum with respect to the crystal axes.

3. Linear spectra of the hydrogen atom in a photonic crystal

Let us now discuss perspectives of observation and application of the effect. The change in the electron mass gives rise to the shift of the energy levels of atoms. This shift cannot be classified as a part of the Lamb shift. Let us demonstrate this point by using the example of the atomic hydrogen. In the approximation where the nucleus is assumed to be a point and infinitely massive the energy levels of atomic hydrogen in the photonic crystal medium are given by the solution of the Dirac equation for the energy eigenvalues. For the energy of the atomic state $|n, j, l, m\rangle$, we have

$$\langle \delta m_{pc} \rangle_{n,j,l,m} = \int d^3p \Psi^*_{njlm}(\mathbf{p}) \delta m_{pc}(\hat{\mathbf{p}}) \Psi_{njlm}(\mathbf{p}) \tag{3}$$

with $\Psi_{njlm}(\mathbf{p})$ being the momentum-space hydrogen wave function.

The energy of the hydrogen state in free space $|a\rangle = |n, j, l, m\rangle$ may be written as $E_a = m_e + \epsilon_a$ with $\epsilon_a = -\frac{1}{2}\frac{\alpha^2 m_e}{n^2} + \mathrm{O}(\alpha^4)$. Here the rest energy part of E_a is distinguished. The frequency ω_{ab} of the transition between the state $|a\rangle$ and the state $|b\rangle = |n', j', l', m'\rangle$ is given by

$$\omega_{ab} = \epsilon_a - \epsilon_b. \tag{4}$$

The transition frequency ω_{ab} is equal to $\epsilon_a - \epsilon_b$ because the rest energy contributions are the same for both the states. The situation is dramatically changed in the case when the atom is placed in the PC medium. As we have seen, in this case the rest energy part of the total energy of the bound electron depends on the orbital angular momentum and the angular momentum z component m. As a result, the rest energy parts of the total energies of the states $|a\rangle$ and $|b\rangle$ make a contribution to the transition frequency

$$\omega^{pc}_{ab} = \langle \delta m_{pc} \rangle_a - \langle \delta m_{pc} \rangle_b + \epsilon^{pc}_a - \epsilon^{pc}_b. \tag{5}$$

Thus, in contrast to the free-space case, in the case of the PC medium the rest energy parts of the total energies of the states $|a\rangle$ and $|b\rangle$ make the contribution to the frequency of the transition between these states. Moreover, the difference between $\langle \delta m_{pc} \rangle_a$ and $\langle \delta m_{pc} \rangle_b$ makes a predominant contribution to the PC correction $\delta\omega^{pc}_{ab}$ to the transition frequency

$$\delta\omega^{pc}_{ab} = \left(\langle \delta m_{pc} \rangle_a - \langle \delta m_{pc} \rangle_b\right)\left(1 + \mathrm{O}(\alpha^4)\right) \tag{6}$$

provided $l_a \neq l_b$ and/or $m_a \neq m_b$. Such a surprising appearance of the contribution from the electron rest energy to the atom transition frequencies in the case when the atom is placed in the PC medium gives rise to the fact that the corrections to these frequencies may be very significant. From Eq. (3) it follows that δm_{pc} is independent of the position of the electron in the PC voids but depends on the direction of the electron momentum with respect to the photonic crystal axes. This mass dependence on direction has a significant effect on the structure of the atomic energy levels because it gives rise to the fact that the mean PC medium correction δm_{pc} in states with different orbital angular momenta and/or angular momentum z components are different. This in turn results in the appearance of the term $\langle \delta m_{pc} \rangle_a - \langle \delta m_{pc} \rangle_b$ in the expression (5) for the atomic transition frequency ω_{ab}. Thus, despite that the modification of the interaction of the electron with its own radiation field in the PC medium gives rise to relatively small corrections to the rest electron mass (in our model they are of order $10^{-6} m_e$), it results in the appearance of the term in the expressions for atomic transition frequencies that is absent in the free-space case. This term makes the contribution to ω_{ab} of order $10^{-6} m_e$, while the transition frequency in free space determined by Eq. (4) is given by $\omega_{ab} = \frac{\alpha^2}{2} m_e(\frac{1}{n_b^2} - \frac{1}{n_a^2}) + \mathrm{O}(\alpha^4)$. Thus, the shifts of the energy levels of the atom in the PC medium actually may be comparable to the atomic transition frequency in free space, and for this the modification of the electromagnetic field in this medium need not to be extraordinary. This point may be demonstrated by using the results of the calculations of the mass corrections δm_{pc} to the energy levels of atomic hydrogen in a high-index-contrast photonic crystal. In this way for the transition frequency $2s - 3p, m = 0$

of hydrogen atom in the PC with the $n_{eff} = 3$, where n_{eff} is the refractive index of the corresponding effective homogeneous medium, we get value $\nu_{pc}(2s - 3p, m = 0) = 4.20 \cdot 10^{14}$ Hz, while this frequency in free space is $\nu_{free}(2s - 3p, m = 0) = 3.83 \cdot 10^{14}$ Hz. Thus, the PC medium correction $\delta\nu_{pc} = \nu_{pc} - \nu_{free} = 0.37 \cdot 10^{14}$ Hz is comparable to the transition frequency in free space.

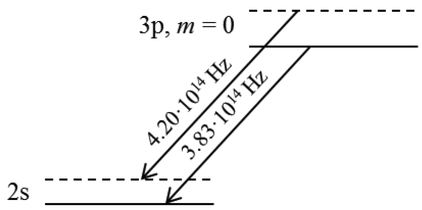

Figure 1. Energy levels of atomic transition frequencies of hydrogen in free space and in the PC medium. The dashed lines denote the energy levels of hydrogen atom placed in a PC.

It must be emphasized that we consider only the atoms placed in a PC's air voids. In this case, in contrast to electrons in ordinary crystals, the atomic electrons are not in contact with the vibrational degrees of freedom of the dielectric host. In this case the influence of the PC medium on atomic electrons is reduced to the influence on the propagation of virtual photons emitted and absorbed by the electrons. This influence can give rise only to the change in the electron mass, and, as a consequence, atoms in the PC should have the same energy-level structure as atoms in free space. Another important point is that the PC medium correction δm_{pc} to the electron mass and hence the atomic frequencies are independent of the position of an atom in a PC's void. As a consequence, spectrum of the light emitted by atom enclosed in PC's voids must be similar to the spectrum of the atoms in free space with the spectral lines shifted in frequency. As we have shown, this shift may be very significant. This gives us the hope that the shift will be observed in near future and the experimental investigations of the effect will give rise to the development of novel-type light sources with the line spectrum. Every spectral line of such sources could be shifted in a wide range by changing properties of a photonic crystal and this is a manifestation of the strong modification of the interaction of a charged particle placed into a PC with its own radiation field. This modification can also have a significant effect on the resonance fluorescence spectrum from emitters placed into a PC. For example, in this case the effect of involving vacuum modes into the strong laser-atom interaction predicted in [11] can take a place. This may be important for quantum-information science implementations since the Mollow triplet in the fluorescence spectrum provides a natural way to spectrally isolate the photons of interest [12].

4. Outlook

We have shown that spectrum of the light emitted by atoms may be profoundly modified if, instead of being in free space, atoms are enclosed in air voids of a PC. Spectral lines of atoms in the PC medium are shifted in frequency, and this shift may be comparable to the atomic transition frequencies in free space. At the same time, we have to keep in mind that this result has been obtained by using the one-loop approximation in describing the electron self-interaction. In solving problems with which the theory of quantum electrodynamics usually deals this approximation obeys a high accuracy. However, because of strong modification of the density of states (DOS) in a photonic crystal from that in free space in some cases the interaction of an atomic electron with its own radiation field may turn out to be effectively strong, and the one-loop approximation may become unapplicable. In this case the change of the electron mass and hence the shift of energy levels of atoms in the photonic crystal medium may be even more significant. Some reason to expect this is given by the effect obtained in Ref. [13] where interaction of an atom with its own radiation field affected by the atomic surrounding was investigated from a general point of view. In Ref. [13] it was shown that for some conditions of the surrounding the interaction of the atom with its own radiation field becomes effectively strong and gives rise to a spectral line splitting. In the case of the photonic crystal medium playing the role of the surroundings, these condition could be realized just because of the singular behavior of the DOS in the PC medium. This provides a way to drive the atomic energy levels. In this way, in particular, light sources with the line spectrum of a new type could be developed.

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