

Bandwidth of a Nonlinear Harvester with Optimized Electrical Load

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Abstract. Many researchers have investigated the possibility of amplifying ambient vibrations and converting the associated kinetic energy into usable electric energy. The vast majority of vibration harvesting devices use mechanical oscillators to boost the amplitude of vibration; however, this can result in a rather narrow band of excitation over which the harvesting device is effective. One approach proposed to overcome this limitation is to substitute the conventional linear oscillator with an oscillator featuring a non-linear compliance characteristic: these mechanisms produce broader frequency responses. The design and optimization of non-linear energy harvesting devices is however not trivial and there is no consensus among the published works that the benefits of non-linear oscillators can be realized in the energy harvesting context. This work attempts to further develop understanding of nonlinear energy harvesters by investigating the optimum resistive load. The definition of an optimal load for the nonlinear device is first considered, given due consideration to bandwidth and stability of the operating point, and comparisons with linear devices is shown. Finally, the issue of multiple solutions in the frequency response is addressed.

1. INTRODUCTION

Energy harvesting from vibrations has become a viable option for powering small devices or low power electronics. Harvesters based on resonant oscillators, in particular, are widely used to obtain high efficiency devices that deliver maximum power to the electrical load. A key strength of this technique is that it can be adapted to different transduction mechanisms. However there are also disadvantages: for the oscillator to amplify efficiently the input excitation, its damping coefficient has to be small. This results in the device having a narrow bandwidth of operation.

Methods to improve the bandwidth involve changing the geometrical characteristics of the oscillator [1], using external mechanism like magnets [2] or piezoelectric materials [3] to change the stiffness of the oscillator, and designing the system so that multiple frequency are amplified (multiple degree of freedom systems) [4]. Another possible solution to the problem is to exploit the electrical force reflected on the oscillator by the transducer [5, 6]. Finally, exploiting the characteristics of nonlinear oscillator to widen the frequency response of energy harvesting, has been studied in several works [7, 8]. Compliance characteristics allowing changes from hardening to softening [3, 9] and bistable configurations [10] have been proposed.



This work focuses on energy harvesters with hardening compliance - one of the most common forms of nonlinearity. In the literature it is suggested that the presence of hardening nonlinearity results in an increase of bandwidth. We first discuss the optimal electrical resistive load for harvesting devices with a hardening nonlinearities, following[11]. We then provide information on how to select both the stiffness and the resistive load of the nonlinear harvester to achieve maximum power. A comparison of the bandwidth of an optimal linear and nonlinear device is then presented. One major drawback with the nonlinear device is the presence of a low power stable solution. A potential method of forcing the nonlinear harvester response onto the desirable high power solution is presented.

2. Optimal load for a nonlinear harvester

For energy harvesters the efficiency of the device is heavily influenced by characteristics of the load it is powering, as this governs the current and hence the force that is reflected back on the mechanical oscillator.

2.1. Optimal resistive load

Considering a purely resistive load, for a linear harvester featuring an electromagnetic transducer, it has been shown that maximum power is delivered to the load when the energy absorbed in the electrical side is equal to that dissipated mechanically. This is not necessarily true for a nonlinear device, where the response often depends on the input excitation and initial conditions [12, 13]. In addition, the frequency response is highly influenced by the forcing and dissipation levels. Here we consider a hardening nonlinearity with the compliance governed by $F_e = kx + k_{nl}x^3$, where x is the displacement of the oscillator. Specifically we set $k = 300\text{N/m}$, $k_{nl}=1.02\times 10^8\text{N/m}^3$, the mechanical damping to be 4.8Ns/m and $\theta=8.9\text{Vs/m}$. A frequency response of the harvester in terms of the relative displacement between the coil and stator is shown in figure 1(a) for the case where the sinusoidal input excitation has amplitude $1.75\text{ }\mu\text{m}$. When the harvester is excited open circuit, such that only mechanical damping losses occur, the displacement of the oscillator reaches its maximum. With a 30Ω resistive load, the additional electrical energy losses result in a reduced amplitude of the response. Note that the maximum amplitude is reached at different frequencies for different electrical loads. This implies that the optimal load resistance varies with the characteristics of the harvester a also with the amplitude and frequency of excitation. This has been shown in [11], where an approximated expression for the optimal load was found to be

$$R_L = \frac{\theta^2}{\frac{m Y_0 \Omega_t \sqrt{3\alpha}}{2 \sqrt{\Omega_t^2 - \omega_n^2}} - c_m}, \quad (1)$$

where θ is the electromechanical coupling coefficient, c_m is the mechanical damping coefficient, m is the moving mass of the oscillator, ω_n is the underlying linear natural frequency and $\alpha = k_{nl}/m$. Y_0 is the amplitude of the sinusoidal base input. The frequency Ω_t is the tuning frequency (the frequency at which maximum power is harvested). This frequency varies with resistive load as shown in figure 1(b). Note that at the two limits, open and short circuit, the power delivered to the load is zero.

2.2. Optimal linear natural frequency

To widen the range in which the harvester can be tuned to the frequency of excitation, without decrease the power harvested, one possible solution is to change the underlying natural resonant frequency. Using the optimal resistance, equation 1, the optimal natural frequency and the

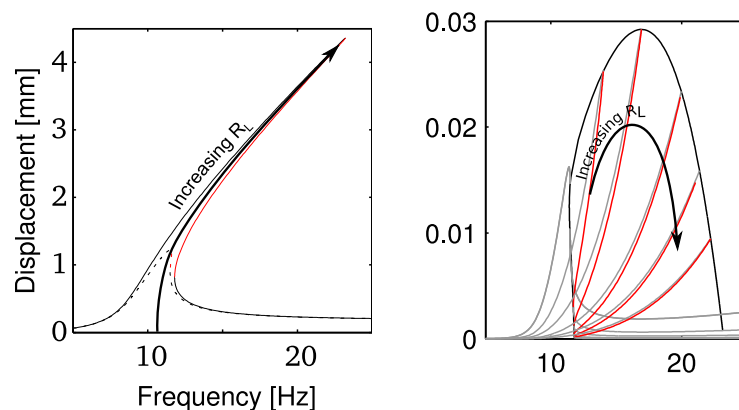


Figure 1. Frequency response of the nonlinear energy harvester in terms of relative displacement between the coil and the stator (a) and of the power delivered to the load (b). In panel (a) the displacement occurring open circuit (solid line) and powering a resistance of 30 Ω (dashed line) is shown. The red lines are representative of unstable solutions. Panel (b) shows the power vs frequency response of the harvester powering different load resistances.

optimal power can be found to be

$$\omega_n^2 = \Omega_t^2 - \frac{3}{16} \frac{Y_0^2 \alpha}{c_m^2}; \quad P_{opt} = \frac{1}{8} \frac{\Omega_t^2 Y_o^2 m}{c_m}. \quad (2)$$

Note that the maximum power does not depend on the nonlinear coefficient whereas the optimal linear natural frequency is a function of both the nonlinear coefficient and the amplitude of excitation. Interestingly, the maximum power delivered by a tunable nonlinear harvester to the optimized load is the same produced by a tunable linear harvester. The power envelope

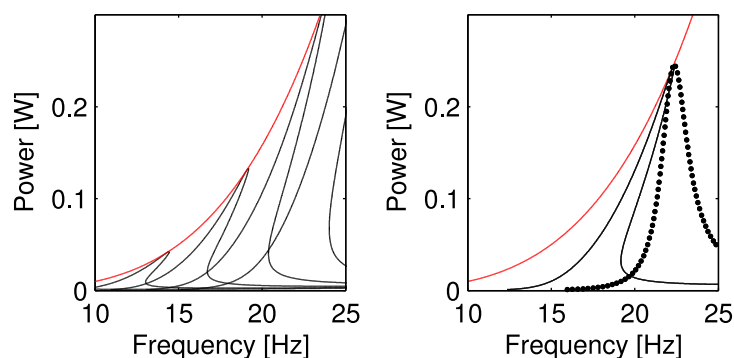


Figure 2. Maximum power with tunable harvesters: envelope of the power vs frequency curves tuned at different frequencies (a) and comparison between the response of a linear (bullets) and nonlinear (solid line) device tuned at the same frequency (b). In both panels the red curve represent the envelope of the maximum achievable power.

described by equation (2) is plotted in red in figure 2, where the amount of power delivered to the load are the same for the linear and nonlinear devices, however for the nonlinear device the underlying linear frequency is significantly smaller than ω_t .

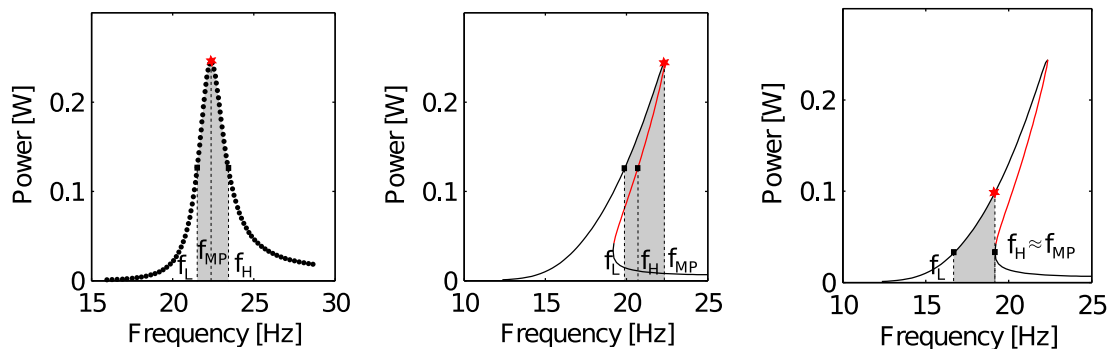


Figure 3. 3dB-bandwidth for the linear (a) and nonlinear (b) device. The linear power vs frequency curve is drawn with black bullets. The nonlinear stable oscillations are drawn with a black solid line whereas the unstable branches are drawn in red. Panel (c) gives a graphical representation of the MPUS definition. The maximum power is marked with a red star and the half power with black squares.

3. Bandwidth: definition and comparison

In the previous section we demonstrated that, when optimally tuned for maximum power extraction, the linear and nonlinear harvester generate the same amount of power. We now consider the bandwidth of the optimized linear and nonlinear harvesters. Firstly, the classical definition of 3dB-bandwidth is extended to nonlinear harvester, where the frequency response is strongly asymmetric about the maximum power.

3.1. Bandwidth definition for nonlinear harvesters

The definition of 3dB-bandwidth requires that the frequencies at which half the maximum power is delivered to the load are found. Due to the near-symmetry of the resonant peak of the linear device, see figure 3(a), these frequencies, Ω_L (or f_L) and Ω_H , span the resonant frequency giving the bandwidth $[\Omega_L : \Omega_H]$.

In contrast, for the nonlinear device, figure 3(b), $\Omega_L < \Omega_H < \Omega_{MP}$ hence the maximum power occurs at one extreme of the bandwidth $[\Omega_L : \Omega_{MP}]$. Note that over a portion of the bandwidth of the nonlinear energy harvester more than one stable solution exists. This definition of bandwidth does not take into account that the response could be on the lower branch. An alternative definition of bandwidth is to limit it to the region where only one solution exists is possible [14], which we term MPUS-bandwidth (Maximum Power Unique Solution - bandwidth), see figure 3(c). Note that as for the previous definition, the maximum power is delivered to the load at one edge of the bandwidth but in addition the maximum power achieved with this bandwidth is lower than the maximum achievable power and hence lower than the maximum power harvested with the equivalent linear device.

3.2. Comparison with linear devices

Figure 4(a) compares the bandwidth of the optimal linear and nonlinear devices; however; this does not tell the full story due to the presence of lower amplitude solutions within the bandwidth of the nonlinear device. To assess this we consider the MPUS-bandwidth definition. Here a comparison with the linear device bandwidth in figure 4(a) would be unfair, since the linear harvester produces more power than the nonlinear device. For this reason the MPUS-bandwidth is compared with the bandwidth of a linear device where the mechanical and the electrical damping have been increased so that its maximum power is the same as the maximum

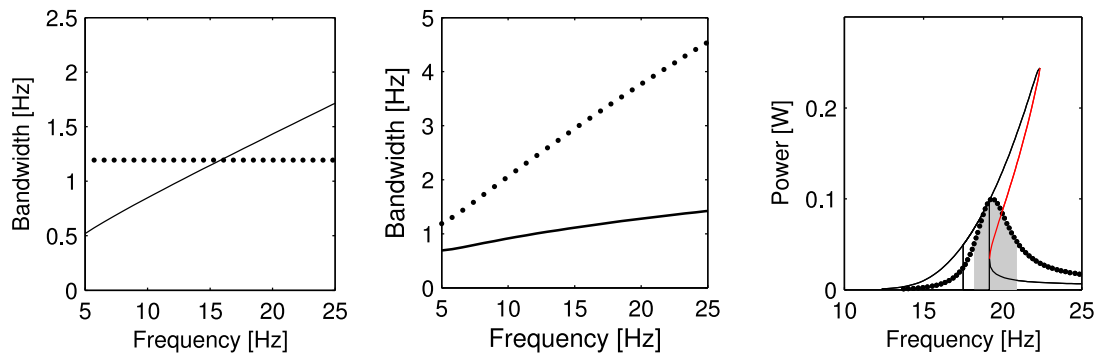


Figure 4. Comparison between the bandwidth of the linear (dots) and the nonlinear (solid line) device. The bandwidth of the response in of the linear and nonlinear devices are shown in (a) for the case where the nonlinear bandwidth in figure 3(b) is used. A comparison using the MPUS-bandwidth, figure 3(c), is shown in (b) for the case where the linear harvester has been tuned to achieve the same maximum power as the nonlinear harvester at the same frequency, as shown in (c).

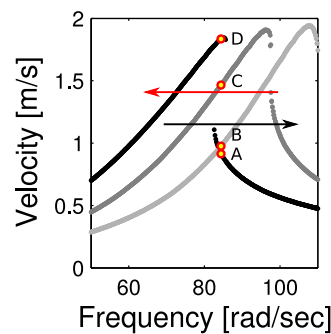


Figure 5. The effect on the frequency response of a nonlinear device when subjected to modification of the underlying linear natural frequency ω_n

P_{MPUS} (see figure 4(b)). In this case the linear device out-performs the nonlinear device in terms of bandwidth over the whole range of tuning frequencies. Hence, to make use of a wider bandwidth via nonlinearity, it is necessary to operate in the region where multiple solutions exist.

4. Ensuring a higher solution response

As shown in the previous section, a nonlinear harvester offers real advantages over the linear device only if it is possible to exploit the region in which multiple solutions exist. In order to operate in this region, a controller that induces the device to respond with high amplitude oscillations is required. Here, a possible method to force the response of the nonlinear harvester on the high energy branch is suggested. The method is based on the observation that at low frequency only one solution exists and that if the frequency is slowly increased from this point, the solution climbs the high energy solution. As the excitation frequency cannot be controlled, the suggestion here is to shift the underlying natural frequency instead.

This is shown graphically in figure 5. Consider the black line as the normal operation response of the device, which is tuned to operate close to 82rad/s. If the response drops, due to a perturbation, onto the lower branch then underlying natural frequency is adjusted upwards such

that the gray response is achieved. Here there is only one solution at the excitation frequency. Now the natural frequency is slowly reduced to its normal value, hence the curve moving back towards and onto the black curve. During this reduction in the natural frequency the device response ‘climbs’ the response curve back to the desired operation point.

5. CONCLUSIONS

In this work a comparison between optimized linear and nonlinear energy harvester is provided. It is discussed that in the case of tunable harvesters, a linear and nonlinear device deliver the same maximum power to the optimal resistive load. Using this result, the bandwidth of optimally tuned linear and nonlinear harvesters are compared. The results showed that there exists a specific frequency range over which the nonlinear harvester has a wider bandwidth than the linear one. However it requires that the device operating is a regime where a lower response is also possible. Removing this possibility by restricting operation to regions where only one solution exists results in a narrower bandwidth for the nonlinear device.

Considering the importance of operating in the multiple solution region, for the nonlinear device, a possible strategy to force the harvester to response with high amplitude oscillations is suggested. Although this is presented as a preliminary study, the results achieved are encouraging. In future work we will discuss a mechanism for implementing the linear frequency shifting and a complementary control strategy to ensure the harvester stays on the upper branch.

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