

Magnon Bose-Einstein condensation at inhomogeneous conditions

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Abstract. The Spin Supercurrent and Bose-Einstein condensation of magnons, similar to an atomic BEC, was discovered in superfluid $^3\text{He-B}$, which is characterized by absolute purity. Later this phenomena were observed in a few magnetically ordered materials with different types of impurities. In this article we will review the properties of magnon BEC in a presence of impurities and defects.

1. Introduction

The conventional magnon BEC, the phase-coherent precession of magnetization was discovered for the first time in superfluid $^3\text{He-B}$ in 1984 [1]. It manifests itself by a region, where the magnetization is deflected on a large angle (about 104°) and is precessing coherently even in an inhomogeneous magnetic field. This state was named Homogeneously Precessing Domain (HPD). The transverse component of magnetization in HPD is described by the wave function $S_\perp e^{i\omega t + \phi}$. It possesses all the properties of the spin superfluidity. The spatial gradient of phase ϕ leads to a spin supercurrent which transports the magnetization. There were observed: phase-slip processes at the critical current [2]; spin current Josephson effect [3]; spin current vortex [4]; Goldstone modes [5, 6, 7], etc. The long standing quest of nontrivial magnetic relaxation in superfluid ^3He have been resolved [8]. The comprehensive review of spin superfluidity in $^3\text{He-B}$ one can find in [9] and recent one in [10, 11, 12].

There is another phenomenon named in some publications as magnon BEC (see [13]). In reality it is the special case of magnetic phase transitions which could be described in terms of magnon's equilibrium density. Near the phase transition, the magnon density increases and, at a certain temperature, a new magnetic state appears. Owing the Bose statistics of magnons, this phenomenon is described via similar mathematics as that for the conventional atomic BEC. However, in fact, there exists the principal difference. In the case of a conventional BEC, we deal with the state of excitations (atoms) in a quantum vacuum (Universe). The BEC state of atoms does not change the vacuum background. Similarly, in the case of magnon BEC we are describing here, the ground magnetic states do not change. What is being condensed is the non-equilibrium magnons which were created by NMR. The conventional BEC is known to be forbidden for the equilibrium gas of excitations (see [14]). The use of the term "magnon BEC" for magnetic phase transitions, described in review [13], is thus misleading.

There are many new physical phenomena related to the Bose condensation of magnons, which have been observed after the discovery of HPD. For the last 25 years there was found 5



different magnon BEC states in superfluid ^3He [15]. These include in particular compact objects – coherently precessing states trapped by orbital texture [16]. At small number N of the pumped magnons, the system is similar to the Bose condensate of the ultracold atoms in harmonic traps, while at larger N the analog of the Q -ball in particle physics develops [17].

A Q -ball is a non-topological soliton solution in field theories containing a complex scalar field Ψ . Q -balls are stabilized due to the conservation of the global $U(1)$ charge Q [18]. They are formed due to suitable attractive interaction that binds the quanta of Ψ -field into a large compact object. In some modern SUSY scenarios Q -balls are considered as a heavy particle-like objects, with Q being the baryon and/or lepton number. For many conceivable alternatives, Q -balls may contribute significantly to the dark matter and baryon contents of the Universe, as described in review [19]. Stable cosmological Q -balls can be searched for in existing and planned experiments [20].

The Q -ball is a rather general physical object, which in principle can be formed in condensed matter systems. In particular, Q -balls were suggested in the atomic Bose-Einstein condensates [21]. In $^3\text{He-B}$, the Q -balls are formed as special states of phase coherent precession of magnetization. The role of the Q -charge is played by the projection \mathcal{S}_z of the total spin of the system on the axis of magnetic field, which is a rather well conserved quantity at low temperature, or which is the same the magnon number N . At the quantum level, this Q -ball is a compact object formed by magnons – quanta of the corresponding Ψ -field.

In $^3\text{He-B}$ the Q -balls are formed at low temperatures, when homogeneous magnon BEC in the form of Homogeneously Precessing Domain (HPD) becomes unstable due to parametric Suhl instability [22, 23, 24]. At low temperatures the condensate can be formed only in a trap, similar to that in atomic gases [25], and the Q -balls are either formed in these traps or dig their own trap.

The Q -ball in superfluid $^3\text{He-B}$ was discovered by case as a very strange coherent signal of small amplitude (below 10% of HPD) but extremely long (up to minute) [26]. Later it was found that the frequency of the signal grows up, not down, as in the case of HPD [27]. It means that the mechanism of Q -ball formation is very different than HPD. In the works [28, 29] the steady state of Q -balls have been maintained by CW RF pumping. Again, in contrast with HPD, the signal was excited by sweep field up (frequency down). Finally the signal was explained as a formation of a Q -ball in a texture trap [17]. It was found that the Q -ball can be excited even by an off-resonance excitation [16, 30]. The recent detailed experimental investigations of Q -balls formed in the specially prepared and the well controlled traps were made in [31].

2. BEC of ^3He in aerogel

The further progress of the investigations of BEC states in superfluid ^3He is related to some kind of artificial impurities. This became possible by immersing the superfluid ^3He in a very porous material called aerogel. HPD spectroscopy proved to be extremely useful for the investigation of the superfluid order parameter in this novel system – superfluid ^3He confined in aerogel [32, 33, 34, 35, 36]. By squeezing or stretching the aerogel sample, one creates the global anisotropy which captures the orbital vector $\hat{\mathbf{l}}$. This allows to orient the orbital vector $\hat{\mathbf{l}}$ in the desirable direction with respect to magnetic field [37, 38].

In this conditions we are able experimentally reorient the orbital momentum of superfluid $^3\text{He-B}$ perpendicular to a magnetic field. For the transverse orientation of $\hat{\mathbf{l}}$ two new magnon states have been identified. One of them exists at $|\Psi|^2 < S/\hbar$ and has the following form of spin-orbit interaction obtained (we omit for simplicity the constant term):

$$F_{\text{so}}(\Psi)_{l=0} = -\frac{\chi}{4\gamma^2}\Omega_L^2 \left(\frac{|\Psi|^2}{S} - \frac{4}{5} \right)^2, \quad |\Psi|^2 < \frac{S}{\hbar}, \quad (1)$$

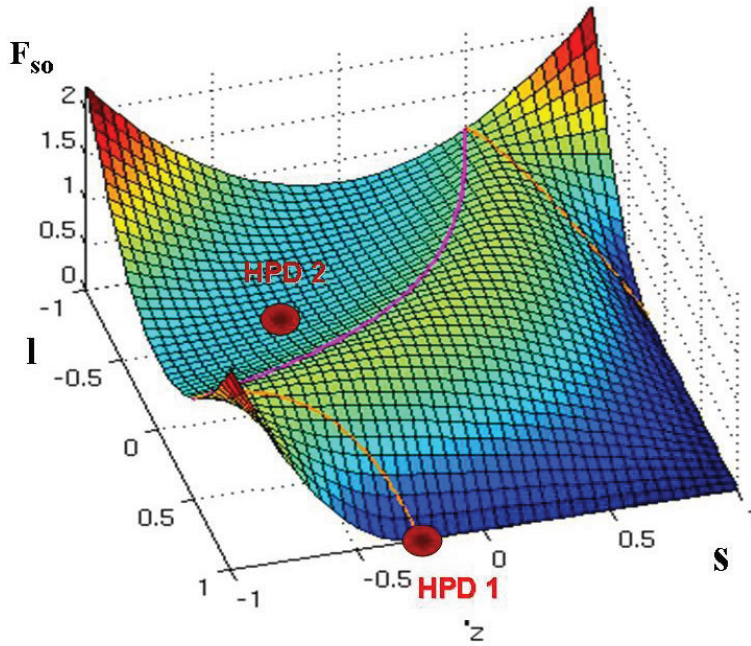


Figure 1. The profile of the spin-orbit energy in $^3\text{He-B}$ as a function of $S = \cos \beta$ and orbital variable l , where β is the tipping angle of precession and l is the projection of the orbital angular momentum of a Cooper pair on the direction of magnetic field. Spontaneous phase-coherent precession emerging at $l = 1$ and $S \approx -1/4$ is called Homogeneously Precessing Domain (HPD). The second BEC state (HPD2) is emerging at $l = 0$ and $S < 0$.

where χ is the susceptibility and Ω_L - Legget frequency, the frequency of the longitudinal mode of magnetic resonance in superfluid ^3He .

This state has an attractive interaction between magnons, and is unstable since the compressibility β_M of the magnon gas is negative: $d^2\epsilon/dn^2 < 0$.

The other state exists at $|\Psi|^2 > S/\hbar$ and has the following form of spin-orbit interaction

$$F_{\text{so}}(\Psi)_{l=0} = \frac{\chi}{20\gamma^2} \Omega_L^2 \left(\frac{|\Psi|^2}{S} - 2 \right)^2, \quad |\Psi|^2 > \frac{S}{\hbar}. \quad (2)$$

This state has repulsive interaction between magnons and is stable. The magnon BEC formation under these conditions has been observed [39].

As in the case of $^3\text{He-B}$, all the information on the $^3\text{He-A}$ order parameter needed to study the coherent precession is encoded in the spin-orbit interaction. For $^3\text{He-A}$, the spin-orbit interaction averaged over the fast precession has the following form [15]:

$$F_{\text{so}}(|\Psi|) = \frac{\chi \Omega_L^2}{4\gamma^2} \times \left[-2 \frac{|\Psi|^2}{S} + \frac{|\Psi|^4}{S^2} + \left(-2 + 4 \frac{|\Psi|^2}{S} - \frac{7}{4} \frac{|\Psi|^4}{S^2} \right) (1 - l^2) \right] \quad (3)$$

In a static bulk $^3\text{He-A}$, when $\Psi = 0$, the spin-orbit energy F_{so} in Eq.(3) is minimized when the

orbital vector $\hat{\mathbf{l}}$ is perpendicular to magnetic field, i.e. for $l = 0$. Then one has

$$F_{\text{so}}(|\Psi|, l = 0) = \frac{\chi\Omega_L^2}{4\gamma^2} \left[-2 + 2\frac{|\Psi|^2}{S} - \frac{3}{4}\frac{|\Psi|^4}{S^2} \right], \quad (4)$$

with a negative quartic term. The attractive interaction between magnons destabilizes the BEC, which means that homogeneous precession of magnetization in $^3\text{He-A}$ becomes unstable. This instability predicted by Fomin [40] was experimentally confirmed in Ref. [41].

However, as follows from (3), at sufficiently large magnon density $n = |\Psi|^2$

$$\frac{8 + \sqrt{8}}{7} S > n > \frac{8 - \sqrt{8}}{7} S, \quad (5)$$

the factor in front of l^2 becomes negative. Therefore it becomes energetically favorable to orient the orbital momentum $\hat{\mathbf{l}}$ along the magnetic field ($l = 1$). For this orientation one obtains the Ginzburg-Landau free energy with

$$F_{\text{so}}(|\Psi|, l = 1) = \frac{\chi\Omega_L^2}{4\gamma^2} \left[-2\frac{|\Psi|^2}{S} + \frac{|\Psi|^4}{S^2} \right]. \quad (6)$$

It corresponds to the conventional Ginzburg-Landau free energy in atomic BEC. The quadratic term modifies the potential U ; the quartic term is now positive.

In the language of BEC, this means that, with increasing the density of Bose condensate, the originally attractive interaction between magnons should spontaneously become repulsive when the critical magnon density $n_c = S(8 - \sqrt{8})/7$ is reached. If this happens, the magnon BEC becomes stable and in this way the state with spontaneous coherent precession could be formed. This self-stabilization effect is similar to the effect of Q -ball, where bosons create the potential well in which they condense. However, such a self-sustaining BEC with originally attractive boson interaction has not been achieved experimentally in bulk $^3\text{He-A}$, most probably because of the large dissipation, due to which the threshold value of the condensate density has not been reached.

Finally the fixed orientation of the orbital vector $\hat{\mathbf{l}}$ has been achieved in $^3\text{He-A}$ confined in aerogel. Silicon strands of aerogel play the role of impurities with local anisotropy along the strands. According to the Larkin-Imry-Ma effect, the random anisotropy suppresses the orientational long-range order of the orbital vector $\hat{\mathbf{l}}$; however, when the aerogel sample is deformed the long-range order of $\hat{\mathbf{l}}$ is restored. Experiments with globally squeezed aerogel [37] demonstrated that a uni-axial deformation by about 1% is sufficient for global orientation of the vector $\hat{\mathbf{l}}$ along the anisotropy axis. When magnetic field is also oriented along the anisotropy axis one obtains the required geometry with $l = 1$, at which the magnon BEC in $^3\text{He-A}$ becomes stable. The first indication of coherent precession in $^3\text{He-A}$ has been reported in [42, 43, 44] and confirmed in [45] by observation of a long induction decay signal.

Contrary to the unconventional magnon BEC in the form of HPD in $^3\text{He-B}$, the magnon BEC emerging in the superfluid $^3\text{He-A}$ is in one-to-one correspondence with the atomic BEC. For $\mu > U$, the condensate density determined from equation $dF/dn = \mu$ continuously grows from zero as $n \propto \mu - U$.

For $l = 1$ the Ginzburg-Landau free energy acquires the standard form:

$$F = \int d^3r \left(\frac{|\nabla\Psi|^2}{2m} + (\omega_L(\mathbf{r}) - \mu)|\Psi|^2 + \frac{1}{2}b|\Psi|^4 \right), \quad (7)$$

where we have modified the chemical potential by the constant frequency shift:

$$\mu = \omega + \frac{\Omega_L^2}{2\omega}, \quad (8)$$

and the parameter b of repulsive magnon interaction is

$$b = \frac{\Omega_L^2}{2\omega S} \quad (9)$$

At $\mu > \omega_L$, magnon BEC must be formed with density

$$|\Psi|^2 = \frac{\mu - \omega_L}{b} . \quad (10)$$

This is distinct from $^3\text{He-B}$, where condensation starts with finite condensate density. Eq. (10) corresponds to the following dependence of the frequency shift on tipping angle β of coherence precession:

$$\omega - \omega_L = -\frac{\Omega_L^2}{2\omega} \cos \beta . \quad (11)$$

The final proof of the coherence of precession in $^3\text{He-A}$ in aerogel was the observation of the free precession after a pulsed NMR and also after a switch off the CW NMR [45]. In conclusion, in contrast to the homogeneously precessing domain (HPD) in $^3\text{He-B}$, the magnon Bose condensation in $^3\text{He-A}$ obeys the standard Gross-Pitaevskii equation.

Indeed, there is one important problem. The aerogel plays a role of impurities, which may be distributed very inhomogeneously. As a result the BEC state may be not a global state but a local state [46]. The example of HPD state in inhomogeneous aerogel is presented in Fig. 2.

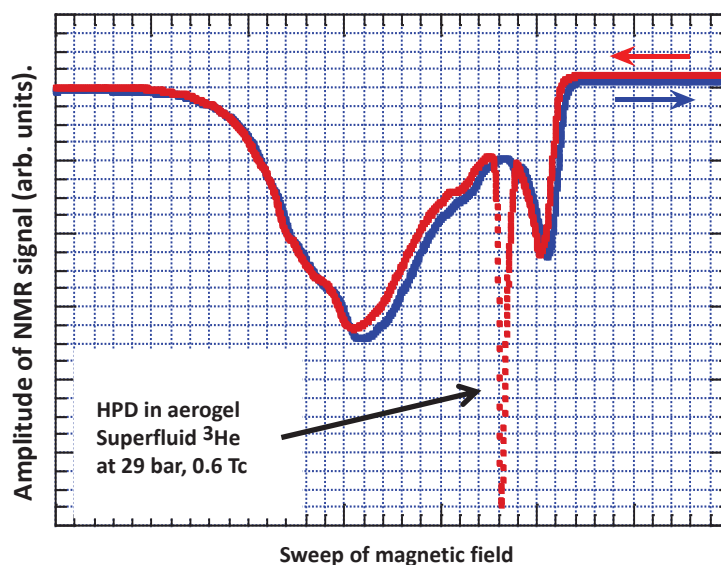


Figure 2. The HPD signal, observed in superfluid $^3\text{He-B}$ immersed in inhomogeneous aerogel. The HPD is formed only in a local place, which position was confirmed by an experiments at different gradient of magnetic field

The similar local states have been observed in $^3\text{He-A}$ and in HPD2 in aerogel. At pulsed NMR a few independent local BEC states are created, which induction signals are beating. The example of signal from a few BEC states of HPD2 are shown in Fig. 3.

Definitely, there are the gradient of magnetic ordering between the local BEC states, which leads to a relaxation due to spin diffusion current [44]. The relaxation may increase also due

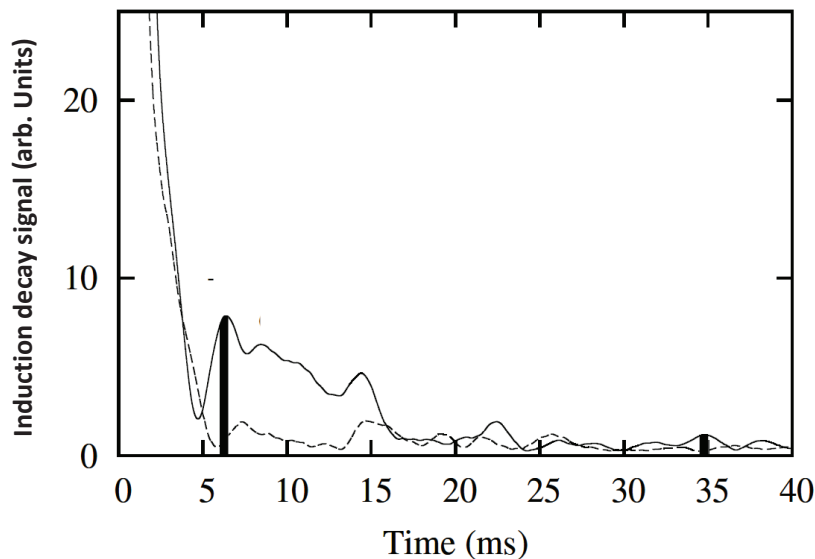


Figure 3. The beating of an induction signals from a few HPD2 states.

to local frequency shift owing to a local concentration of impurities. The topological defects may play an important role for magnetic relaxation as a sink region of magnons energy. All this demonstrate a great importance of the quality of the samples for observation of magnon BEC states. This problem becomes even more important for a magnon BEC in solid antiferromagnets, where the problem of impurities play a crucial role.

3. BEC in solid antiferromagnets

The magnon BEC in terms of coherent spin precession has been discovered in a monocrystals of CsMnF_3 and MnCO_3 [47]. The dynamic properties of coupled nuclear-electron precession in these antiferromagnets is very similar to a $^3\text{He-A}$ in aerogel [48]. It also shows the dynamical frequency shift (pulling). Magnetic systems with pulling were studied intensively in 70th of previous century [49]. This phenomenon takes place in the case when the frequencies of NMR and antiferromagnetic resonance are comparable. The typical system with pulling is the easy plain antiferromagnets with ions of Mn. The NMR frequency of Mn^{55} in hyperfine field is very high, about 600 MHz, and the antiferromagnetic resonance frequency in easy plain and cubic antiferromagnets can be very low. Due to coupling through the hyperfine field, the two modes of resonance appear, low frequency quasi NMR mode (NEMR) and high frequency quasi antiferromagnetic resonance mode (ENMR). The properties of magnetic subsystems change significantly due to the coupled precession. The hyperfine gap appears in a spectrum of antiferromagnetic spin waves. The quasinuclear spin waves appear, with the antiferromagnetic length of coherence. In other words, the nuclear magnetic system gets some properties of magnetically ordered system.

The pulse or CW magnetic resonance can move the spin system out of equilibrium. This deviation can be described in the language of elementary excitation, the spin waves (magnons). Magnons may condense very fast to a new equilibrium state. The density of non-equilibrium magnons depends on the angle of magnetization deflection β :

$$N = \frac{m - m_z}{\hbar} = \frac{\chi H}{\hbar \gamma} (1 - \cos \beta) . \quad (12)$$

The frequency dependence for NEMR mode in CsMnF_3 and MnCO_3 is very similar to that of superfluid $^3\text{He-A}$ (see Eq. 11):

$$\Omega = \omega_n - \omega_p \frac{m_z}{m_0} = \omega_n - \omega_p \cos \beta, \quad (13)$$

where ω_n and ω_p are the non-shifted NMR frequencies in hyperfine field and pulling, m_0 is the equilibrium nuclear magnetization and m_z is its projection on a hyperfine field. It means that both systems may have very similar properties. The condition of dynamic stability of coherent precession [10, 12] for NEMR mode in CsMnF_3 and MnCO_3 are fulfilled.

$$\frac{\partial \Omega}{\partial \cos \beta} < 0, \quad (14)$$

Owing this similarity, the magnon BEC observation in this systems was suggested [48]. The experiments demonstrated the formation of magnon BEC as by CW NMR technics [47] so by pulsed NMR experiments [50]. The last one was performed by a new techniques, developed in Ref. [45]. In this method the BEC is formed by a very long RF pulse. In this case the single BEC state is created. After the switching off of the RF pulse one can see a nice induction decay signal without beats. The details of experiments with CsMnF_3 and MnCO_3 one can find in Ref. [51, 52, 53]. Indeed, the length of this signal strongly depends on the quality of the crystal. The signal is 10 times longer in a new sample of better quality [54]. Definitely, the quality of the sample plays a crucial role for observation of a BEC in solid antiferromagnets.

It is important to point out that a similar experiment was described in Ref. [55]. The qualitative behavior of the signal was very similar. Indeed, it is difficult to figure out all parameters of the experiments, described there. To explain those results authors have used the scenario of saturation (heating) of the nuclear subsystem under a RF pumping. This scenario strongly contradicts the results of our experiments. The nuclear spin subsystem in these antiferromagnets is paramagnetic and after deflection should be dephased due to inhomogeneity of a superfine field. Furthermore, the dynamics of deflected magnetization should be described by Bloch equation. It means that the nuclear system should thermalize in a time T_2 and the transverse component should vanish. In our experiments there was found that it is not a case! In CW NMR experiments the deflected nuclear magnetization precess homogeneously at constant temperature and the NMR line does not saturate. The explanation can be found in the book [56]. This phenomenon was also considered in [57]. Two different scales for inhomogeneity were considered: the microinhomogeneous broadening for a distance smaller than the length of coherence of Suhl-Nakamura interaction (about 10^4 of the lattice size), and the macroscopic broadening for a longer distances. The microscopic broadening can be suppressed in the case when pulling is much bigger than a broadening. As a result the antiferromagnetic magnetization precess coherently at a short distance and force the nuclear subsystem precess coherently. For the longer distance it is not the case. It means that the nuclear subsystem shows the properties of ordered one, like the superfluid $^3\text{He-A}$. Even the coherence lengths are comparable. The magnetization for this mode of resonance follows the Landau-Lifshits scenario. At the RF pumping it deflects but the module of magnetization does not change.

This work was done at the International Laboratory of the Magnetic Superfluidity and Non-linear Magnetic Resonance, Kazan Federal University.

References

- [1] Borovik-Romanov A S, Bunkov Yu M, Dmitriev V V and Mukharskiy Yu M 1984 *JETP Lett.* **40**, 1033
- [2] Borovik-Romanov A S, Bunkov Yu M, Dmitriev V V, Mukharskiy Yu M and Sergatskov D A 1989 *Phys.Rev.Lett.* **62**, 1631
- [3] Borovik-Romanov A S, Bunkov Yu M, de Waard A, Dmitriev V V, Makroczyova V, Mukharskiy Yu M and Sergatskov D A 1988 *JETP Lett.* **47**, 478

- [4] Borovik-Romanov A S, Bunkov Yu M, Dmitriev V V, Mukharskiy Yu M and Sergatskov D A 1990 *Physica B* **165**, 649
- [5] Bunkov Yu M, Dmitriev V V and Mukharskiy Yu M 1986 *JETP Lett.* **43**, 168
- [6] Bunkov Yu M, Dmitriev V V and Mukharskiy Yu M 1992 *Physica B* **178**, 196
- [7] Kupka M, Skyba P, 2012 *Phys. Rev. B* **85**, 184529
- [8] Bunkov Yu M 2004 *J. of Low Temp. Phys.* **135**, 337
- [9] Bunkov Yu M 1995 *Spin Supercurrent and Novel Properties of NMR in ^3He* , (Prog. Low Temp. Phys. **14**, 69) ed W Halperin, (Amsterdam: Elsevier)
- [10] Bunkov Yu M and Volovik G E 2008 *J. of Low Temp. Phys.* **150**, 135
- [11] Bunkov Yu M and Volovik G E 2010 *J. Phys.: Condens. Matter* **22**, 164210
- [12] Bunkov Yu M 2009 *J. Phys.: Condens. Matter* **21**, 164201
- [13] Giamarchi T, Rüegg Ch and Tchernyshyov O 2008 *Nature Physics* **4**, 198
- [14] Kagan Yu *et al.* 2007 *Phys. Lett. A* **361**, 401
- [15] Bunkov Yu M and Volovik G E 2013 *Spin superfluidity and magnon BEC* (Novel Superfluids) ed. K. H. Bennemann and J. B. Ketterson (Oxford: University press)
- [16] Bunkov Yu M 2005 *J. of Low Temp. Phys.* **138**, 753
- [17] Bunkov Yu M and Volovik G E 2007 *Phys. Rev. Lett.* **98**, 265302
- [18] Coleman S R 1985 *Nucl. Phys. B* **262**, 263
- [19] Enqvist K and Mazumdar A 2003 *Phys. Rept.* **380**, 99
- [20] Kusenkov A, Kuzmin V, Shaposhnikov M and Tinyakov P G 1998, *Phys. Rev. Lett.* **80**, 3185
- [21] Enqvist K and Laine M 2003 *JCAP* **0308** 003
- [22] Bunkov Yu M, Dmitriev V V, Mukharskiy Yu M, Nyeki J and Sergatskov D A 1989 *Europhys. Lett.* **8**, 645
- [23] Bunkov Yu M, Lvov V S and Volovik G E 2006 *JETP Lett.* **83**, 530
- [24] Bunkov Yu M, Lvov V S and Volovik G E 2006 *JETP Lett.* **84**, 289
- [25] Pitaevskii L and Stringari S 2003 *Bose-Einstein condensation* (Oxford: Clarendon Press)
- [26] Bunkov Yu M, Fisher S N, Guenault A M and Pickett G R 1992 *Phys. Rev. Lett.* **69**, 3092
- [27] Bunkov Yu M, Fisher S N, Guenault A M, Pickett G R and Zakazov S R 1994 *Physica B* **194**, 827
- [28] Chen A S, Bunkov Yu M, Godfrin H, Schanen R and Scheffer F 1998 *J. Low Temp. Phys* **110**, 51
- [29] Chen A S, Bunkov Yu M, Godfrin H, Schanen R and Scheffer F 1998 *J. Low Temp. Phys* **113**, 693
- [30] Cousins D J, Fisher S N, Gregory A I, Pickett G R and Shaw N S 1999 *Phys. Rev. Lett.* **82**, 4484
- [31] Autti S, Bunkov Yu M, Eltsov V B, Heikkinen P J, Hosio J J, Hunger P, Krusius M and Volovik G E 2012 *Phys. Rev. Lett.* **108**, 145303
- [32] Bunkov Yu M, Collin E and Godfrin H 2005 *J. Phys. Chem. Solids* **66**, 1325
- [33] Halperin W P, Choi H, Davis J P and Pollanen J 2008 *J. Phys. Soc. Jpn.* **77**, 111002
- [34] Dmitriev V V, Zavjalov V V, Zmeev D E, Kosarev I V and Mulders N 2002 *JETP Lett.* **76**, 321
- [35] Bunkov Yu M, Collin E, Godfrin H and Harakaly R 2003 *Physica B* **329-333**, 305
- [36] Kunimatsu T, Matsubara A, Izumina K, Sato T, Kubota M, Takagi T, Bunkov Yu M and Mizusaki T 2008 *J. Low Temp. Phys.* **150**, 435
- [37] Kunimatsu T, Sato T, Izumina K, Matsubara A, Sasaki Y, Kubota M, Ishikawa O, Mizusaki T and Bunkov Yu M 2007 *JETP Lett.* **86**, 216
- [38] Elbs J, Bunkov Yu M, Collin E, Godfrin H and Volovik G E 2008 *Phys. Rev. Lett.* **100**, 215304
- [39] Hunger P, Bunkov Yu M, Collin E and Godfrin H 2012 *J. Physics: Conference Series* **400**, 012019
- [40] Fomin I A 1979 *JETP Lett.* **30**, 164
- [41] Borovik-Romanov A S, Bunkov Yu M, Dmitriev V V and Mukharskiy Yu M 1984 *JETP Lett.* **39**, 469
- [42] Bunkov Yu M and Volovik G E 2009 *JETP Lett.* **89**, 356
- [43] Matsubara A, Sato T, Kunimatsu T, Izumina K, Kubota M, Mizusaki T and Bunkov Yu M 2009 *J. Phys.: Conf. Ser.* **150** 032052
- [44] Sato T, Kunimatsu T, Izumina K, Matsubara A, Kubota M, Mizusaki T and Bunkov Yu M 2008 *Phys. Rev. Lett.* **101**, 055301
- [45] Hunger P, Bunkov Yu M, Collin E and Godfrin H 2010 *J. Low Temp. Phys.* **158**, 129
- [46] Bunkov Yu M, Collin E, Godfrin H and Harakaly R 2003 *Physica B* **329**, 305
- [47] Bunkov Yu M, Alakshin E M, Gazizulin R R, Klochkov A V, Kuzmin V V, Safin T R and Tagirov M S 2011 *JETP Letters* **94**, 68
- [48] Bunkov Yu 2010 *Physics Uspekhi*, **53**, 843
- [49] Borovik-Romanov A S, Bunkov Yu M, Dumesh B S 1977 *Physica B+C* **86** 1301
- [50] Bunkov Yu M, Alakshin E M, Gazizulin R R, Klochkov A V, Kuzmin, V V, L'vov V S and Tagirov M S 2012 *Phys. Rev. Lett.* **108**, 177002
- [51] Bunkov Yu M, Alakshin E M, Gazizulin R R, Klochkov A V, Kuzmin, Nizamutdinov A S, Safin T R and Tagirov M S 2011 *J. Phys.: Conf. Series* **324**, 012006

- [52] Bunkov Yu M, Alakshin E M, Gazizulin R R, Klochkov A V, Kuzmin, Safin T R and Tagirov M S 2012 *J. Phys.: Conf. Series* **400**, 032001
- [53] Alakshin E M, Bunkov Yu M, Gazizulin R R, Klochkov A V, Kuzmin, Rakhmatullin R M Sabitova A M , Safin T R and Tagirov M S 2013 *Applied Magnetic Resonance* **44**, 595
- [54] To be published.
- [55] Tulin V A 1969 *Sov. Phys. JETP* **28**, 431
- [56] Kurkin M I and Turov E A 1990 *NMR in magnetically ordered materials and its applications* (Moscow, Nauka)
- [57] Borovik-Romanov A S, Bunkov Yu M, Dumesht B S, Kurkin M I, Petrov M P and Chekmarev V P 1984 *Sov. Phys. Uspekhi* **142**, 537