

# Multiphysics finite element model of a frequency-amplifying piezoelectric energy harvester with impact coupling for low-frequency vibrations

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**Abstract.** This paper presents experimentally-verified multiphysics finite element model of a wideband vibration energy harvester with impact coupling, which operates on the principle of frequency up-conversion: under low-frequency harmonic base excitation a cantilever-type resonator (with resonant frequency of 18.8 Hz) impacts a high-frequency piezoelectric cantilever, which starts freely vibrate at its resonant frequency of 374 Hz. Such input frequency amplification enables efficient power generation under low-frequency ambient excitations. The model was implemented in COMSOL and the contact between the cantilevers was formulated by using a nonlinear viscoelastic model. Reported results of dynamical and electrical testing of the fabricated vibration energy harvester confirm the accuracy of the model as well as reveal some operational characteristics of the device under varying impact and excitation conditions.

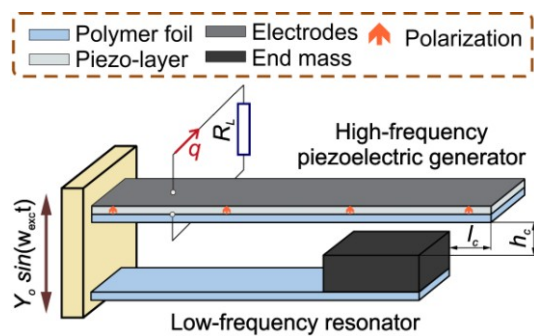
## 1. Introduction

Deriving electrical energy from ambient vibrations through piezoelectric conversion represents one of the most promising approaches to power MEMS-based sensors since kinetic energy is abundant in the environment, while piezoelectric transducers exhibit the highest energy density and are amongst the most cost-effective solutions in terms of fabrication at micro and nano scales [1]. Vibration energy harvesters (VEHs) are mostly based on linear oscillators, generating maximum power only when driven at their resonant frequency. Slight deviation from the resonance leads to a significant drop in power output. Tackling the issue of narrow bandwidth currently represents one of the major challenges in the field [2]. Three main categories of broadband VEHs may be distinguished [3]: i) linear generators (e.g. generator arrays, coupled resonators), ii) nonlinear generators (e.g. mono/bi/multi-stable structures, frequency up-converting systems, generators with resonance tuning), iii) generators exploiting advanced power conditioning circuits. This study is concerned with the frequency up-conversion, whereby low-frequency (LF) resonator impulsively triggers high-frequency (HF) self-oscillations of the piezoelectric generator(s) using either magnetic [4] or impact coupling [5]. The former avoids the issue of contact wear that is inherent to the latter but the usage of ferromagnetic

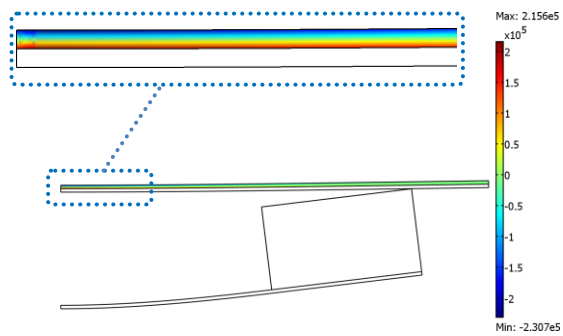


components increases device volume leading to reduced power density. On the other hand, impact coupling is a more straightforward approach for implementing frequency up-conversion, but care must be taken in selecting materials of contacting surfaces to avoid premature failure due to contact fatigue.

This study considers an impact-coupled vibration energy harvester (IC-VEH), where rigid end mass of the LF resonator impacts a soft polymeric substrate of the piezoelectric generator (instead of brittle piezoceramics), providing favorable conditions to prolong contact fatigue lifetime [6]. The paper is devoted to multi-physics finite element (FE) modeling of the IC-VEH. Device characterization results are presented in order to show accuracy of the developed model and provide some insight into the operational characteristics of the harvester. To the best of the authors' knowledge this model is the first attempt to simulate with FE a piezoelectric IC-VEH as a vibro-impact system consisting of two deformable structures as only analytical models are reported in the literature [7,8].



**Figure 1.** Schematics of the modeled and tested impact-coupled vibration energy harvester.



**Figure 2.** FE model of the IC-VEH: close-up shows electric field distribution ( $a = 0.5g$ ).

## 2. Finite element modeling and testing of impact-coupled vibration energy harvester

### 2.1. Device description

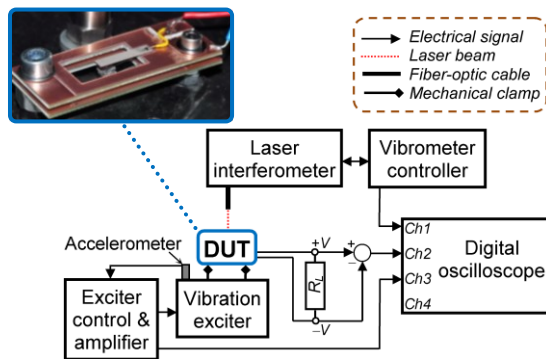
The considered IC-VEH consists of polymeric LF resonator with steel end mass mounted below the HF piezoelectric generator connected to the load  $R_L$  (Figure 1). LF resonator impacts the generator at a contact point that is located at a distance  $l_c$  from the free end of the piezoelectric cantilever. Contact gap  $h_c$  is defined as a distance between the generator and resonator tip when it is displaced downwards due to gravity. Devices design (Table 1) is based on polymeric cantilevers that are patterned by laser cutting. It was realized by a variation of the process flow reported in [6]. LF resonator is fabricated from PET foil and has steel end mass, while piezoelectric generator comprises bulk PZT-5H sheet laminated on PET substrate. Dry adhesive is used for bonding of structural layers.

### 2.2. Model formulation

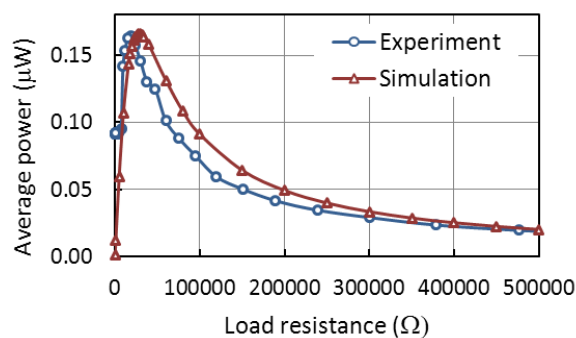
The considered IC-VEH constitutes a nonlinear vibro-impact system comprised of two elastic deformable structures undergoing dynamic contacting under harmonic base excitation. A 2D finite element (FE) model of the harvester (Figure 2) was developed in COMSOL. External resistive load was introduced into the model, enabling prediction of generated electrical power. Mechanical contact between the LF resonator and piezoelectric generator was implemented by using a nonlinear viscoelastic model of Hunt and Crossley. The model assumes small contact surface and is valid for direct central and frictionless contact. The normal contact force in Hunt-Crossley model is represented by the pure elastic Hertz's law combined with a nonlinear damping term [9]:

$$F_c = k_c z(t)^\alpha + c_c z(t)^\alpha v(t) \quad \text{for } z(t) < 0 \text{ and } v(t) < 0 \quad (1)$$

where  $k_c$  – contact stiffness coefficient,  $c_c$  – contact damping coefficient,  $z(t)$  – indentation,  $v(t)$  – indentation velocity,  $\alpha$  – force exponent depending on contact surface geometry ( $\alpha=2$  is assumed).



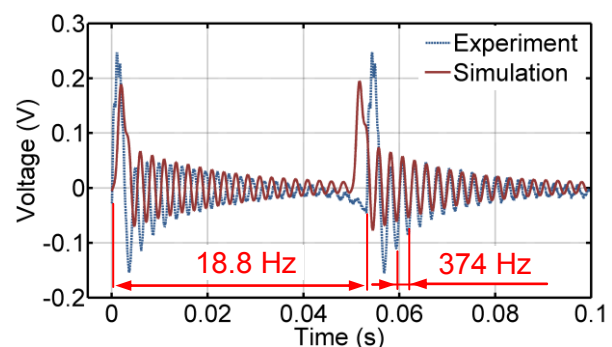
**Figure 3.** Experimental setup for IC-VEH testing.



**Figure 4.** Experimental and simulation plots of average power vs. load resistance ( $a = 0.5g$ ).

**Table 1.** Dimensions of the simulated and tested impact-coupled vibration energy harvester.

Component	LF resonator	Piezoelectric cantilever
	l×w×t (mm)	
Polymeric substrate (PET)	13×4×0.125	15×4×0.125
Piezoelectric layer (PZT-5H)	-	15×4×0.130
Steel end mass	4.8×4×3.2	-



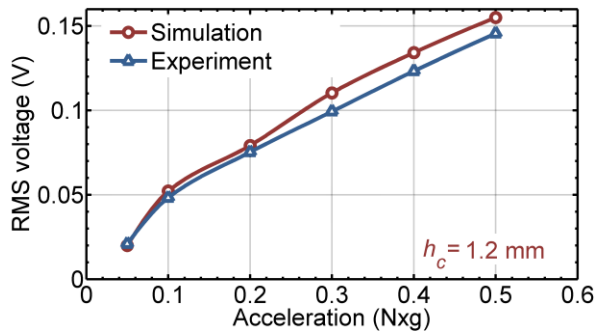
**Figure 5.** Measured and simulated voltage signals ( $h_c = 1.2$  mm,  $a = 0.05$ g,  $R_L = 1$  M $\Omega$ ).

Eq. 1 indicates that damping term depends on the indentation, which is reasonable from physical point of view since contact area increases with indentation and a plastic region is more likely to develop for larger indentations. Another advantage of the model is that the contact force evolves continuously, i.e. it builds up from zero upon impact and returns to zero upon separation [9]. The model includes a single contact pair located on the right edge point of the end mass.

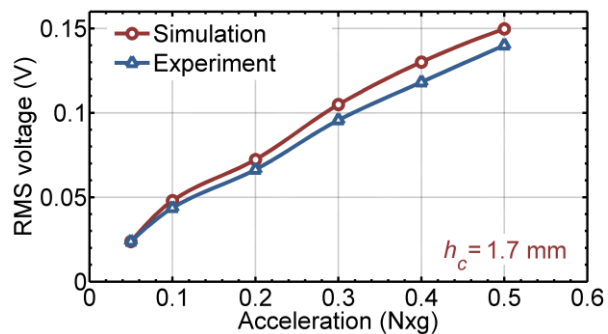
### 2.3. Model verification

Verification of the FE model was conducted in two steps. Firstly, the accuracy of piezoelectric generator model without impacting was tested by comparing results of experimental and numerical frequency response analyses performed by varying resistive load. The responses of the fabricated IC-VEH (Figure 3) were measured by means of automated vibration test bench [10]. Figure 4 indicates good agreement between experimental and simulation plots of average power as a function of load resistance. The relative error between the measured and the computed values of average power at optimal load and voltage at open circuit is less than 2%. Therefore the voltage output at open circuit is further used as a parameter for verifying and adjusting the contact model of the IC-VEH.

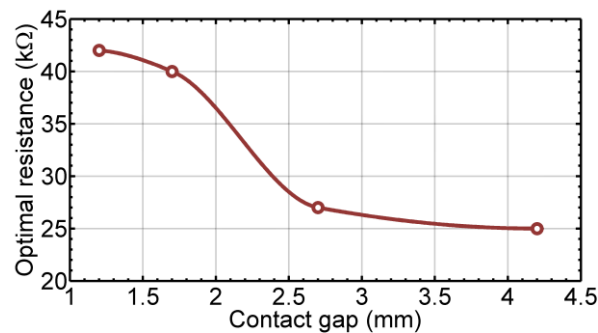
In the second step, values of  $k_c$  and  $c_c$  were determined by trial-and-error through multiple simulation tests. Transient voltage responses measured with the setup shown in Figure 3 were compared with the numerical responses computed under the equivalent excitation conditions. The values of  $k_c \approx 10^6$  and  $c_c \approx 10^{-6}$  were found to provide a good qualitative and acceptable quantitative agreement between the model predictions and measurements (Figure 5). The figure demonstrates the mechanism of frequency up-conversion in the IC-VEH. When the device is subjected to base excitation at 18.8 Hz (resonant frequency of the resonator), high-frequency decaying oscillations (374 Hz) are induced in the generator through impacting. Results of transient simulations performed with gap sizes of 1.2 mm and 1.7 mm indicate that subtle changes in RMS voltage magnitude are well reproduced for different gaps (Figures 6-7).



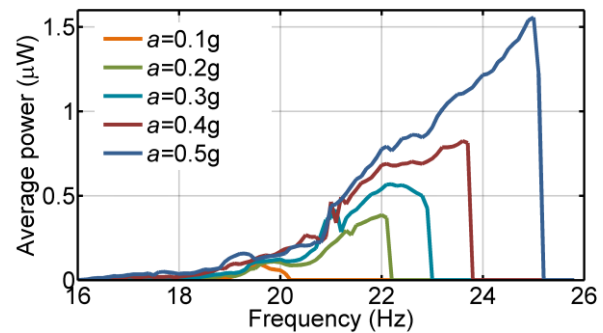
**Figure 6.** Measured and simulated RMS voltage values as a function of applied acceleration ( $h_c = 1.2$  mm,  $l_c = 2$  mm,  $f_{exc} = 18.8$  Hz,  $R_L = 1$  M $\Omega$ ).



**Figure 7.** Measured and simulated RMS voltage values as a function of applied acceleration ( $h_c = 1.7$  mm,  $l_c = 2$  mm,  $f_{exc} = 18.8$  Hz,  $R_L = 1$  M $\Omega$ ).



**Figure 8.** Variation of optimal load resistance as a function of contact gap  $h_c$  ( $a = 0.2g$ ).

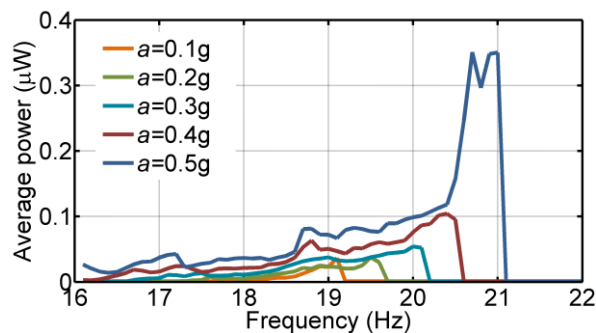


**Figure 9.** Measured power frequency responses during harmonic up-sweep at various acceleration magnitudes ( $h_c = 1.2$  mm,  $R_{opt} = 42$  k $\Omega$ ).

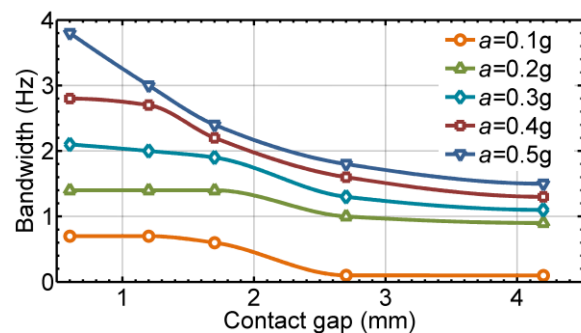
It can be also noted from the figures that variation of discrepancy between experimental and numerical values follows the same trend for both gap sizes, i.e. slightly increases with applied acceleration. Performed verification indicates that an average relative error constitutes less than 10%.

#### 2.4. Characterization results

Contact gap is an important design parameter in IC-VEH therefore experimental study was carried out to examine the influence of gap size on optimal resistance, power output and bandwidth. Measurements show that optimal resistance of the generator under normal resonant operation is about 22 k $\Omega$ . However results derived from frequency responses recorded using different contact gaps in 1.2–4.2 mm range reveal increasing optimal resistance with smaller gaps (Figure 7). It was reported in [7] that characteristic frequency of the coupled vibration phase (when resonator and generator move together after contact) largely determines the power output in IC-VEHs. Therefore it could be reasoned that in our case the variation in contact gap evokes changes in the coupled vibration frequency and its contribution to the resonant frequency of the overall vibro-impact system, which in turn could change optimal load resistance ( $R_{opt} = 1/\omega_r C$ ). The performance of the IC-VEH in terms of power output and bandwidth was examined both for up-sweep and down-sweep excitation cases. Figures 9-10 indicate that up-sweep is considerably more beneficial. Measurements at different contact gaps and acceleration levels demonstrate that for up-sweep excitation the peak power is on average 400% larger than for the down-sweep, while in the case of bandwidth – 300%. Figure 11 illustrates the trend of diminishing bandwidth with larger contact gaps for various acceleration magnitudes during harmonic up-sweep. The results also suggest that the most advantageous contact gap size in terms of both power and bandwidth may be derived for the considered case.



**Figure 10.** Measured power frequency responses during down-sweep at various acceleration magnitudes ( $h_c = 1.2$  mm,  $R_{opt} = 42$  k $\Omega$ ).



**Figure 11.** Variation of bandwidth as a function of contact gap  $h_c$  for different acceleration magnitudes (up-sweep).

### 3. Conclusions

The paper presented multiphysics FE model of the IC-VEH that constitutes an electromechanical vibro-impact system composed of two cantilevers – a low-frequency resonator and a high-frequency piezoelectric generator that undergo impacting when subjected to harmonic base excitation. Such configuration realizes frequency up-conversion mechanism, which enables large input frequency amplification leading to efficient power generation under low-frequency input excitation. Vibro-impact interaction between the resonator and generator is accurately reproduced by implementing into COMSOL a nonlinear viscoelastic (Hunt-Crossley) contact model, yielding a small relative error between the experimental and numerical values of RMS voltage produced by the generator under open circuit condition. Results of performed characterization of the fabricated IC-VEH device in the automated vibration test bench indicate strong influence of the contact gap size on optimal load, average power and bandwidth of the harvester.

The presented FE model constitutes an effective and reliable design tool enabling optimization of mechanical and piezoelectric structures of IC-VEHs, which may have arbitrary geometry and configuration (e.g. multiple resonators and generators, segmented electrodes, etc.).

### Acknowledgments

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