

# Impedance matching for broadband piezoelectric energy harvesting

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**Abstract.** This paper presents a system design for broadband piezoelectric energy harvesting by means of impedance matching. An inductive load impedance is emulated by controlling the output current of the piezoelectric harvester with a bipolar boost converter. The reference current is derived from the low pass filtered voltage measured at the harvester terminals. In order to maximize the harvested power especially for nonresonant frequencies the filter parameters are adjusted by a simple optimization algorithm. However the amount of harvested power is limited by the efficiency of the bipolar boost converter. Therefore an additional switch in the bipolar boost converter is proposed to reduce the capacitive switching losses. The proposed system is simulated using numerical parameters of available discrete components. Using the additional switch, the harvested power is increased by 20 %. The proposed system constantly harvests 80 % of the theoretically available power over frequency. The usable frequency range of  $\pm 4$  Hz around the resonance frequency of the piezoelectric harvester is mainly limited due to the boost converter topology. This comparison does not include the power dissipation of the control circuit.

## 1. Introduction

A piezoelectric harvester (PEH) transforms the kinetic energy stored in mechanical vibrations into electrical energy using the piezoelectric effect. Usually a PEH consists of a piezo material mounted to a mechanical resonator which can be a cantilever beam for instance. In case a PEH is excited near its resonant frequency, a power in the range between 10  $\mu$ W and 10 mW can be harvested [1, 2]. This power is sufficient to supply wireless sensors nodes in a vibrating environment [3, 4, 5]. But due to the mechanical resonance behavior, the harvested power decreases considerably for nonresonant excitation frequencies. Since many vibrational environments do not have an a priori fixed frequency but rather a shifting frequency or even a random behavior, the PEH bandwidth must be increased to conform to the practical conditions [6].

The mechanical design approaches for broadening the bandwidth are summarized in [7, 8] and can be grouped as follows: resonance tuning, multi-modal and multi-frequency harvesting, frequency up-conversion and bistable oscillations. However the power spectrum of the PEH is not only affected by the mechanical resonator but also by the electrical load. Even at resonance the power harvested using a state of the art interface circuit (diode rectifier, SECE or SSHI [9, 10]) varies and depends mainly on the mechanical characteristics of the PEH. The theoretical



power limit is achieved when the electrical load behaves like the conjugate complex electrical impedance of the PEH [11, 12, 13]. Furthermore with a conjugate load impedance the harvested power at nonresonant frequencies could be theoretically the same compared to the excitation at resonance. Previously reported designs use the amplitude ratio and phase shift between the excitation acceleration and the PEH voltage to optimize the harvested power [11, 12]. Therefore an acceleration sensor is needed which is unnecessary in the proposed system.

This paper is organized as follows. First, the conjugate complex impedance matching of the PEH is described. Then, the control of a bipolar boost converter emulating a inductive load impedance for a constant frequency is proposed and simulated. Afterwards the optimization algorithm adjusting the emulated load impedance to maximize the harvested power also for changing frequencies is presented. Finally, the simulation results are compared with a similar system design presented in [13].

## 2. Emulating an inductive load impedance

The electromechanical model of the PEH is shown in figure 1(a), where  $m, k$  and  $d$  are the mass, spring and damping constants of the mechanical oscillator and  $\dot{x}$  its velocity. The electromechanical coupling and the capacitance of the piezoelectric material is modeled by  $\gamma$  and  $C_p$ . The voltage and current on the PEH terminal are denoted by  $v$  and  $i$ .

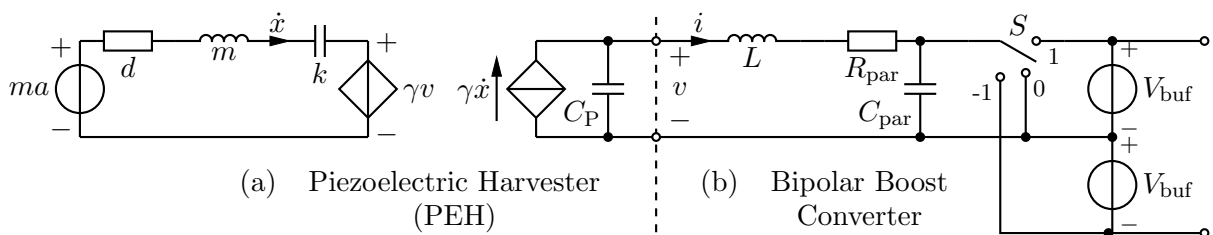
The excitation acceleration  $a$  is assumed to be sinusoidal with the amplitude  $\hat{a}$  and the frequency  $f$ . Then, the maximum power limit  $P_{\text{lim}}$  is harvested when the PEH is connected to a load impedance  $Z_0$  which is equal to the conjugate complex PEH impedance  $Z_{\text{PEH}}$ :

$$P_{\text{lim}} = \max \overline{vi} = \frac{(m\hat{a})^2}{8d} \iff Z_0 = Z_{\text{PEH}}^*(f) . \quad (1)$$

As the PEH impedance is frequency dependent the load impedance must be adjusted whenever frequency changes. Since the power limit  $P_{\text{lim}}$  is frequency independent, the same power for resonant as well as nonresonant excitation could be harvested. The applicable frequency range is limited due to the reactive power flowing alternately between the energy reservoirs of the load and the PEH. It can be shown that the reactive power needed for impedance matching increases for nonresonant frequencies. Since the power transfer between the PEH and the energy storage in the load is associated with losses the usable harvested power decreases.

In order to realize a complex load impedance, the bipolar boost converter in figure 1(b) is able to transfer power from the PEH into the buffer storage  $V_{\text{buf}}$  and vice versa. That means, for positive and negative PEH voltages  $v$ , the current  $i$  through the inductor  $L$  can rise and fall as long as  $|v| < V_{\text{buf}}$ . The resistive and capacitive parasitics of the inductor  $L$  and the switch  $S$  are summarized by  $R_{\text{par}}$  and  $C_{\text{par}}$  modeling the power losses of the converter.

According to equation (1) the load impedance  $Z_0$  must be inductive since the PEH impedance  $Z_{\text{PEH}}$  is capacitive. A possible realization of an inductive load impedance would be  $Z_0 = R_0 + sL_0$  where  $R_0$  and  $L_0$  are adjustable to satisfy equation (1). If the PEH voltage  $v$  is measured then



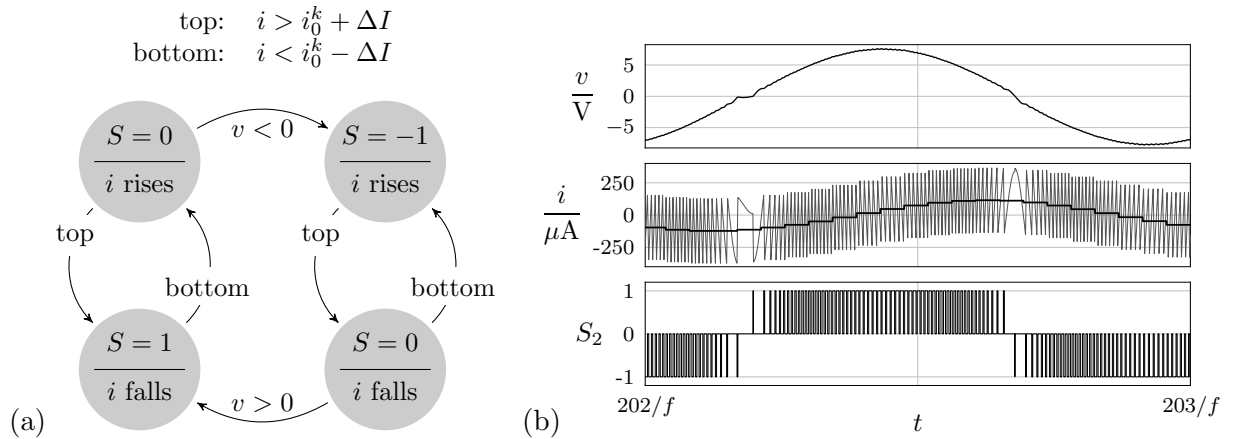
**Figure 1.** Model of the piezoelectric harvester and the bipolar boost converter.

the current  $i_0$  through  $Z_0$  is calculated by

$$I_0(s) = \frac{V(s)}{R_0 + sL_0} = A_0 \frac{V(s)}{1 + s/\omega_0} \quad \text{with} \quad A_0 = \frac{1}{R_0} \quad \text{and} \quad \omega_0 = \frac{R_0}{L_0}.$$

Thus, the current  $i_0$  through the emulated load impedance is determined by low pass filtering the PEH voltage  $v$ . Since the gain  $A_0$  and the cutoff frequency  $\omega_0$  must be adjustable, a digital filter realization with the sampling time  $t^k = kT_s$  is preferred. Before the measured PEH voltage  $v^k$  is low pass filtered it is extrapolated a half time step into the future compensating for the time delay of the sampled output  $i_0^k$ . The extrapolated PEH voltage  $\tilde{v}^k \approx v^k(t^k + T_s/2)$  is estimated by a cubic polynomial fitted to the last eight measurements.

After determining the sampled current  $i_0^k$  flowing through the emulated load impedance  $Z_0$ , the bipolar boost converter in figure 1(b) is used to pull this current out of the PEH in average. This is achieved by a hysteresis control which sets the switch position  $S$ . Using only the two switch positions  $S \in \{-1, 1\}$ , the current  $i$  can be easily controlled in a hysteresis window  $\Delta I$  around  $i_0^k$  as follows. In the switch position  $S = -1$  the current  $i$  rises as long as  $|v| < V_{\text{buf}}$ . If  $i$  reaches the upper hysteresis boundary  $i_0^k + \Delta I$ , the switch position  $S = 1$  forces the current  $i$  to fall towards to the lower boundary  $i_0^k - \Delta I$ . With that two-switch hysteresis control the capacitive energy loss in each switching cycle is equal to  $4V_{\text{buf}}^2 C_{\text{par}}/2$ . This energy loss can be reduced by a factor of four if the additional switch  $S = 0$  is used. The finite state machine of the three-switch hysteresis control is shown in figure 2(a). If the PEH voltage  $v$  is positive then the current  $i$  is controlled by the switch positions  $S \in \{0, 1\}$ , otherwise the switch positions  $S \in \{-1, 0\}$  are used. A simulation of the proposed system design is shown in figure 2(b). The applied parameters are given in Table 1 in which  $R_{\text{par}}$ ,  $C_{\text{par}}$  and  $L$  are extracted from available



**Figure 2.** (a) Finite state machine of the three-switch hysteresis control. (b) Quasi steady state of the proposed system. The thick line in the middle plot shows the sampled current  $i_0^k$ .

**Table 1.** Applied numerical parameters for available discrete components.

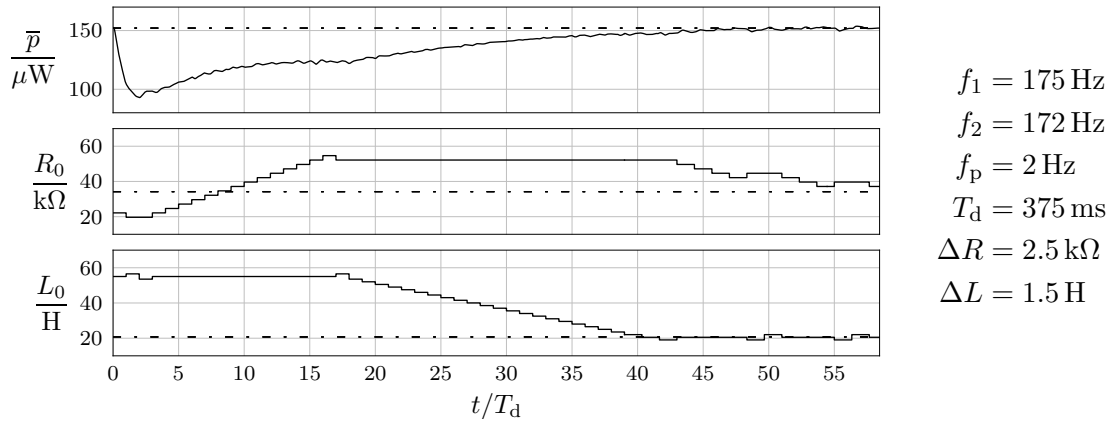
Piezoelectric Harvester (Mide V22B)		Bipolar Boost Converter		Load Impedance
$m = 1.8 \cdot 10^{-3} \text{ kg}$	$\hat{a} = 4 \text{ m/s}^2$	$V_{\text{buf}} = 10 \text{ V}$	$L = 180 \text{ mH}$	$f = 175 \text{ Hz}$
$d = 0.0426 \text{ kg/s}$	$C_{\text{P}} = 23 \text{ nF}$	$R_{\text{par}} = 15 \Omega$	$\Delta I = 250 \mu\text{A}$	$R_0 = 19.7 \text{ k}\Omega$
$k = 2076.5 \text{ N/m}$	$\gamma = 1.07 \cdot 10^{-3} \text{ C/m}$	$C_{\text{par}} = 15 \text{ pF}$		$L_0 = 55 \text{ H}$
$f_{\text{res}} = 170.94 \text{ Hz}$	$P_{\text{lim}} = 152.11 \mu\text{W}$			$T_s = 250 \mu\text{s}$

discrete components. The mean power flowing out of the harvester is  $0.99 P_{\text{lim}}$ . The gap to  $P_{\text{lim}}$  results from a small time shift between the emulated load current  $i_0$  and the mean of the modulated current  $i$ . This is caused by the irregular switching at the zero crossings of  $v$ .

### 3. Broadband impedance matching

According to equation (1), the load impedance  $Z_0 = R_0 + sL_0$  must be adjusted whenever frequency changes since the PEH impedance  $Z_{\text{PEH}}(f)$  is frequency dependent. If the PEH model parameters are known in advance the frequency  $f$  can be measured and  $R_0$ ,  $L_0$  are determined by a simple look-up table. However, in practice the model parameters are not sufficiently known and slightly depend on the bias point of the PEH. For this reason the averaged output power  $\bar{p} = \overline{vi}$  is maximized by adjusting the load parameters  $R_0$  and  $L_0$  with a two-dimensional optimization method. The utilized algorithm is known as coordinate-search method [14]. Alternately, one of the two coordinates  $R_0$ ,  $L_0$  is changed with the constant step size  $\Delta R$  or  $\Delta L$  as long as the averaged power  $\bar{p}_0$  increases. If a local maximum along one coordinate is found further optimization is done along the other coordinate. The time delay between two steps  $T_d$  must correspond to the settling time of the total system which is shorter for smaller step sizes. For this reason the coordinate-search method is chosen over the simplex algorithm where both coordinates are changed at the same time resulting in higher transient overshoots.

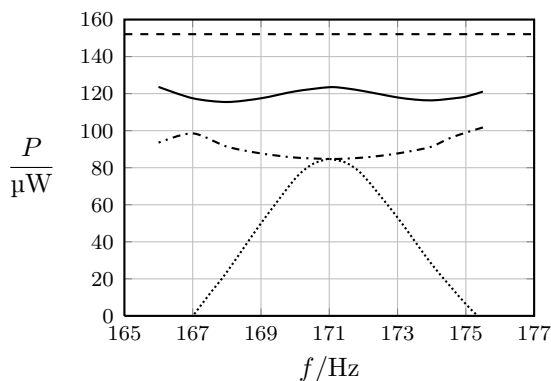
The averaged output power  $\bar{p} = \overline{vi}$  is approximated by averaging the instantaneous power through the emulated load impedance  $p_0 = R_0(i_0^k)^2$ . Using the approximation  $\bar{p}_0 \approx \bar{p}$  which is caused by the phase shift between  $i_0$  and  $i$ , an extra measurement of the fast changing current  $i$  is avoided. The averaging of  $p_0$  is done by a third order Bessel low pass filter with a cutoff frequency  $f_p \ll f$ . Figure 3 shows the averaged output power  $\bar{p}$  while optimizing  $\bar{p}_0$  after a frequency step from  $f_1$  to  $f_2$  at  $t = 0$ . The steady state power ripple is necessary to detect the next frequency change, but can be further reduced with smaller step sizes.



**Figure 3.** Illustration of the coordinate search algorithm after a frequency step from  $f_1$  to  $f_2$  at  $t = 0$ , maximizing the averaged PEH output power  $\bar{p} = \overline{vi}$ . The dash dotted lines mark  $P_{\text{lim}}$ ,  $R_0 = \text{Re } Z_{\text{PEH}}(f_2)$ ,  $L_0 = -\text{Im } Z_{\text{PEH}}(f_2)/(2\pi f_2)$ , from top to down.

By means of this optimization the emulated load impedance is matched to the conjugate complex impedance of the PEH for arbitrary frequencies around the PEH resonant frequency. Figure 4 compares the harvested power flowing into the two buffer sources  $V_{\text{buf}}$  for three different operating conditions. This comparison does not consider the power consumption of the required control circuitry. The dotted line shows the power which is harvested with the system concept presented in [13]. By using only the two-switch hysteresis control this concept emulates a *constant* load impedance which is equal to the conjugate complex impedance of the harvester at resonance.

The next two cases show the improvements of the proposed system. First, the dash dotted line illustrates the improvement by using an optimized load impedance. Finally, the latter case is further improved (solid line) by using the three-switch hysteresis control which reduces the capacitive power losses. In contrast to the description of equation (1) the harvested power spectrum is approximately flat and limited to the shown frequency range. Since the reactive power needed for impedance matching increases for nonresonant frequencies the amplitude  $\hat{v}$  of the PEH voltage also increases. The bipolar boost converter stops working properly if  $\hat{v} \geq V_{\text{buf}}$ . However, if  $\hat{v} \lesssim V_{\text{buf}}$  then the efficiency increases due to the lower switching frequency.



**Figure 4.** Theoretical power limit  $P_{\text{lim}}$  of the PEH (dashed), harvested power with the three- (solid) and two- (dash dotted) switch hysteresis control, harvested power with a similar system concept presented in [13] (dotted) using a hysteresis window of  $\Delta I = 1.5 \text{ mA}$ .

#### 4. Conclusion

A system design has been presented optimizing the power harvested from a nonresonant, sinusoidally and continuously excited PEH. The proposed concept harvests a power of  $0.8 P_{\text{lim}}$  which is approximately constant over frequency. This power is 40 % larger than the peak power of the concept presented in [13]. An experimental validation of the presented system design has to be performed. The power consumption of the required control circuitry is expected to be much higher compared to implementation of the SECE scheme which needs less than  $5 \mu\text{W}$  [9]. Hence the switching frequency must be decreased with a more sophisticated hysteresis control.

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