

Analytical Method for Selecting a Rectification Technique for a Piezoelectric Generator based on Admittance Measurement

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Abstract. AC-DC converters employed for harvesting power from piezoelectric transducers can be divided into linear (i.e. diode bridge) and non-linear (i.e. synchronized switch harvesting on inductor, SSHI). This paper presents an analytical technique based on the measurement of the impedance circle of the piezoelectric element to determine whether either diode bridge or SSHI converter harvests more of the available power at the piezoelectric element.

1. Introduction

AC-DC converters that perform the rectification of the power harvested by piezoelectric transducers are divided in two different groups: non-linear (i.e. parallel synchronized switch harvesting on inductor, SSHI) and linear (i.e. diode bridge). The paper presents a method to estimate the electrical impedance of AC-DC converters in order to predict the harvested power to be obtained with a certain piezoelectric transducer from the measurement of its impedance circle. Thus, it is possible to estimate which AC-DC converter harvests more power.

The paper is divided as follows: Section 2 describes how to calculate the elements of the piezoelectric equivalent circuit. In section 3, the impedance of the mechanical part of the piezoelectric transducer as well as the electrical part are computed when a resistive load is connected. These values are employed to calculate the harvested power ratio. Section 4 presents an analytical method to compute the electrical impedance of a diode bridge. In Section 5, the expression for the inversion voltage factor of the parallel SSHI converter is given. Finally, Section 6 summarizes the conclusions.

2. Piezoelectric Equivalent Electrical Impedance in Open Circuit

Measurements of the admittance of different piezoelectric transducers have been done with an AUTOLAB PGSTAT302N potentiostat/galvanostat. The piezoelectric equivalent circuit at the resonance frequency on the secondary side has the electrical components L'_n , C'_n , R'_n in series and C_p connected in parallel to them. The admittance of a piezoelectric element is expressed as:

$$Y(s) = \frac{s^2 + s \frac{R'_n}{L'_n} + \frac{C'_n + C_p}{C_p C'_n L'_n}}{s^2 + s \frac{R'_n}{L'_n} + \frac{1}{L'_n + C'_n}} \quad (1)$$



A fitting curve process using the measured data and equation (1) is employed to estimate the value of L'_n , C'_n , R'_n and C_p .

The ratio of the modulus of the maximum conductance to the modulus of the susceptance at the series resonance frequency corresponds to the figure of merit M [1]:

$$M = Q_m K = Q_m k^2 = \frac{|Y_{m0}|}{|Y_d|} = \frac{1/R'_n}{\omega_s C_p} \quad (2)$$

where ω_s is the series resonance frequency.

This figure of merit is employed by Guyomar et al.[2] to decide when a non-linear rectifier will harvest more power than a diode bridge since if $M \leq \pi$, the non-linear technique harvests more power than the linear technique. However, there is no prediction about the percentage of the harvested power that will be gained employing one or the other technique. Two piezoelectric cantilevers based on the QP40W and V21BL piezoelectric elements from Mide are employed to present the results obtained with a value of M equal to 0.42 and 1.13, respectively.

3. Piezoelectric Equivalent Electrical Impedance with a Resistive Load

The analysis done by Brufau-Penella et al. [3] allows calculating the power ratio that is defined as the ratio of the maximum power harvested with resistive load R_L to the maximum power harvested with a complex load employing the elements of the piezoelectric equivalent circuit. However, the argumentation related to the Thevenin equivalent model is not valid when a load is connected to the piezoelectric element since in this case the load is placed in parallel to the piezoelectric electrical capacitor C_p as it is exposed in [4].

The electrical equivalent impedance of the mechanical part of the piezoelectric transducer is the same for all the circuits under consideration and is given by the series connection of L'_n , C'_n , R'_n :

$$Z_{mech}(j\omega) = R'_n + j\omega L'_n + \frac{1}{j\omega C'_n} = R_{mech} + jX_{mech} \quad (3)$$

If a resistive load is connected to the piezoelectric transducer, the electrical equivalent impedance corresponds to the impedance of capacitor C_p in parallel with R_L :

$$Z_{elec}(j\omega) = \frac{R_L}{1 + \omega^2 C_p^2 R_L^2} (1 - j\omega C_p R_L) = R_{elec} + jX_{elec} \quad (4)$$

The harvested power ratio of a resistive load to a complex load is defined as the ratio of the harvested power employing resistor R_L as a load to the harvested power employing the complex conjugate of electrical impedance Z_{mech} , which corresponds to the maximum harvested power:

$$P_{ratio} = \frac{4R_{elec}R_{mech}}{(R_{mech} + R_{elec})^2 + (X_{mech} + X_{elec})^2} \quad (5)$$

Thus, there will be piezoelectric elements for which the power ratio is close to one and others for which is far from one. The peak value of the power ratio for two different piezoelectric cantilevers based on the QP40W and the V21BL piezoelectric elements is 0.58 and 0.91, respectively.

4. Piezoelectric Equivalent Electrical Impedance with a Diode Bridge

The connection of a diode bridge circuit with a resistive load and a filtering capacitor to a piezoelectric generator, see figure 1, has two equivalent circuits, as shown in figure 2, depending if the diodes are not conducting (circuit A) or are conducting (circuit B).

The electrical impedance for circuits A and B is:

$$Z_{elcda}(j\omega) = \frac{1}{j\omega C_p} \quad (6)$$

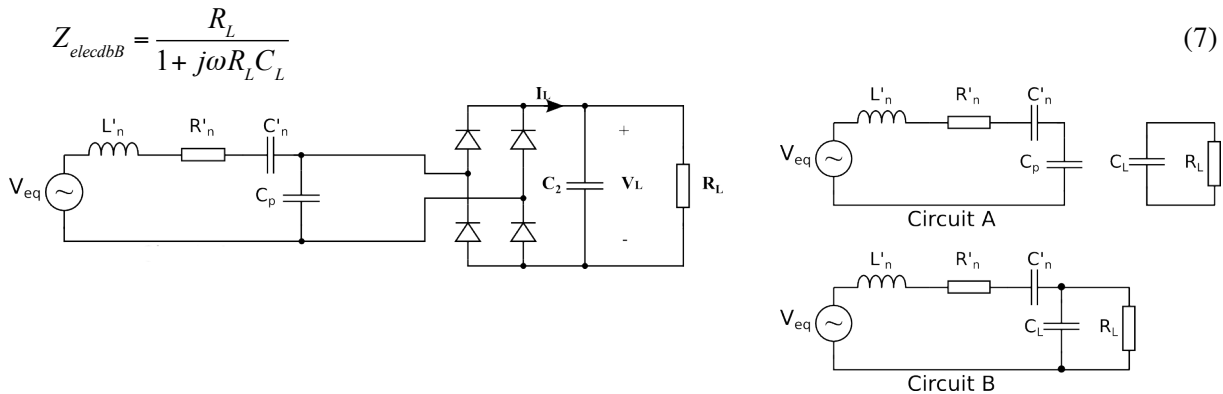


Figure 1. AC-DC converter circuit composed by a piezoelectric element connected to a diode bridge.

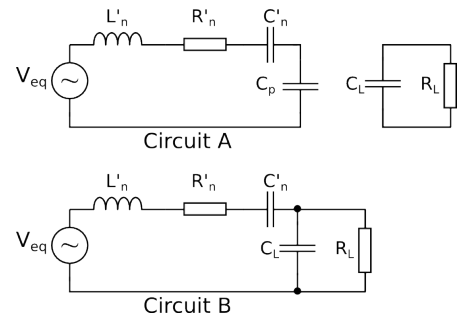


Figure 2. Two equivalent circuits of figure 1 depending on the fact that the current flows through the diodes (circuit B) or not (circuit A).

Therefore, the transfer function of the electrical impedance of a diode bridge connected to resistor R_L and capacitor C_L is given by:

$$Z_{elecdb}(j\omega) = \frac{Z_{elecdbA}(j\omega)(1 - e^{-j\theta}) + Z_{elecdbB}(j\omega)(e^{-j\theta} - e^{-j\omega T/2})}{1 - e^{-j\omega T/2}} \quad (8)$$

where between 0 and θ only C_p is connected to Z_{mech} and between θ and $T/2$ the current flows through the diodes and C_L and R_L are also connected to the piezoelectric element. The transfer function of the electrical impedance has a period $T/2$. The value of θ is calculated combining the following two equations:

$$V_L = I_L R_L = \frac{1}{T/2} \int_{\theta/\omega}^{T/2} \frac{V_{eq} \omega}{L'_n} \frac{1}{\left(\left(\frac{C_p + C'_n}{L'_n C_p C'_n} - \omega^2 \right)^2 + \left(\frac{\omega R'_n}{L'_n} \right)^2 \right)^{1/2}} \sin \omega t \, dt \quad (9)$$

$$\cos \theta = 1 - 2V_L / V_{eq} \quad (10)$$

where V_{eq} corresponds to the open circuit voltage of the piezoelectric element.

The harvested power ratio for this case is defined as the ratio of the harvested power connecting a diode bridge to a piezoelectric element to the maximum harvested power:

$$P_{ratiodb} = \frac{4R_{elecdb}R_{mech}}{\left(R_{mech} + R_{elecdb} \right)^2 + \left(X_{mech} + X_{elecdb} \right)^2} \quad (11)$$

If the power ratio is close to one, then a linear technique like the diode bridge rectifier would give a result as good as a non-linear technique without the complexity at the circuit level. The harvested power is the power of R_{elecdb} that is not equal to the power of R_L . The power of the load is a fraction of the harvested power. Figures 3 and 4 show the harvested power ratio for piezoelectric elements QP40W and V21BL respectively as a function of frequency and resistor R_L when a diode bridge is employed as AC-DC converter. The peak value of the power ratio is 0.57 and 0.92 for figures 3 and 4, respectively. The results obtained from an electrical simulation of the piezoelectric transducers connected to a diode bridge have been compared to the results obtained from the equations of this section and match.

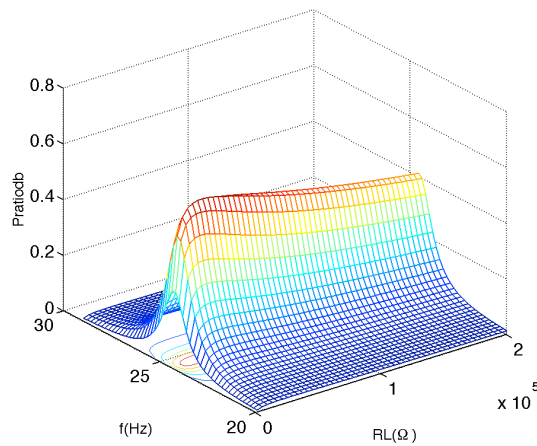


Figure 3. Power ratio of the diode bridge as a function of RL and frequency for the cantilever based on the piezoelectric element QP40W.

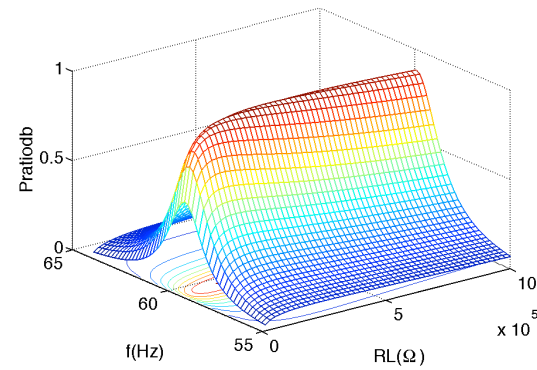


Figure 4. Power ratio of the diode bridge as a function of RL and frequency for the cantilever based on the piezoelectric element V21BL.

5. Piezoelectric Equivalent Electrical Impedance with a SSHI Converter

The connection of a SSHI circuit with a resistive load and a filtering capacitor to a piezoelectric generator, as shown in figure 5, has three equivalent circuits depending if the diodes are not conducting (circuit A), are conducting (circuit B) or the current is flowing through inductor L (circuit C), see figure 6.

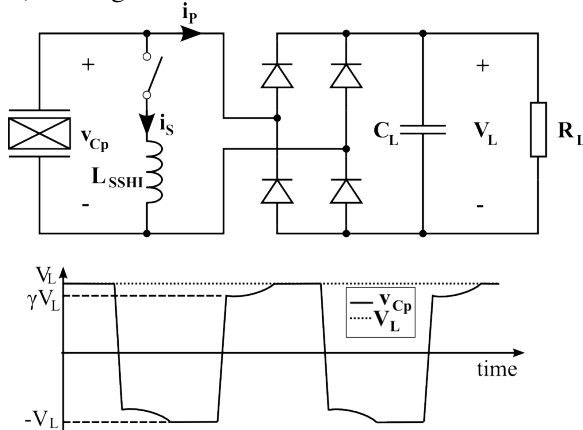


Figure 5. AC-DC converter circuit composed by a piezoelectric element connected to parallel SSHI circuit.

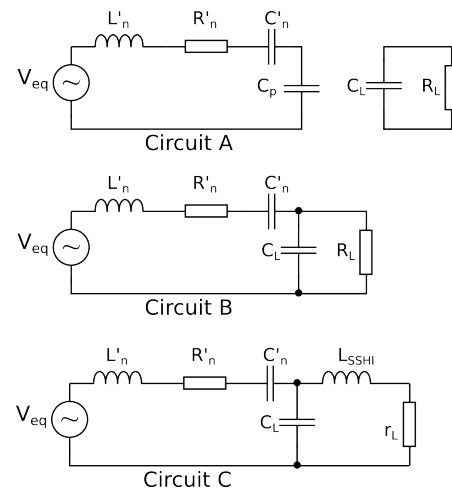


Figure 6. Three equivalent circuits of figure 1 depending on the fact that the current does not flow through diodes (circuit A), flows through diodes (circuit B) or through inductor L_{SSH} (circuit C).

A non-linear technique implies the inversion of the piezoelectric voltage through inductor L_{SSH} when the maximum displacement of the piezoelectric element is reached. The voltage inversion factor γ is calculated considering that in circuit C of figure 6, capacitor C_p has an initial voltage V_L that corresponds to the rectified voltage on R_L . The value of γ is given by the following expression:

$$\gamma = -C_p L_{SSH} \omega_n e^{-\xi \omega_n T_d / 2} \left(\frac{\alpha^2 - 2\alpha \xi \omega_n + \omega_n^2}{1 - \xi^2} \right)^{1/2} \sin \left(\omega_n (1 - \xi^2)^{1/2} T_d / 2 + \theta \right) \quad (12)$$

where r_L is the effective associated series resistance of inductor L_{SSH} , $\alpha = r_L / L$, $\xi = r_L / 2(C_p / L)^{1/2}$, $\omega_n = (1 / (LC_p))^{1/2}$, $T_d / 2 = \pi / (\omega_n \xi)$ and $\theta = \tan^{-1}(\omega_n (1 - \xi^2) / (\alpha - \xi \omega_n))$.

Figures 7 and 8 show the voltage on capacitor C_p as a function of time for cantilevers based on piezoelectric elements QP40W and V21BL, respectively, for different values of inductor L_{SSH} and r_L . Piezoelectric element V21BL obtains a better voltage inversion factor γ than piezoelectric element QP40W.

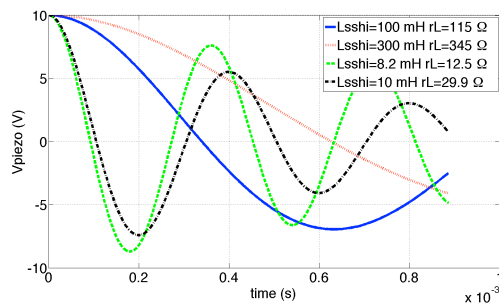


Figure 7. Inversion voltage on C_p as a function of time for cantilever based on piezoelectric element QP40W.

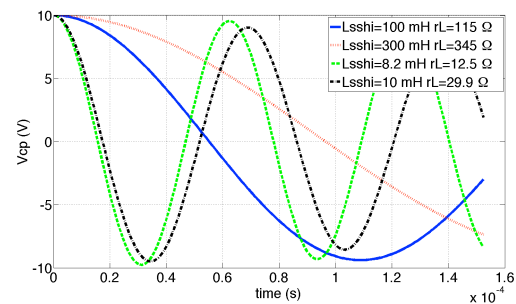


Figure 8. Inversion voltage on C_p as a function of time after closing MOSFET T_1 for cantilever based on piezoelectric element V21BL.

6. Conclusions and Future Work

The electrical equivalent impedance of a resistor and a diode bridge has been calculated. Then, the power ratio defined as the harvested power by each one of the two circuits to the maximum harvested power has been calculated and compared to the results obtained from electrical simulations. One of the piezoelectric elements harvests only a 0.57 of the available harvested power while the other one has a power ratio close to 1. If the harvested power of the diode bridge is close to the harvested power using the complex conjugate of the mechanical impedance as a load, a non-linear technique will not provide a benefit in terms of harvested power.

Further work will include the calculation of the electrical impedance of the parallel SSHI converter to determine the power ratio of this rectifying technique.

References

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